A Nonparametric Procedure for Estimating Agricultural Cost Functions

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A. E. Research Series, No. 91-4

April 1991

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Paper presented as part of the 1990-91 Applied Statistics in Agriculture series, College of Agriculture, University of Idaho, April 16, 1991.

Abstract

Diewert's quadratic approximation lemma is shown mathematically and graphically to be equivalent to a second-order Taylor's series expansion of a quadratic unit cost function. It is also shown to equal the geometric mean of the first-order "Laspeyres" and "Paasche" approximations of a cost function. Using Diewert's lemma and USDA/FEDS data, it is determined that there was no increase in cost efficiency among very-large, Washington-Palouse, soft-white-wheat-following-fallow producers between 1974 and 1983.

Key Words

cost functions, cost efficiency, technological change, price indexes

I. Cost Efficiency

As new technology is introduced and adopted in agricultural production, a shift in the production and cost functions occurs. To measure this shift is to measure the effect of a new technology on total factor productivity or, its inverse, cost efficiency. Typically, the procedures for measuring technological change cluster at the poles of extreme simplicity and complexity. The simple procedures are appropriate for only the most narrowly defined production functions or circumstances. The complex approach is appropriate for general functional forms but requires a substantial investment in the knowledge of econometric procedures. An advancement in measuring productivity developed by Diewert (1976) claims to be both simple and accurate.

This article will examine four ways to estimate cost efficiency. All four techniques are variations of a Taylor series expansion for a quadratic function.[1] The first two approaches are "Laspeyres" and "Paasche" type, first-order approximations of cost efficiency. The third method uses a second-order Taylor series approximation to measure cost efficiency. Finally, Diewert's "quadratic approximation

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[1] A Taylor's series expansion of a quadratic function is $f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$ (Thomas, p. 786-87).

lemma" is presented as a simple and accurate alternative to the other three ways of estimating cost efficiency. It will be shown graphically and mathematically that the Diewert approximation is equal to either a second-order Taylor series approximation or the geometric mean of the Laspeyres and Paasche cost efficiency indexes.

II. Estimating Unit Cost Functions

A. First-Order "Laspeyres" Approximation

Consider a quadratic unit cost function that is continuous, linearly homothetic, non-decreasing, and concave in input prices and subject to discrete changes in technology.

(1) $C = f(P_i, T)$.

C is the average cost per unit of output,[2]

P is the price of inputs i (i = 1 ... n).

T is the adoption of new technology through time.

A first-order Taylor-series expansion of unit cost C_1 is estimated from unit cost C_0 at price P_0. Where $P_1 > P_0$ and 0 is the initial and 1 the subsequent period.

(2) $C_1 = C_0 + \Sigma_i(S_{i0})(P_{i1}-P_{i0}) + (\alpha_0)(T_1-T_0)$ + remainder.

Where S_{i0} is the change in unit cost for a change in initial input prices and equals the factor share of total

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[2] In this study, unit costs (C) equal total costs per acre (TC) divided by yield per acre (Y) , i.e., C = TC/Y.

expenditures on input i in the initial time period (Shephard, p. 11).

α_0 is the change in unit cost from a change in initial technology. Equation (2) is a first-order approximation of C_1.

(3) $(C_1-C_0) \approx \Sigma_i(S_{i0})(P_{i1}-P_{i0}) + (\alpha_0)(T_1-T_0)$

Equation (3) is a first-order "Laspeyres" approximation of change in unit cost in terms of a change in input prices and a change in cost efficiency between the initial and subsequent time periods.

If technological change is constant, then equation (3) states that a change in unit costs is approximated by a change in input prices alone. Equation (3) can be rewritten as line segments (see figure 1) such that:

(4) $BP_1-CP_1 \approx (CD/P_0P_1)\ P_0P_1$;

$CB \approx CD$;

$CB < CD$.

Where $C_1 = BP_1$; $C_0 = AP_0 = CP_1$; $S_{i0} = CD/P_0P_1$.

CD is the Laspeyres index of the change in input prices in that it includes the subsequent higher input price P_1 and the initial greater quantities of the subsequently more expensive inputs. It implies that no substitution of relatively less costly inputs is made for the relatively more costly input.[1] Using the Laspeyres approach, constant technology appears as a increase in cost efficiency. Therefore, a Laspeyres index of cost efficiency over-estimates the measure of technological changed because a change in unit cost from a change in input prices would be attributed to changes in efficiency.

B. First-Order "Paasche" Approximation

The reverse of the Laspeyres process results in a Paasche estimate of cost efficiency. This time a first-order Taylor-series expansion of unit cost C_0 is estimated from unit cost C_1.

(5) $C_0 = C_1 + \Sigma_i(S_{i1})(P_{i0}-P_{i1}) + (\alpha_1)(T_0-T_1)$ + remainder

By multiplying both side of equation (5) by minus one and rearranging terms, results in an equation similar in form to equation (3).

(6) $(C_1-C_0) \approx \Sigma_i(S_{i1})(P_{i1}-P_{i0}) + (\alpha_1)(T_1-T_0)$.

S_{i1} is the change in unit cost for a change in subsequent input prices and equals the factor share of total expenditures on input i in the subsequent time period.

α_1 is the change in unit cost from a change in subsequent technology.

Equation (6) is also presented graphically in figure 2.

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[1] Laspeyres price index is $P_{la} = \Sigma_i p_{i1} x_{i0} / \Sigma_i p_{i0} x_{i0}$, (i = 1 ... n)).

(7) $AP_0 - FP_0 \approx (FE/P_0P_1)\ P_0P_1$

$AF \approx FE;$

$AF > FE.$

Where $C_0 = AP_0$; $C_1 = BP_1 = FP_0$; $S_1 = FE/P_0P_1$.

FE (or GB) is the Paasche price index. Lesser quantities of the subsequently more expensive inputs continue to be used even at the lower initial prices. This again implies that there is no substitution effect.[2]

Now, the Laspeyres result can be restating for a Paasche measure of cost efficiency. Using the Paasche approach, constant technology appears as a decrease in cost efficiency. Therefore, a Paasche index of cost efficiency under-estimates the measure of technological changed because the change in unit cost from a change in efficiency would be attributed to a change in input prices. This is just the reverse of the Laspeyres case. Comparing equations (4) and (7) reveals that,

(8) $CD > BC > BG$.

Where $AF = CB$ and $FE = BG$. See Figure 3.

The first-order Laspeyres and Paasche approximations of a unit cost function will over- and under- estimate the change in cost efficiency, respectively. Neither approach takes the substitution effect from changing input prices into account and thereby fail to accurately separates the change in unit

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[2] Paasche price index is $P_{pa} = \Sigma_i p_{i1} x_{i1} / \Sigma_i p_{i0} x_{i1}$.

cost into the change in input prices and the change in efficiency from new technology. (Perhaps an average of the two would work?)

C. Second-Order Taylor Series Approximation

In contrast to the simplicity and inaccuracy of the Laspeyres and Paasche approaches is the complexity and precision of an econometrically-estimated second-order Taylor series expansion of a quadratic cost function. Consider a second-order expansion of cost function C_1 expanded at C_0.

$$(10)\quad C_1 = C_0 + \Sigma_i S_{i0}(P_{i1}-P_{i0}) + \tfrac{1}{2}\Sigma_i(\delta S_{i0}/\delta P_{i0})(P_{i1}-P_{i0})^2$$
$$+\tfrac{1}{2}\Sigma_j(\delta S_{i0}/\delta P_{j0})(P_{i1}-P_{i0})(P_{j1}-P_{j0})$$
$$+ \alpha_0(T_1-T_0) + \tfrac{1}{2}(\delta\alpha_0/\delta T_0)(T_1-T_0)^2$$
$$+ \tfrac{1}{2}\Sigma_i(\delta S_{i0}/\delta T_0)(P_{i1}-P_{i0})(T_1-T_0).$$

A second-order approximation of a cost function requires estimating the coefficients for the first and second derivatives for each argument of the function. The large number of coefficients pose a degrees of freedom problem. A way to decrease the number of coefficients that need to be estimated at one time is to estimate the coefficients of the "factor share equations."

$$(11)\quad \delta C_1/\delta P_{i1} = S_{i0} + \tfrac{1}{2}(\delta S_0/\delta P_{i0})(P_{i1})$$
$$+ \tfrac{1}{2}(\delta S_{i0}/\delta P_{j0})(P_{i1})(P_{j1})$$
$$+ \tfrac{1}{2}(\delta S_{i0}/\delta T_0)(P_{i1})(T_1),\ i = 1 \ldots n\text{-}1.$$

Equation (11) is the first derivative of the equation (10) with respect to the subsequent input price for each input.

(12) $S_{i1} = S_{i0} + \tau_{ii}P_{i1} + \tau_{ij}P_{i1}P_{j1} + \beta_i P_{i1}T_1$.

Where $\tau_{ii} = \frac{1}{2}\Sigma_i(\delta S_0/\delta P_{i0})$, $\tau_{ij} = \frac{1}{2}(\delta S_{i0}/\delta P_{j0})$, and $\beta_i = \frac{1}{2}(\delta S_{i0}/T_0)$, ($S_{i0} = \Gamma_i$ if unknown).

The coefficients of the factor share equation have to meet a set of restriction if they are to reflect to the underlying assumptions of a "well-behaved" cost function. These include (1) linearly homogeneous in input price restrictions: $\Sigma_i\Gamma_i = 1$, $\Sigma_i\beta_i = \Sigma_{i.}\tau_{ij} = \Sigma_{.i}\tau_{ij} = \Sigma_i\Sigma_i\tau_{ij} = 0$ and (2) symmetry restrictions: $\tau_{ij} = \tau_{ji}$.

Once estimated the coefficients are then substituted into equation (10) and solved for the change in cost efficiency.

(13) $\alpha_0(T_1-T_0) + \frac{1}{2}(\delta\alpha_0/\delta T_0)(T_1-T_0)^2 = C_1-C_0 - (\Sigma_i S_{i0}(P_{i1}-P_{i0}) + \Sigma_i\tau_{ii}(P_{i1}-P_{i0})^2 + \Sigma_j\tau_{ij}(P_{i1}-P_{i0})(P_{j1}-P_{j0}) + \beta_i(P_{i1}-P_{i0})(T_1-T_0))$.

It is possible to show this result graphically if equation (10) is simplified to one input and assuming constant technology.

(14) $C_1-C_0 = S_0(P_1-P_0) + \frac{1}{2}(\delta S_0/\delta P_0)(P_1-P_0)^2$.

Where, $\delta S_0 = S_0-S_1$; and $\delta P_0 = -(P_1-P_0) = -P_1P_0$.

Substituting the appropriate line segments, (see figure 3),

(15) $BP_1-AP_0 = (CD/P_1P_0)(P_1P_0) - \frac{1}{2}(S_0-S_1)((P_1P_0))(P_1P_0)^2$;

$BC = CD - \frac{1}{2}(CD-FE)$;

$2BC = 2CD - (CG+GB+BD-GB)$

$2BC = 2CD - CG - BD$

$2BC = 2CG + 2GB + 2BD - CG - BD$

$2BC = CG + 2GB + BD$

$2BC = BC + GD$

$BC = GD.$

Estimating a second-order Taylor expansion of a cost function, constant technology results in a measure of constant cost efficiency. Therefore, a second-order index of cost efficiency correctly estimates the measure of technological changed because the change in unit cost are proportioned correctly between changes in input prices and changes in efficiency.

D. Diewert's Quadratic Approximation

Diewert's quadratic approximation lemma for a quadratic function f is

(16) $f(z^1)-f(z^0) = \frac{1}{2}[\nabla f(z^1)+\nabla f(z^0)]^T(z^1-z^0)$

$\nabla f(z^r)$ is the first derivative of f evaluated at r (Diewert, p 118). This lemma claims that the second derivatives can be dropped and, using only the geometric mean of the first derivatives, a second-order or quadratic approximation of a function can still be obtained.

Applying Diewert lemma to the cost function,

(17) $f(P_{i1},T_1)-f(P_{i0},T_0) =$

$C_1-C_0 =$

$\Sigma_i \frac{1}{2}(S_{i1}+S_{i0})(P_{i1}-P_{i0}) + \frac{1}{2}(\alpha_1+\alpha_0)(T_1-T_0)$.

For Diewert's claim to be true equation (17) must equal a second-order Taylor series expansion, i.e., equation (14), under the same set of assumptions. By substituting $\delta S_0 = S_0-S_1$; and $\delta P_0 = -(P_1-P_0) = -P_1P_0$ into (14) and rearranging terms,

(18) $C_1-C_0 = S_0(P_1-P_0) - \frac{1}{2}(S_0-S_1)/(P_1-P_0)(P_1-P_0)^2$;

$= S_0(P_1-P_0) - \frac{1}{2}(S_0-S_1)(P_1-P_0)$;

$= S_0(P_1-P_0) - \frac{1}{2}(S_0)(P_1-P_0) + \frac{1}{2}(S_1)(P_1-P_0)$

$= \frac{1}{2}(S_0)(P_1-P_0) + \frac{1}{2}(S_1)(P_1-P_0)$

$= \frac{1}{2}(S_1+S_0)(P_1- P_0)$. Q.E.D.

Equation (18) shows that Diewert's lemma applied to a cost function is corresponds one to one with a second-order Taylor series expansion of the function.

This lemma is also equal to the geometric mean of the Laspeyres and Paasche estimates of the cost function. The sum of equations (3) and (6) equals,

(19) $2(C_1-C_0) = \Sigma_i(S_{i1}+S_{i0})(P_{i1}-P_{i0}) + (\alpha_1+\alpha_0)(T_1-T_0)$

$C_1-C_0 = \Sigma_i \frac{1}{2}(S_{i1}+S_{i0})(P_{i1}-P_{i0}) + \frac{1}{2}(\alpha_1+\alpha_0)(T_1-T_0)$.

The expression $\frac{1}{2}(\alpha_1+\alpha_0)(T_1-T_0)$ is a Diewert index of cost efficiency. In equation (19), the change in unit costs is proportioned correctly between changes in input prices and the change in efficiency.

Graphically, it has been shown that, (see figure 3)

(20) CD > BC = GD > BG.

GD is the Fisher input price index between costs C_1 and C_0 as derived by Diewert's lemma. By dividing both GD and BC by AC, the slope of AB is equals the slope of ED. When these slopes are equal, then Diewert's quadratic lemma becomes a Lagrange remainder for a zero-order Taylor's expansion, which is an exact measure of the price index of the cost function (Chiang, p. 272-3).

Diewert has also shown that his quadratic lemma is generalized to non-homothetic functions as well (pp. 122-23). This is important since non-homothetic functions allow for factor bias. Diewert's quadratic lemma for a non-homothetic function includes the geometric mean of the change in factor shares due to factor bias, i.e., the same procedure used to adjust for factor substitution alone in linearly homogeneous functions (p. 122).

III. A General Model of Cost Efficiency

Consider now a continuous, twice differentiable, concave, non-decreasing, non-homothetic cost function in which per-unit average cost is a function of input prices and discrete variables for time, region, and enterprise size.

(21) $C_{jtru} = f(P_{itru}, T, R, U)$.

Where C_{jtru} is unit average cost of commodity j in time t, region r, and enterprise size u; T is time; R is region, and U is enterprise size; P_{itru} is price per unit of input i to produce commodity j in time t, region r, and size u; i is the input category for capital (k), labor (l), energy (e), fertilizer (f), materials (m), and land (a); all inputs within input categories are complements; input categories may be either complements or substitutes; all input categories are variable.

Equation (16) can be expressed in translog form and approximated using Diewert's quadratic lemma as the geometric mean of C_1 and C_0 expanded around points 0 and 1 respectively.

(22) $\ln(C_{jtru1}/C_{jtru0}) =$

$$\tfrac{1}{2}\Sigma_i(S_{itru1}+S_{itru0})\ln(P_{itru1}/P_{itru0})$$

$$+ \tfrac{1}{2}(\alpha_{t1}+\alpha_{t0})(T1-T_0) + \tfrac{1}{2}(\alpha_{r1}+\alpha_{r0})(R_1-R_0)$$

$$+ \tfrac{1}{2}(\alpha_{u1}+\alpha_{u0})(U_1-U_0).$$

Where Si is the factor share of expenditure on input i, and α_t is the first partial derivative of the cost function with respect to time, e.g., $\alpha_{t1} = \delta \ln C1/\delta T_0$; and similarly for regions (α_r) and enterprise size (α_u).

Solving equation (21) for the measures of cost efficiency by time, region, and size

$$(23)\ \tfrac{1}{2}(\alpha_{t1}+\alpha_{t0})(T_1-T_0) + \tfrac{1}{2}(\alpha_{r1}+\alpha_{r0})(R_1-R_0) + \tfrac{1}{2}(\alpha_{u1}+\alpha_{u0})(U_1-U_0) = \ln(C_{jtru1}/C_{jtru0}) - \tfrac{1}{2}\Sigma_i(S_{itru1}+S_{itru0})\ln(P_{itru1}/P_{itru0}).$$

Equation (22) can be interpreted as meaning that any difference between per unit costs and input prices is credited to differences in cost efficiency across time, region and size of enterprise.

By holding the region and size variables constant equation (22) can be rewritten as:

$$(24)\ \tfrac{1}{2}(\alpha_{t1}+\alpha_{t0})(T_1-T_0) = \ln(C_{t1}/C_{t0}) - \Sigma_i\tfrac{1}{2}(S_{it1}+S_{it0})\ln(P_{it1}/P_{it0}).$$

Equation (23) can be rewritten in antilogs,

$$(25)\ e^{\frac{1}{2}(\alpha t1+\alpha t0)(T1-T0)} = C_{t1}/C_{t0} \div \pi_i(P_{it1}/P_{it0})^{\frac{1}{2}(Sit1+Sit0)}$$

Equation (24) measures changes in cost efficiency from differences in technology across time.

IV. An Empirical Measure of Intertemporal Cost Efficiency

To determine cost efficiency using Diewert's lemma, data are needed on total expenditures per acre, yield per acre, individual input expenditures per acre, and the input prices per unit of input. The original survey data for wheat from the USDA/Farm Enterprise Data System (FEDS) survey of 1983 were used. Enterprise data were selected from Washington Palouse (area 400) for soft white winter wheat, following fallow produced in 1974 and 1983. The data were sorted by total planted acres and, using a budget generator, generated two representative enterprise budgets from a composite of data for the 91-100 percentiles, the enterprises which we designated as "very large" for this region. These two representative budgets for the "initial" 1974 and the "subsequent" 1983, very large wheat enterprises in Washington are used to illustrate Diewert's method of measuring changes in intertemporal cost efficiency.

See table 1. Rows (1) and (2) give the expenditures on inputs per acre. The sum of these rows equals total cost for the respective enterprises. Rows (3) and (4) present factor shares as the percent of an input's cost to total cost. Row (5) is the average of the factor shares.

The 6th and 7th rows give the input prices as \$/hour for capital and labor, \$/gallon for energy, \$/pound for fertilizer, \$/wt ave unit for materials, and \$/acre for land. The 8th row is the input price ratio in logs for the two enterprises. The 9th row is the product of rows (5) and (8). The last element in row (9) is the sum of the elements in that row and equals a Fisher input price index in logs.

Rows (10) and (11) present the five-year average yields per acre for wheat in the Palouse from 1972 to 1976 and 1981 to 1985. The 12th row is the log of the yield ratio. Row (13) is the log of the total expenditure ratio. Row (14) is the log of the unit cost ratio and is equal to row (13) minus row (12). Row (15) is intertemporal cost efficiency in logs and equals row (14) minus the Fisher price index in row (9). Row (16) is the index of intertemporal cost efficiency in the Palouse for winter wheat following fallow between 1974 and 1983 and equals the antilog of row (15) multiplied by 100.

The Washington Palouse intertemporal cost efficiency index between 1974 and 1983 is 100. This means that "the change in unit costs is proportioned correctly between changes in input prices and the change in efficiency." In this case, total costs increased 217%, yields increased 14.5%, average cost per unit increased 177%, and input prices increased by 177%. Thus, all of the change in unit costs are explained by changes in input prices. Consequently, Palouse wheat producers have increased inputs use at the same rate as yield increased resulting in no increase in cost efficiency.

V. Warranted Assertions

The Laspeyres, Paasche, second-order Taylor-series, and Diewert quadratic approximation ways of estimating cost efficiency were presented graphically and mathematically. It has been shown that the Diewert's and second-order Taylor-series indexes are equivalent and equal to the geometric mean of the Laspeyres and Paasche procedures.

Finally, in an empirical example, Diewert's lemma was used in conjunction with USDA/FEDS enterprise data to determine that very large Palouse wheat enterprises did not increase their the cost efficiency between 1974 and 1983.

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Table 1. Intertemporal Cost Efficiency for Very Large Washington Wheat Enterprises: 1974 to 1983

	Capital	Labor	Energy	Fert.	Mat'ls	Land	Total
--Cost							
1 WA VL 1983 (\$/acre)	60.93	9.85	8.31	16.31	22.01	189.48	306.89
2 WA VL 1974 (\$/acre)	16.70	14.11	1.94	9.03	24.09	40.91	96.78
---Cost Share							
3 WA VL 1983 (%)	.20	.03	.03	.05	.07	.62	1.00
4 WA VL 1974 (%)	.17	.04	.02	.09	.25	.42	1.00
5 $\frac{1}{2}(S_{83}+S_{74})$ (%)	.19	.04	.02	.07	.16	.52	1.00
---Price & Price Index							
6 WA VL 1983 (\$/Unit)	36.41	5.13	1.11	.28	5.19	88.54	
7 WA VL 1974 (\$/Unit)	12.26	2.51	.33	.20	8.68	19.06	
8 $\mathrm{Ln}(P_{83}/P_{74})$ (\$/Unit)	1.09	.71	1.21	.31	-.51	1.54	
9 $\frac{1}{2}(S_{83}+S_{74})\mathrm{Ln}(P_{83}/P_{74})$	.22	.03	.03	.02	-.08	.80	1.02
---Yield, Cost & Results							
10 WA VL 1981-85 (Ave Bu/Acre)							39.98
11 WA VL 1972-76 (Ave Bu/Acre)							34.92
12 $\mathrm{Ln}(Q_{83}/Q_{74})$ (Bu/Acre)							.13
13 $\mathrm{Ln}(TC_{83}/TC_{74})$ (Cost/Acre)							1.15
14 $\mathrm{Ln}(C_{83}/C_{74})$ (Cost/Bu)							1.02
15 $\frac{1}{2}(\alpha_{83}+\alpha_{74})(T_{83}-T_{74})$							.00
16 $100e^{1/2(\alpha 83+\alpha 74)(T83-T74)}$ Index of Cost Efficiency (1974 = 100)							100

Figure 1

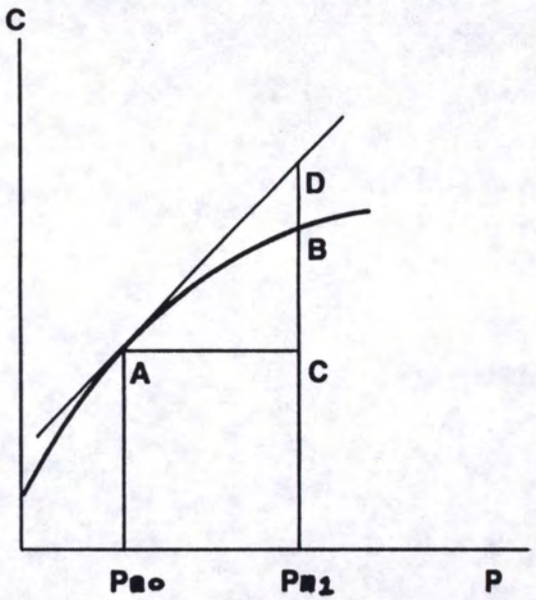

Figure 2

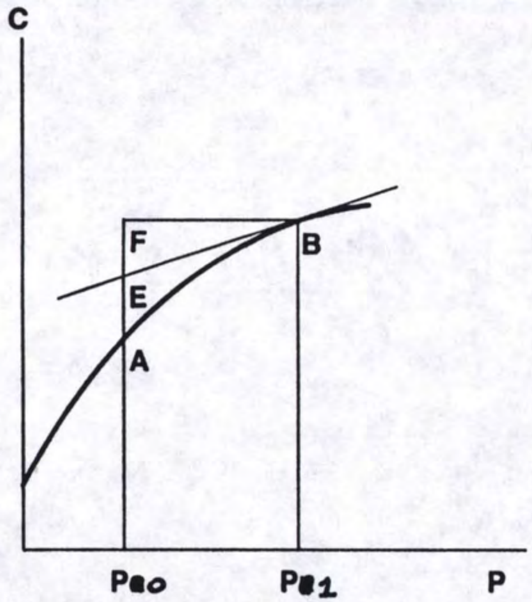

Figure 3