

**A Technique for Making Upward Adjustments  
in National Coefficients in Pool-Quotient  
Non-Survey Regional I-O Models**

**Ag Econ Research Series 92-10**

**November 24, 1992**

**M. H. Robison**

Regional input-output (I-O) analysis has become an all but standard element in regional economic decision making. The high cost of survey-based models has spawned development of a variety of non-survey techniques. Currently popular are the supply-demand-pool (SDP) technique (Schaffer and Chu, 1969), the Regional Purchase Coefficient (RPC) technique (Stevens et. al., 1983).

The SDP technique assumes borrowed technology, usually national, and estimates regional exports as the excess of regional production over requirements. Crosshauling is not permitted. The RPC permits crosshauling by reducing SDP row-adjustment-factors and thereby raising, in-effect, selected SDP export estimates. Both SDP and RPC approaches preserve national technology through regional imports, i.e., regional demands not met by regional production are obtained via purchases from outside.

While regional exports in excess of SDP estimates are easily modeled, adjustments in the opposite direction, i.e., regional exports less than SDP estimates are more difficult. This case might be indicated by survey or published data, or it might be indicated by the RPC estimating equation. IMPLAN (US Department of Agriculture, 1986), for example, abandons RPC export estimates in favor of the SDP estimate when less than the SDP estimate. Whatever the source, export estimates less than SDP estimates suggests a substitution of the locally produced good for other inputs and thereby a departure from national technology. And it

is this departure from national technology that raises the modeling difficulty.

The object of this paper is to modify the general SDP technique-mechanics to allow for departures from national technology, i.e., to allow for local input substitution. Our focus is not on export estimation itself, but rather on how to incorporate observed exports less than SDP export-estimates into the general framework of SDP models.

### The Supply-Demand-Pool Technique

The standard SDP technique begins with an estimate of "regional requirements" (Isard, 1953). Assuming national technology at the regional level, regional requirements for commodity  $i$  are given as follows:

$$(1) \quad R_i = n_i X$$

where:

$n_i$  = row vector of national coefficients for sectors present in the region.

$X$  = column vector of regional total gross outputs.

Sectors are defined broadly to include investment, personal consumption, and government. Accordingly, assuming  $n$  industrial sectors in the region, vector terms in (1) are of dimension  $n+3$ .

Regional exports are estimated as the excess of regional production over regional requirements:

$$X_i - R_i > 0$$

$$(2) \quad \tilde{E}_i = \begin{cases} X_i - R_i & \text{if } X_i - R_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

where:

$$\tilde{E}_i = \begin{cases} \text{the SDP estimate of regional commodity } i \\ \text{exports.} \end{cases}$$

The regional I-O model is obtained through the formation of scalars as follows:

$$(3) \quad \rho_i = \frac{X_i - \tilde{E}_i}{R_i}$$

Premultiplying both sides of (1) by (3) provides base period equilibrium for commodity i:

$$(4) \quad X_i = \rho_i \mathbf{n}_i \mathbf{X} + \tilde{E}_i$$

#### Supply-Demand-Pool Technique Extended to Permit Crosshauling

Crosshauling is incorporated into the structure of the SDP model through an estimate of regional exports greater than the SDP estimate (2). That is, for sectors i such that:

$$(5) \quad E_i > \tilde{E}_i$$

where:

$E_i =$  exports estimated independent of the SDP technique.

Export estimate  $E_i$  might result from an application of the RPC technique, e.g., IMPLAN, or it might be obtained in some other fashion, from survey or published data, or even based on simple assumption.

Scalars (3) are now replaced by scalars:

$$(6) \quad \rho_i = \frac{X_i - E_i}{R_i}$$

and base period equilibrium is obtained by premultiplying both sides of (1) by (6) providing:

$$(7) \quad X_i = \rho_i n_i X + E_i$$

Inasmuch as scalars (6) are always less than scalars (3), and scalars (3) are never greater than 1.0, scalars (7) are always less than 1.0. Crosshauling is indicated in (8) by the simultaneous export and import of commodity  $i$ . The export of commodity  $i$  is indicated by  $E_i$ , while the otherwise implicit import of commodity  $i$  is given by  $R_i(1 - \rho_i)$ .

### Preservation of National Technology

In compact notation, the regional I-O coefficients matrix appears as follows:

$$(8) \quad \mathbf{A} = \hat{\rho} \mathbf{N}_r$$

where:

$\mathbf{A}$  = regional I-O coefficients matrix of dimension  $n \times n+3$ .

$\hat{\rho}$  = diagonal matrix of scalars estimated as in (3) or (7).

$\mathbf{N}_r$  =  $n \times n+3$  partition of national coefficients matrix for sectors present in the region.

The SDP technique, either standard or extended to permit crosshauling, preserves national technology. Regional requirements in excess of regional production is assumed to be imported. Formally, the vector of regional imports is computed as follows:

$$(9) \quad \mathbf{m} = (\mathbf{1}) \{ \mathbf{I} - \hat{\rho} \} \mathbf{N}_r \hat{\mathbf{X}} + (\mathbf{1}) \mathbf{N}_{N-r} \hat{\mathbf{X}}$$

where:

$\mathbf{m}$  = row vector indicating total imports by each regional sector.

$N_{N-R}$  = partition of national coefficients matrix with rows for national industries not present in the region, and  $n+3$  columns for regional sectors.

Regional imports appear as the sum of two components on the right-side of (9). These are commonly referred to as "competitive" and "non-competitive" imports respectively.

#### Deviation From National Technology: Observed Exports Less than SDP Exports

Consider now the case where observed exports are less than exports estimated with the SDP technique. Formally:

$$(10) \quad E_i < \tilde{E}_i$$

where  $E_i \geq 0$ . In this case regional production absorbs more commodity  $i$  than indicated by national technology, and thereby a departure from national technology.

Our procedure for adjusting for departures from national technology begins with the formation of scalars:

$$(11) \quad \sigma_i = \frac{X_i - E_i}{R_i}$$

Given (11) and the derivation of  $\tilde{E}_i$  in (2), it is apparent that:

$$(12) \quad \sigma_i > 1.0$$

A set of coefficients indicating a departure from national technology are given as follows:

$$(13) \quad \epsilon_i = (\sigma_i - 1) n_i$$

where:

$\epsilon_i$  = coefficients indicating row-wise proportional absorption of commodity  $i$  in excess of national technology.

#### Incorporating Input Substitution into the Non-Survey Model

For regional industries  $i$  other than those where (10) is observed, the non-survey regionalizing procedure presented in (4) and (7) allocates regional production to regional absorption and exports. If this allocation is accepted, excess regional absorption for industries  $i$  where (10) is observed must be met by a reduction in imports and/or factor services, i.e., the substitution of local commodity  $i$  for imports and factor services. This substitution can be expressed by revised imports and factor services as follows:

$$(14) \quad m^* + v^* = m + v - s$$

where:

$m^*$  = revised vector of regional imports.

$v^*$  = revised vector of regional factor payments.



$V =$  vector of regional factor payments derived from national coefficients.

$S =$  vector of import and factor service substitution.

and  $m$  is imports estimated with national coefficients as in (9).

To be acceptable as a vector of import and factor service substitution, vector  $S$  must meet, at a minimum, the following necessary condition:

$$(15) \quad S \leq m + V$$

The task before us then is to estimate a vector  $S$  that meets condition (16) while accommodating excess local absorption of commodities  $i$  where condition (11) is observed.

#### A Crude Estimate of Import and Factor Service Substitution

The total demand by regional sectors for commodities absorbed in excess of national technology is given by the following:

$$(16) \quad g = (1) e \hat{X}$$

where:

$g =$  demand by each industry  $j$  for commodities absorbed in excess of national technology.

$e =$  matrix formed from row vectors  $e_i$

We might now simply let vector (16) serve as our estimate of import and/or factor service substitution vector  $\mathbf{S}$ . Assuming vector  $\mathbf{g}$  satisfies condition (15), then our final regional input-output coefficients matrix appears simply as:

$$(17) \quad \mathbf{A}^g = \mathbf{e} + \mathbf{A}$$

where:

$\mathbf{A}^g$  regional I-O coefficients matrix revised to reflect import and factor service substitution as indicated by vector  $\mathbf{g}$ .

and rows of matrix  $\mathbf{e}$  are assumed to be arranged with the row structure of  $\mathbf{A}$ . It will be observed that use of  $\mathbf{g}$ , as an estimate of substitution vector  $\mathbf{S}$ , in the manner of regional I-O coefficients matrix-estimate  $\mathbf{A}^g$ , is tantamount to allowing SDP scalars  $\rho_i$  take on values greater than 1.0 without restriction.

Aside from the fact that vector  $\mathbf{g}$  limits the opportunities for applying the import and factor service substitution technique, i.e., limits application to those cases where vector  $\mathbf{g}$  meets condition (15), there are other strong objections to  $\mathbf{g}$  as an estimate of substitution vector  $\mathbf{S}$ , and therefore of  $\mathbf{A}^g$  as an estimate of the regional I-O coefficients matrix. In particular, note that  $\mathbf{S} = \mathbf{g}$  permits import and factor service substitution by a given industry  $j$  up to and including the point where

$S_j = m_j + V_j$ . Arguably, some imported commodities, particularly those comprising the set of non-competitive imports, and some factor services, are limitational in the production of regional commodities  $j$ . Aside from meeting condition (15), use of  $g$  as an estimate of  $S$  puts no limits on the degree of import and factor service substitution.

#### Variable Elasticity of Import and Factor Service Substitution

We hypothesize that import and factor service substitution is a function of two things, an industry's demand for commodities absorbed in excess of national technology,  $g_j$ , and that industry's ability to substitute, as indicated by the size of its would-be imports,  $m_j$ , and factor payments,  $V_j$ . Industries  $j$  who otherwise import a substantial portion of their inputs, and/or have substantial factor payments, have a wider range of opportunities to substitute local inputs for factor services and non-local inputs than do industries with smaller would-be imports and factor payments. Accordingly, we offer the following as a general expression for industry  $j$ 's substitution of imports and factor services:

$$(18) \quad \check{S}_j = f(g_j, m_j + V_j)$$

with the following properties:

$$\frac{\partial \check{S}_j}{\partial g_j} > 0$$

and

$$\frac{\partial \check{s}_j}{\partial (m_j + v_j)} > 0$$

We will also require that industries  $j$  with no demand for commodities absorbed in excess of national technology, do not participate, as it were, in our process of regional import and factor service substitution. That is:

$$\check{s}_j = 0 \quad \text{if} \quad g_j = 0$$

Assume  $\mathbf{s} = \check{\mathbf{s}}$  satisfies necessary condition (15), then our procedure for estimating the revised regional I-O coefficients matrix, though more complex than the case where  $\mathbf{s} = \mathbf{g}$ , is nonetheless simple. We first form the following vector indicating the absorption of each commodity  $i$  in excess of national technology:

$$(19) \quad \mathbf{f} = \epsilon \mathbf{X}$$

where:

$\mathbf{f}$  = vector of regional commodities with excess absorption.

Clarifying the contents of (19), it is easily shown that:

$$(20) \quad \tilde{E}_i - E_i > 0$$

$$f_i = \begin{matrix} 0 & \text{otherwise} \end{matrix}$$

Given (19), we next perform an otherwise standard RAS bi-proportional adjustment on matrix  $\epsilon$ , yielding a second matrix  $\epsilon^*$ , with the following properties:

$$(21) \quad \mathbf{f} = \epsilon^* \mathbf{X}$$

and

$$(22) \quad \mathbf{s} = (\mathbf{1}) \epsilon^* \hat{\mathbf{X}}$$

where:

$\epsilon^*$  = revised matrix of coefficients indicating local absorption in excess of national technology.

Our revised regional I-O coefficients matrix, adjusted now to accommodate the vector of import and factor services substitution  $\mathbf{s}$ , appears as:

$$(23) \quad \mathbf{A}^* = \epsilon^* + \mathbf{A}$$

where:

$A^*$  = regional I-O coefficients matrix revised to reflect import and factor service substitution as indicated by vector  $\check{s}$ .

### An Approach for Estimating Import and Factor Service Substitution with Variable Elasticity

There may be any number of explicit expressions exhibiting the properties of general expression (18). We offer the following as a simple approach in the mechanically-efficient spirit of the SDP technique:

$$(24) \quad \check{s} = ((m+v) \hat{\gamma}) \psi$$

where:

$$\gamma = g(1/(1)g[1]) = \text{vector } g \text{ normalized.}$$

$$\text{and: } \psi = \frac{(1) f}{(m+v) \gamma'}$$

Postmultiplying both sides of (29) by a sum vector provides substitution necessary to absorb local production equal to

$(1) f$ , the overall excess local absorption indicated in (24), an obvious requirement for  $s = \check{s}$  to be accepted as our substitution vector.

Work is in progress to test the general properties and overall applicability of ( ) as an estimator of import and factor services substitution. We do note that a sufficient condition for passing necessary condition ( ) is that:

$$(25) \quad \psi \leq 1.0$$

We note that the likelihood that sufficient condition ( ) is met increases as the overall need for substitution,  $(1) f$ , diminishes, reducing the numerator in (30), and as the industries  $j$  demanding substantial commodities with regional absorption in excess of national technology, industries  $j$  with relatively large  $g_j$ , tend also to be industries with greater opportunity for import and factor service substitution, i.e., industries  $j$  with relatively large  $m_j + v_j$ . Even if sufficient condition ( ) fails, the likelihood that less stringent but necessary condition ( ) is met increases as the  $g_j$  are spread between the industries of the region, thus spreading the burden of import substitution across industries.

**References**

Stevens, B.H., G.I. Treyz, D.J. Ehrlich, and J.R. Bower, 1983. "A New Technique for the Construction of Non-Survey Regional Input-Output Models," International Regional Science Review, 8(3), 271-186.

Schaffer, W. and K. Chu, 1969. "Nonsurvey Techniques for Constructing Regional Interindustry Models." Papers of the Regional Science Association, 23, 83-101.

US Department of Agriculture, Forest Service, \*1986. "IMPLAN Version 2.0: Methods Used to Construct the 1982 Regional Economic Data Base," General Technical Report RM-000, Rocky Mountain Forest and Range Experiment Station, Fort Collins.

Isard, W., 1953. "Regional Commodity Balances and Interregional Commodity Flows," American Economic Review, 43, 167-180.