

**Comparing the Cost of Erosion Damage In
Idaho With A Nonlinear Yield Function**

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ABSTRACT: A formal statistical test of linear versus nonlinear crop yield response to eroding topsoil depth was conducted. The significantly nonlinear estimated relationship was employed to compare the cost of erosion damage in two Idaho regions. The results have important implications for soil conservation and water quality policy.

To determine the correct mix of practices for soil conservation and water quality policy it is important to know the cost of erosion damage. The cost of erosion damage depends greatly on how crop yield responds to topsoil loss. This crucial question demands an answer: is the crop yield-topsoil relationship linear or nonlinear? Agronomic theory suggests a nonlinear relationship with diminishing marginal product for topsoil. Crop yield data sets indicate both functional forms and studies with implications for water quality and erosion control policy have also used both forms.

Serious policy errors could result if analysis is based on a simple linear model when the relationship is really nonlinear. In that case, the linear model overestimates erosion damage on deep soils where the slope of the linear function exceeds the slope of the nonlinear. Perhaps more seriously, the linear underestimates damage on shallow, already eroded soils, where the linear slope is less than the nonlinear (See Figure 1). A simple linear model may not give proper emphasis to erosion control on shallow, eroded soils.

If the true relationship is nonlinear, the simple linear model also ignores the dynamic aspect of erosion damage, that is erosion damage accelerates as topsoil declines. With a nonlinear relationship, further loss of topsoil results in increasing incremental yield loss as shown in Figure 2. Ignoring accelerating erosion damage could spell critical, costly consequences for delaying erosion control policies.

The magnitude of the cost of erosion damage is also important for analysis of water quality practices and policy. Two broad types of practices exist for improving water quality. Preventive practices, such as reduced tillage, improve water quality by preventing (reducing) erosion and runoff and thereby reducing sediment and other agricultural residuals in surface water. In addition to offsite water quality benefits, these practices also enjoy onsite benefits from reducing long term productivity and income loss from erosion. Treatment practices, such as sediment basins and filter strips, improve water quality by treating runoff to reduce sediment, nutrient and pesticide concentrations. Because these practices do nothing to reduce erosion, they derive only offsite benefits from improved water quality.

If the cost of erosion damage is high, the benefits of preventive practices will compare more favorably relative to treatment practices. In this case, the cost-effective mix of water quality practices will emphasize treatment practices more heavily than if erosion damage were not costly.

We don't know of any definitive statistical tests of the nature of the yield-topsoil relationship, a relationship that has great relevance for soil conservation and water quality policy. The analysis in this paper, based on a ten year data set, provides this important test for linearity for two crops in two Idaho dryland grain producing regions. The analysis also compares the cost of erosion damage for the prevailing rotations in the two regions. One region is the Palouse in north Idaho, one of the highest yielding dryland wheat producing regions in the country.

Review of the Literature

We use topsoil depth as a key variable in studying the cost of erosion damage. Topsoil depth is an indicator for the complex of soil properties that are affected by erosion and in turn influence crop yield. Soil erosion reduces crop yield by altering rooting zone depth, soil structure, bulk density, organic matter content, water infiltration, water storage capacity and soil nutrient level (Langdale and Shrader).

Some soil properties like organic matter and soil nutrients can be restored by proper management; incorporating crop residue and increasing fertilization. Other soil properties like root zone depth and soil structure are more permanently damaged by erosion. Those properties that can be remedied by proper management respond better when there is deep friable subsoil. The existence of restrictive layers or other impediments in the root zone result in more dramatic permanent damage to crop productivity from erosion. Because many of these yield influencing soil properties are correlated with topsoil depth, that single measure can statistically explain much of the impact of erosion on crop yield. Topsoil depth is a relatively good indicator of the physical properties of the soil (Buckman and Brady). In a recent review of erosion impact studies, ten out of fifteen used topsoil depth as a proxy for the soil properties affected by erosion that in turn influence crop yield (Young).

Both linear and nonlinear crop yield response functions appear in the literature of erosion impacts. Examples of linear models are found in Krauss and Allmaras, Thomas et al. and Shelton and Bailey. Nonlinear models are found in Pawson et al, Frohberg and

Swanson, and Burt. While the linear functional form may be easier to work with because the marginal product of topsoil is constant, there are serious limitations for policy applications as mentioned earlier.

Mitscherlich-Spillman Nonlinear Form

Agronomic theory has long contended that additional inputs generally produce a diminishing increment to yield. Numerous studies have indicated this result for fertilizer and water. Some of the early studies are reported in a classic 1924 book by Spillman and Lang. They called this response the Law of Diminishing Returns or the "law of the soil." It seems plausible that the yield response to topsoil depth should also exhibit diminishing marginal returns. Much data also suggests that additional topsoil depth beyond the crop rooting depth might not boost yields nor will that topsoil be deleterious to yields. Finally, there is evidence that zero topsoil or subsoil after the loss of all topsoil can still support positive though diminished yield (Wetter; Larson et al.; Shrader). Based on agronomic theory, a suitable yield-topsoil response function should: (1) have a non-zero intercept, (2) exhibit nonnegative but diminishing marginal returns to topsoil, and (3) have an upper limit which yield on deep topsoil approaches asymptotically.

The Mitscherlich-Spillman functional form (M-S) exhibits all of these properties (See Figure 2):

$$Y = A + B (1 - \exp(-CX))$$

where: Y = crop yield

X = topsoil depth

A = intercept term, expected yield from zero topsoil (subsoil)

B = maximum yield increment from topsoil (A+B is the asymptotic upper limit on yield with deep topsoil)

C = coefficient that influences the curvature of the yield response function

Theoretically, this nonlinear model is well-suited but it can be used to estimate yield response only if the data indicate a nonlinear form.

To our knowledge there has not been any research that conducts a statistically significant test of linearity in yield response. The question is whether the relationship between yield and topsoil depth is better represented by the theoretically preferred Mitscherlich-Spillman nonlinear functional form or a default linear equation form. Constructing a formal statistical test that would distinguish these two alternatives is difficult because the alternative models are non-nested. That is, neither model can be constructed from the other by transformations such as addition or deletion of variables whose significance could be tested. Tests for distinguishing non-nested models are difficult because of lack of determinacy in outcomes with small samples, lack of symmetry with respect to hypothesis specification and test behavior comparisons derived by experiment rather than analytical solutions (Fomby, Hill and Johnson, p.422).

Method for Resolving Linear Versus Nonlinear

Statistical Test with Splined Regression Model

The difficulty of directly comparing the nonlinear M-S model with a linear model led us to test first for linearity within a nested model context; comparing a spline functional form to a linear functional form. If the test indicates the nonlinear spline model better represents the data, then the theoretically preferred Mitscherlich-Spillman nonlinear equation form can be estimated. A splined regression model can be nonlinear in that it allows different segments to have different slopes. If the segment slopes are not significantly different, then the model can be collapsed back into a simple linear model.

We used the following splined regression model to test for linearity.

$$Y_i = A + B1 * \text{Min}(X_i, X_p) + B2 * \text{Max}(X_i - X_p, 0) + e_i$$

This model consists of two linear segments that join at the breakpoint topsoil depth, X_p (See Figure 3). The first segment, with slope B_1 , runs from the intercept to the spline and represents crop yield for topsoil depths ranging from zero to the breakpoint, X_p . The second segment begins at the spline and slope B_2 represents the additional yield obtained from the incremental topsoil depth above X_p . The slopes of the two segments can be equal, in which case the yield function reverts to a simple linear relationship. Alternatively, the slopes can be different ($B_1 > B_2$ for a crop yield function), representing a nonlinear relationship.

The test for linearity consists of the following one-tailed T-test.

$$H_0: B_1 \leq B_2$$

$$H_a: B_1 > B_2$$

If the test t is less than the tabled value we accept the hypothesis and conclude that the data support a linear relationship. If the test t exceeds the tabled value we reject the hypothesis and accept the alternative, concluding that the data indicate a nonlinear relationship. With evidence that the nonlinear model is appropriate, the theoretically preferred Mitscherlich-Spillman nonlinear equation form could be adopted and estimated with regression analysis.

Study Areas

Two study areas, both dryland grain producing regions, were selected for evaluating erosion damage; the Palouse region of north Idaho and the Downey area in southeast Idaho. The ample precipitation in the Palouse near Moscow, Idaho, about 22 inches, coupled with fertile soils and moderate climate, produce some of the highest wheat yields under non-irrigated production in the U.S. The elevation is about 2500 feet. The average frost-free period is about 140 days with average air temperature of 48° F. Palouse soil is formed in loess and has deep silt loam subsoil. The predominant rotation is soft white winter wheat followed by dry peas. In the Downey area, the higher elevation (4800 to 5800 feet) shortens the growing season slightly and offers less precipitation, 14 inches annually. Average air temperature is 44° F. The typical rotation is hard red winter wheat and a year of fallow because of limited moisture. The soils, of loessial and silty alluvium have a calcareous layer in the soil horizon. The existence of this layer in the root zone suggests that topsoil could be more critical for crop growth and implies that yield damage from erosion may be more severe.

Results

We estimated splined regression models with annual dummy variables to reflect annual weather differences for soft white winter wheat and dry peas in the Palouse and for hard red winter wheat in the Downey area. Breakpoints were chosen for topsoil depth that minimized the error sum of squares. For both wheat and peas in the Palouse the breakpoint was 15 inches. For wheat near Downey the breakpoint was 4 inches. Estimated splined regression models for both areas with weighted intercept were:

Palouse

$$\text{Wheat Yld} = 66.87 + 2.07 B_1 + .30 B_2 \quad R^2 = .726$$

(.3095) (.1219)

$$\text{Pea Yld} = 1135.9 + 50.81 B_1 + 17.44 B_2 \quad R^2 = .402$$

(12.31) (5.88)

Ririe

$$\text{Wheat Yld} = 14.2 + 1.5031 B_1 + .4664 B_2 \quad R^2 = .903$$

(.3908) (.1067)

Standard errors appear in parentheses.

Results of the T-test showed that two segments of each yield function had different slopes. Therefore all crops exhibited a nonlinear response to topsoil depth. These test results are presented in Table 1.

Table 1. Results of T-test for linearity

Crop	B1	B2	Contrast	Std Error	DF	Test t	Tabled t 5%	Decision
Palouse								
Pea	50.81	17.44	33.37	15.95	90	2.09	1.99	Nonlinear
Wheat	2.07	0.30	1.75	0.38	204	4.66	1.98	Nonlinear
Ririe								
Wheat	1.50	0.47	1.03	0.44	63	2.33	2.00	Nonlinear

With this evidence that the nonlinear form is more appropriate, the M-S model for each crop was estimated with annual dummy variables for weather. The following are the estimated equations with weighted intercepts.

Palouse

$$\text{Wheat yld} = 62.6 + 44.3 * (1 - \exp(-0.08652 * \text{depth}))$$

(5.01) (0.0208)

$$\text{Pea yld} = 1140.5 + 1431.4 * (1 - \exp(-0.0443 * \text{depth}))$$

(310.2) (0.0242)

Ririe

$$\text{Wheat yld} = 14.5 + 13.0 * (1 - \exp(-0.1196 * \text{depth}))$$

(2.48) (0.0554)

Asymptotic standard errors appear in parentheses.

A graphical comparison of the splined model and M-S model, both with weighted average intercepts, for wheat in the Palouse is presented in Figure 4. Note the close conformance of the two models suggesting the appropriateness of the splined regression linearity test and the suitability of the M-S model given the nonlinear result from that test.

The estimated M-S equations for each crop with weighted intercept were used as yield estimating equations in the damage function. The cost of erosion damage is estimated as the present value of the lost income from reduced yield due to one year of erosion. The lost yield and income for the prevailing crop rotation due to one year of erosion is projected over a 75 year damage horizon. To calculate present value we used a 4% real discount rate that is based on the return to farm assets during a period of stable prices.

The rate of erosion for conventional tillage with the wheat-pea rotation in the Palouse on a typical slope (16% slope, 200 foot length) is 12.59 tons per acre per year. One inch topsoil loss reduces wheat yield about 6%. The erosion rate for conventional tillage with the wheat-fallow rotation in the Downey area is 12.07 tons per acre per year for the typical slope (9% slope, 400 foot length). One inch soil loss reduces wheat yield by 10% in the south study area.

While the proportional yield damage from erosion is greater in the Downey area as expected from the restrictive subsoil, the cost of erosion damage is greater in the Palouse. The cost of one year of erosion damage is \$7.82 per acre in the Palouse and \$1.45 per acre in the Downey area. The cost of erosion damage is much less in the Downey area because wheat yields, absolute yield declines and crop prices are all lower. Also the Downey area is a summer fallow region whereas the Palouse is an annual cropping area. The cost of erosion damage is higher with annual cropping because income is lost every year of the rotation as opposed to every other year over the damage horizon.

Because of the nonlinear yield response function, the projected relative cost of erosion damage between north and south Idaho could change with cumulative erosion over time. We hypothesized that the cost of damage, while currently lower in the south than the north, would increase in the south relative to the north because of the calcic horizon in southern soils.

To test this hypothesis we projected erosion for 100 years at current rates. Topsoil would decline to 4.3 inches in the south and to 6.4 inches in the north. We then projected the cost of erosion damage 100 years from now on those depleted soils. The cost of erosion damage in the south increased slightly faster than in the north to \$18.56 and \$63.35, respectively. The ratio of southern damage to northern damage increased from .18 currently to .29 after 100 years of erosion. In comparison elsewhere, with greater differences between subsoils, the relative cost of erosion damage between two soil series may change even more dramatically with cumulative erosion. To incorporate changing erosion damage in soil conservation policy analysis, calls for dynamic analysis.

Conclusions and Policy Implications

The relationship between crop yield and topsoil depth was found to be nonlinear with statistical significance at the 5% level. While the proportional yield damage from erosion was higher in the Downey area, the cost of erosion damage was higher in the Palouse. The cost of onsite erosion damage and offsite damage costs can be used for targeting soil and water conservation efforts and for choosing the cost-effective mix of treatment and preventive practices for improving water quality. Because of the nonlinear yield response function and accelerating erosion damage, it might be a mistake to base soil conservation policy on current erosion damage. It would be better to project erosion damage into the future with a dynamic analysis. A low damage area now might become a high damage area later. In the face of accelerating erosion damage, delaying erosion control could be costly.

Selected References

- Buckman, H.O. and N.C. Brady. The Nature and Properties of Soils. New York: The McMillan Company, 1969.
- Burt, O.R. Farm Level Economics of Soil Conservation in the Palouse Area of the Northwest. Amer. J. Agr. Econ. 63: 83-92, 1981.
- Fomby, T.B., R.C. Hill, and S.R. Johnson. Advanced Econometric Methods. New York: Springer-Verlag Inc., 1984.
- Frohberg, K.K. and E.R. Swanson. A Method for Determining the Optimum Rate of Soil Erosion. Res. Rpt. No. 161, Ill. Agr. Exp. Stn., Urbana, Illinois, 1979.
- Krauss, H.A., and R.R. Allmaras. Technology Masks Soil Erosion Effects on Soil Productivity. In B.L. Schmidt, R.R. Allmaras, J.V. Mannering, and R.J. Papendick (eds.) Determinants of Soil Loss Tolerance. Soil Science Society of America, Madison, Wisconsin, 1982.
- Langdale, G. W., and W.D. Shrader. Soil Erosion Effects on Soil Productivity of Cultivated Cropland. In B.L. Schmidt, R.R. Allmaras, J.V. Mannering, and R.J. Papendick (eds.) Determinants of Soil Loss Tolerance. Soil Science Society of America, Madison, Wisconsin, 1982.
- Larson, W., F. Pierce, R. Dowdy, and W. Graham. Soil Erosion and Soil Productivity. Paper presented for Farm Agricultural Resources Management Conference, March 17-18, 1982, Iowa State Univ.
- Pawson, W.W., et al. Economics of Cropping Systems and Soil Conservation in the Palouse. Wash. State Univ. Agr. Exp. Stn. Bul. No. 2, Pullman, WA., 1961.
- Shelton, J.R. and F.G. Bailey. Present Value of Loss of One Ton of Surface Soil in Idaho Dry Croplands. Boise, ID: USDA SCS Tech Notes, Nov. 1980.
- Shrader, W. Effect of Erosion and Other Physical Processes on Productivity of U.S. Croplands and Rangelands. Paper prepared for the Office of Technology Assessment, Congress of the United States, Washington, D.C., 1980.
- Spillman, W. and E. Lang. The Law of Diminishing Returns. Yonkers-on-Hudson, 1924.
- Thomas, H.R., W. Lodwick and J. Shelton. Society's Stake in Maintaining Productivity Through Erosion Control. Unpublished paper. USDA-ESS, Oregon State Univ., Corvallis, OR, 1979.
- Wetter, F. The Influence of Topsoil Depth on Yield. Technical Note Agronomy 10, USDA-SCS, Colfax, Washington, 1977.

Xu Feng. Crop Yield Trends and Variability Levels in Washington State and Canada. Unpublished Master's Thesis. Washington State Univ., Pullman, WA, 1987.

Young, D.L. "Modeling Agricultural Productivity Impacts of Soil Erosion and Future Technology." Proceedings of RCA Symposium: Future Agr. Tech. and Res. Cons., Washington, D.C., Dec. 5-9, 1982, Iowa State Univ. Press.

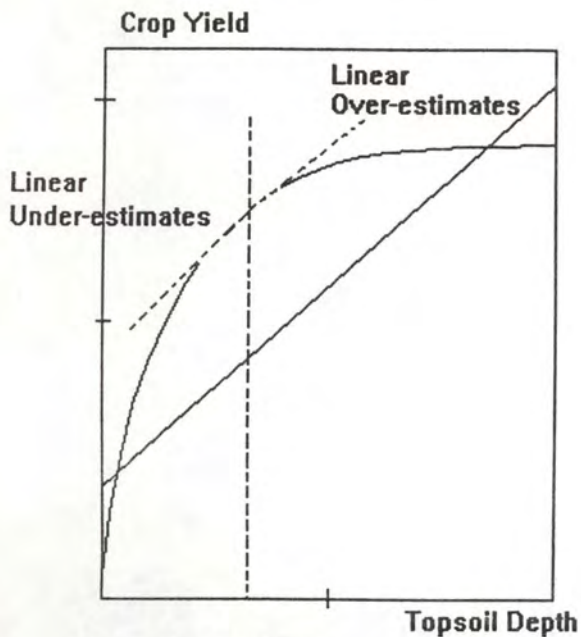


Figure 1. Erosion Damage Estimate If True Relationship Is Nonlinear

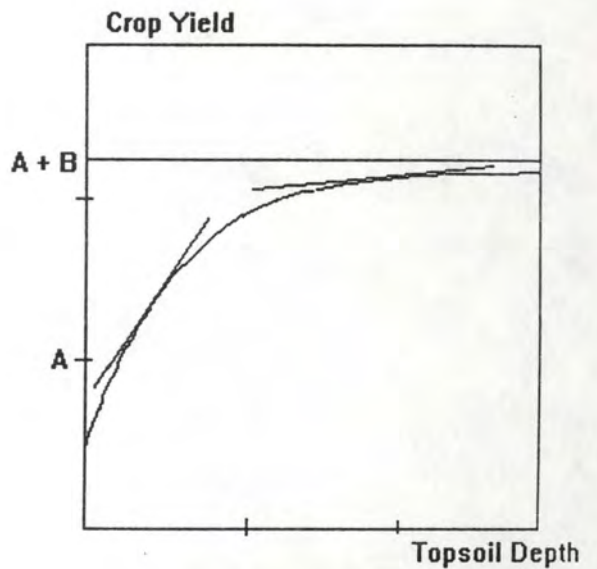


Figure 2. Mitscherlich-Spillman Functional Form

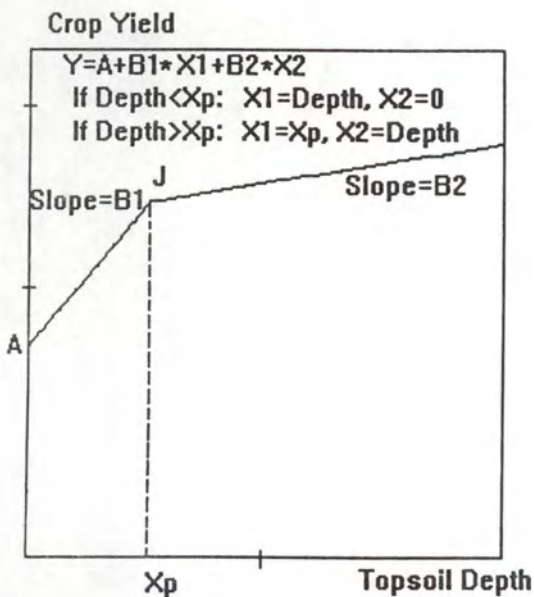


Figure 3. Splined Regression Model

Xp : Breakpoint for Depth
 J : Spline Point for Slope Segments

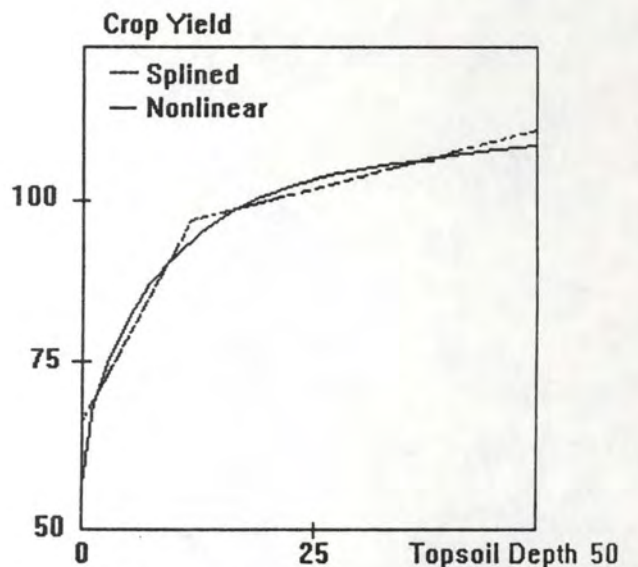


Figure 4. Nonlinear and Splined Regression Models of Wheat Yield On Palouse Soil