REVERSE ENGINEERING LEGACY MECHANISMS FOR MANUFACTURING

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by

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Authorization to Submit Thesis

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Abstract

Reverse-engineering (R-E) mechanical equipment and systems is less discussed than other disciplines such as computer software. There is often a reliance on simply reproducing the net-shape of a part. Replicating the net-shape of a completed part will often produce a functioning facsimile, but is not the most economical to R-E the functions of a part. A goalbased methodology should be applied parallel to established design processes to produce functioning recreations of parts. The R-E methodology developed for this study consists of three parts and is intentionally similar to new-product design: Identify functional purpose and goals of the part/system, quantify the precision and accuracy of the original part's dimensions and tolerances, and isolate the original method of manufacturing from the proposed recreation. For this research, case studies spanning over 2000 years were used at different levels of depth: a scale model of a 1950's hydroelectric turbine, a 1900's desktop Stirling engine, and an over 2000 year old orrery (the Antikythera Mechanism). The hydro-turbine model contrasted the modern goal of the recreation against the original project. Experience with the Stirling engine showed that features necessary to its function were not clearly distinguished in its drawings. Finally, the successfully recreated Antikythera mechanism was evidence that perfect knowledge of the original is unnecessary. Overall, the successful application of a common R-E methodology to these diverse cases endorses the process established in this study.

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Dedication

I would like to dedicate this work to my parents David and Patricia Olson. They cultivated my endless curiosity and taught me how to appreciate and apply knowledge for the betterment of myself and others. I would never have become the person I am today, nor chose to pursue graduate degree, if it were not for their undying support and encouragement. I will always endeavor to carry their values into the future.

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Chapter 1: Introduction to Reverse Engineering 1.1 What is Reverse Engineering?

Generically, reverse engineering is the process of extracting the design intent or manufacturing processes of an existing product. Reverse engineering of software programs and physical objects are similar but differ on several key factors. The focus of this review is on a reverse engineering (R-E) methodology for mechanical components and assemblies. There are often two major applications of reverse engineering: One, identify the techniques or technology in a part or system with the goal of reproduction. Two, understand its design for informational purposes. The sophistication of modern reverse engineering techniques and technology allows almost any product to be analyzed and understood, given enough time and resources. Fortunately, the patent process and associated legal structures are designed to protect truly groundbreaking technology. However, a significant portion of most products is simply trade secrets, such as a manufacturing techniques or material choice.

In the business sector, investigating the products of competitors can provide an advantage. For example, knowing that a competitor's part is injection molded can provide insight into the per-part costs and by inspecting the tolerances you can infer the techniques in tool and die manufacturing. Businesses can use this information to market their own products in areas that their competitors are weak or adjust pricing based on estimates of others' profitability. These are often considered forms of industrial espionage and have various legal ramifications. Alternatively, reverse engineering is a necessity to repair or replace equipment or machines that are obsolete or simply no-longer manufactured. For example, museum restoration efforts require numerous parts that are simply unobtainable.

1.2 Timeliness

Reverse engineering is especially useful in manufacturing maintenance and is becoming increasingly important. In the United States, the average age of manufacturing equipment is now greater than 10 years (Bereau of Economic Analysis, 2016). This number is concerning when the average life expectancy of industrial equipment is 10-15 years (Walton, 1987). This means much of the nation's manufacturing equipment is at or near endof-life. The success of factories, especially small businesses, often hinge on a handful of major capital equipment. The cost to wholly replace said equipment can be financially devastating. This forces companies to extend the life of existing equipment through careful maintenance, periodic overhauls, and occasional retrofits. In my personal career, I have experienced, first-hand, the difficulty maintaining equipment that is greater than 10 years old. As time passes the Original Equipment Manufacturers (OEMs) either cease supporting older models or go out of business entirely. This creates, out of necessity, a climate requiring extensive reverse engineering.

1.3 Existing Processes and Research

There is abundant research on the engineering design process. While there are several strategies and methods to successfully reverse engineer software (pioneered in the 1980's) there is little in the realm of mechanical equipment. A review of existing literature on the topic has identified an opportunity for a more design-orientated approach to reverse engineering that focuses on the design intent of the product instead of simply the net shape of the existing product.

Contrary to popular belief engineering design is a creative process. Just as two artists will produce unique paintings of the same subject, two engineers, will not design identical products even with identical goals and requirements. This creative component is one of the primary confounding factors of reverse engineering. Contemporary literature on the topic focuses on the exact measurement of articles and converting that data into reproducible models. This is somewhat a result of the dilution of the term "reverse-engineering" to refer to obtaining geometric shapes and dimensions from raw measurements (Buonamici & Carfagni, 2016). This is more apparent when examining the advertising for companies that perform reverse engineering. They focus on the technology they use (ASTRO Machine Works Inc,

2018) and the accuracy it allows them to measure parts. This is only suitable for producing a wholesale copy of a machine. Regardless of the significant legal implications, this method is not an efficient method for producing working recreations of the original. Alternatively, some sources seem to treat reverse engineering the same as new product design just with a significantly more complex scoping process (Otto & Wood, 1998).

A single-path methodology of reverse engineering focuses entirely on an individual step in the process, often metrology, without a cohesive interconnected methodology will have difficulty producing useful results. Do not discount the importance of metrology in reverse engineering, it is important to understand the net-shape and materials of the product. Similarly, while an exhaustive statistical analysis of manufactured part tolerances may suggest the designed tolerances, it cannot distinguish between the required dimensions or tolerances from those simply born from convenience. Reverse engineering should be treated less as an analysis and more as a creative design. There is significant literature discussing the engineering design process and philosophy. Unfortunately, reverse engineering is uniquely different from new-product design, enough so that existing methodologies are mostly inapplicable.

Chapter 2: Reverse Engineering Methodology

2.1 Overview of Method

My proposed methodology consists of three interconnected steps: Identification of purpose, Metrology, and Manufacturability or more simply "Why? What? And How?" Each step of my methodology has to be applied at both the assembly and individual component level. At the assembly level the identification of purpose seeks to determine the function of the machine and the desired goals of the reverse engineered reproduction. At the component level, identification of purpose requires identifying the design intent of each feature and the part's interfaces. At the assembly level metrology is applied to determine the net features of a product such as total weight or center of gravity. At the component level materials science should identify the part composition and Geometric Dimensioning and Tolerancing should be applied to specify the features needed fit and finish for proper function. At the assembly level, manufacturability mostly focuses on the ability of a machine to be assembled effectively or efficiently. At the component level, manufacturability examines the pros and cons of various manufacturing techniques and their consequences. I explored this methodology using several case-studies each focusing on one or more of the steps.

2.2 Identification of Purpose

The first component identification of purpose is the "Why" of reverse engineering. The engineer performing a reverse engineering process needs to know why they are doing it. What is the goal of the activity and how will they know if they are successful?

First it is important to distinguish the differences between replicas, reproductions, forgeries, and knock-offs. A replica is when both the form and function of the original are duplicated. A high-quality replica will often be made using the same techniques as the original and can be nearly indistinguishable from an original. A reproduction is a product that mimics the function of a product without necessarily the same features or manufacturing methods. Alternatively, a forgery is the replica with the intent to trick or deceive people into thinking it is the original product. Replica products can be produced innocently then sold as forgeries by the unscrupulous. Similarly, a knock-off is a reproduction intended to be sold in place of the original often to usurp market-share from the originator.

Forgery is, by its nature, illegal. However knock-offs can be crafted carefully to avoid violating the law by omitting or altering controlled intellectual property. While sometimes clear, the difference between a knock-off and a similarly designed product can be unapparent. As an example, while most modern violins share many of the same stylistic and functional features of a Stradivarius, they would not be considered knock-offs (Figure 2.1).



Figure 2.1: (Left to Right) Renaissance, "Francesca" Stradivarius, and Modern Violins Sources: (Rakić', 2018) (The Metropolitan Museum of Art, 2018) (Skinner Inc, 2018)

Sometimes you will choose that the reversed engineered design will be serving a different purpose from that of the original. In these cases it becomes even more important to identify design intent. Scaling is an excellent example of this case. Due to the square cube law, many physical properties do not scale equally. The simplest example is that of spheres. As the size of the sphere increases, the ratio of its surface area to its internal volume is not constant. Since this ratio will impact heat transfer, manufacturing cost, or any number of variables, you cannot simply take an existing engineering design and "make it bigger".

Once you have decided what the goal of your reverse engineering is you can tailor your analysis appropriately. For example, if the goal is producing a replica you will need to pay more careful attention to identifying the original manufacturing methods and materials than if you want a functional reproduction.

2.3 Metrology

Metrology is the study of measurement. In the context of my methodology it consists of a few major components: Geometric Dimensioning and Tolerancing (GD&T), materials, and statistics.

GD&T is an important factor to my reverse engineering methodology. At the highest level GD&T is simply a set standard nomenclature used to specify engineering tolerances. Used improperly GD&T adds nothing to a design, but when used effectively it facilitates the communication of design intent. A key application of this is interfacing: When two components have to fit together they must have interfacing features. A simple example is a pipe flange with a bolt pattern. For it to fit correctly (water or airtight) three conditions must be met simultaneously: The holes must align, the bolts must fit through the combined holes, and the surfaces must be in direct contact. For even this simple example there is a wide variety of required GD&T specifications such as flatness, true-position, cylindricity, and parallelism (Figure 2.2). Some might look at this example and expect that the manufacturing cost would be astronomical. However, in most cases the opposite is true. Without proper GD&T feature callouts the design intent is not communicated and it is left open to interpretation. It is important to note the difference between the presence of a tolerance and the magnitude of it. To convey design intent it is almost as important to specify loose tolerances as tight ones, otherwise you are relying on the manufacturer's guess at a feature's importance. In the case of highly complex assemblies it is unreasonable to expect that the person building it will know what exactly the part even does.



GD&T principles are applied to reverse engineering to identify the goal of each dimension and its needed tolerance, and then to specify it accordingly. This will minimize wasted effort in analysis. If a feature's function is known by the investigator they should know the desired tolerance before measuring and use this knowledge to apply the correct technology. It is simply wasteful to use a sub-micron accurate coordinate measuring machine to measure the length of the electrical cord on a toaster. Similarly, scanning and digitizing an entire device in 3D is not always a value-added activity. An additional complicating factor is that sometimes the only available specimen is a worn out or broken part. This makes direct measurement and duplication worthless as it would simply result in a second broken part.

Material selection is as important in reverse engineering as it is in new-product design. As with all the steps of my methodology it needs to be considered holistically. For a reverse engineered design you want to make a decisions guided by evidence collected from the original and it is often an exercise in empathic intelligence. The standard method is a "rationality assumption". This means that you assume that the original designer chose each feature for a purpose and not randomly. If analysis shows a part is made from a particular alloy of steel the engineer must decide: Were the properties of the alloy important? Or perhaps was it the cheapest option? This again shows why it is necessary to identify design intent if the goal is different from a simple copy. Since major advancements have been made in the world of materials science and production techniques the era of manufacture is often a major factor in material selection. For example: The cap on the Washington monument was made from aluminum which at the time was a precious metal more valuable than silver. Unlike the 1880's, choosing aluminum today is not considered a prestigious choice.



Whenever possible it is best to remove uncertainty when reverse engineering. Therefore, it is almost always beneficial to analyze multiple items of the same component when practical. When presented with a large enough sample of produced components an investigator can infer certain tolerances, quality control, and manufacturing techniques. This is because in manufacturing, like most natural processes, produced parts will inevitably have variation. The same statistical methods used by industrial, process or reliability engineers to identify and improve manufacturing can be used in reverse engineering. The range in the variation and even the shape of the distribution can provide critical details about the OEM's process. At the simplest level, the range of variation can be known to be within the manufacturer's acceptable range otherwise it would have been rejected when originally built. Additionally, the shape of the distribution can provide information about the OEM's QA techniques, manufacturing equipment, or process control methodologies (Figure 2.3). If the location and/or date/time of manufacture can be determined, often through analysis of factory markings or serial numbers, even more can be learned since data trends also carry valuable information.

2.4 Manufacturability

Manufacturability refers to the engineering of a component for efficient manufacturing. This applies to reverse engineering in two ways: First, how was the original made and what does this tell us? Second, how will it be remade? Manufacturing methods and their costs and benefits is a deep subject and is outside the scope of this analysis. However, the identification of the OEM's methods can aid in identifying GD&T and material selection. Additionally, it will impact efforts to reproduce.

The design of tooling or dies can drastically change the resulting product's quality and cost. Sometimes the manufacturing equipment and processes themselves are the most valuable intellectual property of a product. For example, the ingredients for Cola-cola are written on every can and the exact quantities can be determined scientifically. Given enough time and funding a molecularly exact copy of the soda could be created chemically. Therefore, in this case it is not the makeup of the product that is valuable but the formula and process for producing it quickly and cheaply.

It is impossible for a reverse engineer to determine the OEM's entire production process from only the finished product. Therefore, they must apply the same principles as new-product design to choose the appropriate techniques. It is important to remember that differences in capability or technology may make the techniques used by the OEM undesirable in favor of alternatives. This is especially true if the original product is old as new technologies have rendered some manufacturing methods obsolete.

2.5 Case Studies

The scientific method teaches that conclusions can only be drawn from repeatable data. In order to test my proposed methodology I chose to apply it, on a series of diverse case-studies (Figure 2.4) to varying degrees. The goal of this experimentation being that a diverse selection will minimize the risk of extrapolating the method's effectiveness broadly.

My first case study was reverse engineering hand-drawn manufacturing drawings of the Noxon Rapids hydroelectric dam. The goal was to create a modern set of drawings and then create a scale-model for display and demonstration. This study was part of a senior design project working in conjunction with a local engineering firm (WAGSTAFF INC) working to retrofit the dam. This study highlights two main points: First it shows that when reverse engineering it is necessary to consider the desired new purpose and not just the original purpose since it was used to recreate an assembly with a totally new purpose (display/education instead of power generation). Secondly, it is a great example of considering the method of manufacturing and its limitations for both the original and recreation. The design considerations of large scale welded structures are different from small scale conventionally machined parts.

My second case study was a further refinement to the documentation and manufacturing of the University of Idaho's student built desktop Stirling engines. These engines are built as a student project to teach manufacturing principles. It is a Gamma-Type Stirling engine. The original design came from the Massachusetts Institute of Technology (MIT). Since then, the design has been modified by University of Idaho students to reduce cost and improve manufacturability (Allen, 2002). While the original source drawings of the design are now lost, the first iteration for the University of Idaho was documented. However, there is little documentation of the changes made since then, making the purpose of each iterative design change unclear. While we had a complete set of drawings for the current revision, the design intent was not ensured. The goal of this case-study was to examine several current and past iterations of the Stirling engine to isolate the critical dimensions and features and to then refine the drawing package to reflect the appropriate tolerances.

My third case study was the reverse engineering of the Antikythera Mechanism. The Antikythera Mechanism is a mechanical orrery dated to about 100 BC. It mechanically computes the position of the visible heavenly bodies. Even by modern standards it is a highly sophisticated mechanical device and being from the ancient world is even more impressive. It predates similar devices by more than 1000 years. The goal of this case-study was to reverse engineer the mechanism to produce a functional recreation for display and educational purposes. This recreation would specifically focus on the application of modern manufacturing technology to the ancient device and reverse engineering off of incomplete data.



Chapter 3: Noxon Rapids Hydro Turbine

3.1 Background and History of Noxon Rapids Dam

The Noxon rapids dam is an earth filled gravity dam located on the Clark Fork River in Montana. It was completed in July of 1959 and currently is operated by Avista Utilities with a generating capacity of 562MW. It is driven by a series of single-runner verticalshaft Francis-style turbines. The turbines operate up to 100 rpm to produce a total of 130,800 horsepower. Over the years, various improvements have been made to the hydroelectric dam since its construction including adding a 5th generator and improved flow stay vanes. Recently Avista utilities began an extensive retrofit of turbine units 2 and 4. Wagstaff Inc. reverse engineered and manufactured much of the replacement equipment.



Figure 3.1: Photo of Noxon Rapids Dan Source: (Groundspeak, Inc, 2018)

In parallel with the refurbishment Wagstaff Inc. funded a mechanical engineering capstone design project with the University of Idaho. The University was provided with scans of the dam's original drawings. The University's goal was to reverse engineer and recreate the drawings and built a scale model of one of the turbines. A team of four undergraduates was formed plus me as a graduate student mentor.

3.2 Application of R-E Methodology

According to the R-E methodology, we needed to first identify the goal of the reverse engineering. While Wagstaff engineers had separately been developing replacement components for the dam's retrofit, the goal of the University was to digitally recreate the original drawings. Additionally, we were tasked with producing a physical model for display purposes. Capturing the critical details, dimensions, and the design intent of the original drawings was key for the recreating the drawings. However, the physical model needed to only retain recognizable features and major components while still conveying the visual essence and design style of the original.

Due to dam being built in the 1950's the drawings were originally created by hand and the copies provided by Wagstaff were in a slightly degraded state. The computer aided design (CAD) software SolidWorks was used to create solid models of the parts. Unlike drawings (by hand or using programs such as AutoCAD) SolidWorks is a parametric feature-based solid-modeling software. Creating a solid model, by its nature, requires all of the necessary dimensions to be defined before it will work. This is very helpful for reverse engineering because it forces you to deliberately decide each feature and associated dimensions. It also helps prevent over-dimensioning a part since, if the geometry is fully-defined, additional dimensions can be contradictory. Some may be specifically defined in the original while others are implied by shape or function. The team worked to reproduce the drawings as 3D solid models. Unfortunately since older drawings such as these were hand-drawn, shapes could be created artistically instead of mathematically. Certain necessary dimensions may be omitted entirely relying on the skill of the craftsmen installing the parts to "field-fit" them. In these cases the students had to infer the intent of the original. As an example (Figure 3.2) is a drawing of the wicket gate control levers. As you can see there are very few dimensions for a shape as complex as it is. However, as was common practice for the era, the drawing itself is true to scale and therefore dimensions can be extracted by directly measuring distances. However, this technique is not fool-proof since when hand drawn there is an inherent error, while small, on the shape and placement of curves and lines. Therefore, when measurements are taken directly off the drawing we had to apply principles of inductive reasoning to come up with a guess the most likely true dimension. Again using the assumption of rationality, dimensions will normally wall on round numbers or common fractions. You can see evidence

of this when comparing (Figure 3.2) the original drawing to (Figure 3.3) the 3D recreation of the same part. Fortunately, the functionally critical dimensions, such as the minimum and maximum angles and bolt pattern distances are labeled making the functional aspects of the recreation more accurate.





3.3 Scaling

Once the team had reproduced the original drawings, we moved on to producing the scale model. As discussed earlier, you cannot simply rescale a design and maintain the same engineering parameters. Mechanical safety factors and manufacturability will be directly changed based on the scale of parts. In this case-study, our goal included making a desktop sized display model of the turbine. This is a significant reduction in scale from the original. We had to determine the limit of our downscaling on the smallest necessary component of the design and our capability to reproduce it. For example a one inch bolt on the original when scaled down would be essentially non-existent. However, for the purpose of a display model, they are not necessary components. The function or essence of the design is not altered with different attachment techniques. On the hydro-turbine the smallest identified feature for the model was the wicket-gate control linkage.



These links are what attach the wicket gates, also known as guide vanes (Figure 3.4), to the control ring (operating ring) and synchronize their movement. In full-scale these links are individually adjustable to improve the fit between wicket gates and ensure a watertight seal. The next decision point is the largest component. The students identified a dual limitation of both the runner and the stay rings, the circular rings that holds the stay vanes.



Figure 3.5: Francis Turbine Runner Source: (Wagstaff Inc., 2017)

The blades on the runner are curved hydrofoils with a non-uniform cross-section. Due to this complex geometry, it would be impractical to manufacture using the university's 3-axis mill, This quickly lead to identifying that it would be 3D printed and therefore its size was limited to the printable area of our machines. Secondly, the stay rings were to be CNC machined and therefore their size was limited to our CNC mill's working area.

Once a scale of 1:40 was chosen, lying between the identified upper and lower limits, each part must be rescaled. Instead of simply resizing each part by a constant ratio they must be individually reverse engineered for the new scale. As discussed strengths and stresses do not scale linearly, so at smaller scales certain material choices or strengthening features become unnecessary. However, inversely manufacturability often becomes more limiting with smaller sizes and was of great concern for this project. The goal was to manufacture the model in-house using the university's mechanical engineering machine shop. This enforced practical limitations on the tooling sizes and achievable tolerances. An example of these tradeoffs is the control ring, which allows synchronized the rotation of the wicket gates. As displayed in Figure 3.6 and Figure 3.7 changes were necessary. Since weight and handling of components during assembly were less restrictive the lifting eyes were removed from the top of the ring. The ring was made solid by removing the internal ribs since light weighting was unnecessary and to simplify manufacturing. The flanges on the top (where the hydraulic

control rams would attach) were also omitted. Lastly the material was changed from steel to aluminum.

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Similar changes had to be performed on each component during the rescaling process. Some of the most significant changes necessary were the result of hardware. Since we desired that the model spin easily we had to incorporate bearings into the model. To avoid the cost of custom built bearings, we had to make space for standard size ball bearings.

3.4 Results

Using the described methodology the team was able to produce a reasonable 3D reproduction of the original drawings and even a functional digital assembly (Figure 3.8 and Figure 3.9). Additionally, we successfully fabricated a scale model of the turbine body (Figure 3.10), but were unsuccessful producing a transparent inlet casing using the available tools. A clear spiral casing was only achieved at an even smaller scale (Bailey, 2018). Overall, the biggest difficulty was identifying what features to remove for the scale model and identifying how each part would be manufactured at small scale.







Figure 3.10: Photo of Physical Scale Model (1:40 Scale Version)



Figure 3.11: Photo of completed spiral casing on smaller (1:80) scale model

Chapter 4: Stirling Engine

4.1 Background and History of University of Idaho Stirling Engine

The University of Idaho mechanical engineering department has had students manufacturing desktop sized Stirling engines for years. Over time the design has been refined and improved to reduce the cost of manufacturing, reduce failures, and better target learning goals. The goal of the latest design iteration was to update the drawing package to include reasonable and achievable dimensions and tolerances. Since the design has evolved so many times it was advantageous to apply my reverse-engineering methodology in identifying the necessary tolerances and manufacturing techniques.



Figure 4.2: Photo of Current (2018) Design of University of Idaho Stirling Engine

4.2 Application of R-E Methodology

All of the parts of the current iteration of the Stirling engine were already modeled in SolidWorks. However, like many CAD designs they suffered from prefect dimensioning: meaning that the design drafted and assembled with the assumption that every part was built to exact dimensions. Obviously this is impossible. Fits and tolerances are needed to be applied in order to compensate for this reality.

My work consisted of evaluating each part: how does it interface with its connected pieces, how does it impact the whole design, and how it will be manufactured in the university shop. Then drawings communicating these decisions were created. The audience of the design drawings is University of Idaho undergraduate students building the engine. Therefore, it was necessary to limit GD&T callouts to avoid confusion to those unfamiliar with their use.

4.3 Critical & Non-Critical Tolerances

A good set of engineering drawings should provide all the information necessary to build a functional component. This is achieved by adding the dimensions and tolerances, which if followed, will ensure the proper function of the part. When using solid-modeling software it is sometimes easy to incompletely or incorrectly dimension a drawing even though the information is there (during construction of the 3D model). Luckily, if the 3D model is available, it is straightforward to extract the dimensions. Tolerances on the other had are not contained in the 3D model and must be specified on drawings.

An example of this process in action is shown in figures (Figure 4.3) and (Figure 4.4). On the simplest level the original drawing didn't specify the length of the base (nor could it be calculated from the given dimensions). Second, the original didn't show the counter-bore on the holes visually or with a note (the holes don't have a callout at all). Lastly, the addition of hidden lines and a 1:1 scale view aids with visualizing the part.





Figure 4.4: Drawing of Stirling Baseplate (after changes)

4.4 Results

Using the described methodology we were able to produce a new drawing package. The new package had improved specification of tolerances, corrections of errors, and included assembly and sub-assembly level drawings to aid assembly and comprehension. This new drawing package was used exclusively by University of Idaho students in fall 2017 to build Stirling engines in small teams. My ability to accurately represent certain tolerances was limited due to student's unfamiliarity with many GD&T callouts. I suggest that in the future, the drawings be updated to use the correct GD&T callouts instead of simply improved coordinate dimensioning. However, even with that limitation, unlike some of the previous iterations, all of the completed engines ran. Overall, the biggest difficulty was identifying what tolerances were achievable in the university's student machine shop and what tolerances were needed for the engine to run.

Chapter 5: The Antikythera Mechanism

5.1 Background and History of the Antikythera Mechanism

On May 17th 1902 divers exploring a shipwreck in the Mediterranean Sea recovered a mechanical device of unknown function and origin. Dating estimates the device was built around 100-200 BC. Additionally, based on the contents of the ship it is likely to have been built in Greece. The fragments (Figure 5.1) were initially a curiosity. Examination of the surface suggested that the fragments were parts of a highly complicated whole. Only recently, with the development of high-resolution x-ray imaging, were the internals able to be non-destructively examined. The imaging, (Figure 5.2), revealed even more complexity and allowed more research into the mechanisms purpose and function.




Figure 5.2: High Resolution X-Ray of Largest Antikythera Fragment Source: (The Antikythera Mechanism Research Project, 2018)

Inscriptions on its surface refer to astronomical objects and calendar dates. This combined with extensive research into the mechanical gear ratios and components lead scientists to conclude the mechanism was an orrery used to predict astrological phenomenon. It is noteworthy that while the astrolabe, a mechanical device to predict the position of the stars was invented shortly before. Astrolabes themselves are impressive, but have only two moving parts. In comparison, the Antikythera mechanism is essentially an analog computer. Adding to its mystery, the only similar devices known come from more than a thousand years later. Additionally, for comparison, while we describe the mechanism as having a clockworkstyle design, mechanical clocks did not yet exist when it was built.

5.2 Ancient Astronomy and the Geocentric Model

Due to its age the Antikythera mechanism is based on a geocentric model of the solar system. A geocentric model means that the planets are orbiting the earth instead of the sun. There are various historical and anthropological reasons for this model being prevalent during this time period. There is a modern misconception that the geocentric model was simple and our ancestors were foolish to use it. This is untrue. Scientific models exist to explain and predict the natural world. Like modern scientific theories, the geocentric model was slowly refined to better fit the observed data. Its predictions are reasonably accurate by today's standards and more than enough for ancient astronomers taking measurements by eye.

Ancient astronomers identified certain heavenly bodies as different from stars based on their motion in the sky. Due to the earth's rotation, from the earth's surface, all of the observable stars appear to rotate on what is known as the celestial sphere. Unlike the uniform rotation of the stars, planets (and the sun and moon) move differently along their own path. They defined the path of the sun relative to the stars as a circle known as the ecliptic. It reality, this path is the result of earth's tilt. Since most planets in the solar system rotate in approximately the same plane (now known as the ecliptic plane) they will mostly follow the ecliptic through the sky with only minor variation.

Until the invention of the telescope only the position of planets in the sky could be determined. Information about their phase and distance were unattainable. So a model was developed that could predict the position of the planets. In order to characterize their movement, astronomers used ecliptic angles. They would record the angle of the planet along the ecliptic relative to the suns position on the solstice.

If you observe the visible planets for long enough periods certain trends will emerge. One of these trends is apparent-retrograde motion. As we now know, as the earth rotates around the sun its orbit passes other planets. This relative motion makes the planets appear to orbit backwards (like passing a car on the highway). In antiquity astronomers struggled to explain this motion. Eventually they developed the system of deferent and epicycle. In this system each planet has a deferent and epicycle. The deferent is the circular path around the earth. The epicycle is a smaller orbit centered on the deferent (Figure 5.3). This system can be visualized today as the path the moon as it travels around the sun, orbiting an unseen earth. The interaction of the sizes of the deferent and epicycle and their rotation rates produces motion approximating the planets. However, the predicted motion of some bodies, such as the moon, didn't fit this model. In reality this is because the moon's orbit is non-circular. There were competing ancient explanations of this proposed by various astronomers. By the second century AD a consistent mathematical approximation of this eccentricity was developed by Claudius Ptolemy in his treatise "The Almagest". However, the Antikythera mechanism predates that by centuries.



5.3 Application of R-E Methodology

Again, based on my methodology, you first need to identify the goal of the reverse engineering. The goal of reverse engineering the Antikythera mechanism was to create a recreation of the device for an educational display that uses modern manufacturing methods but maintains the original kinematics. Manufacturability of the design was a major component of this case study since we built it in a modern machine shop in a short time instead of an ancient workshop by hand over a long period. This case study was also intended to represent the class of projects where information on the original is limited such as the case of a broken or lost component. This makes the Antikythera Mechanism a study in abductive reasoning (Kolko, 2010).

My work consisted of investigating the documentation of the original as well as speculating as to its uses and purpose. Decisions about what features of the original were critical to maintain, and which could be replaced with modern versions, were needed. A series of specifications that the final design would require was constructed. Then using those specifications a new design process was began to create a full assembly in SolidWorks. Undergraduate students were tasked with producing the design of the outer-planet upper assemblies based on a developed generic design and the baseline specs. Next, an efficient manufacturing process for the gears and components had to be developed and executed. Lastly, the design had to ensure the mechanism could be assembled and calibrated.

5.4 Mathematical Basis of Design and Theoretical Accuracy

In order to design an accurate mechanical representation of the geocentric system you must first have a mathematical model. This began by deconstructing the kinematics associated with the geometry of the geocentric model (Figure 5.4). Then an equation (5-1) to represent the angle of a planet as a function of time and six constants was derived.



$$\theta = \left[\frac{\omega_{1}}{r_{1}} \times T + \varphi_{1}\right] + \left[\left(atan\frac{r_{2}}{r_{1}}\right) \times sin\left(\frac{\omega_{2}}{r_{2}} \times T + \varphi_{2}\right)\right]$$
(5-1)

θ	Ecliptic Angle
$\omega_{1,}\omega_{2}$	Rotation Rates
r_1, r_2	Radius
φ_{1}, φ_{2}	Phase Adjustment
Т	Time

This equation can be reduced by combining constants (5-2). While this reduces complexity, it obscures the physical meaning of each constant but it also simplifies the process of curve-fitting.

$$\theta = [C_1 \times T + \varphi_1] + [C_2 \times sin(C_3 \times T + \varphi_2)]$$
⁽⁵⁻²⁾

With sufficient collected data you can curve-fit to determine the constants. This can be done manually by deconstructing the equation into linear and periodic components or by numerical optimization methods.

For clarity, Jupiter can be used as an example of this process. On the ephemeris data (Figure 5.5) of the planet's position, two points on the curve with the same phase can be identified such as the first and last valley. This would have been recorded as the date ending retrogradation in astronomy. Then calculate the slope of the line between these two points and it's Y-offset. This linear component represents the deferent and its constants. By subtracting the linear component (Figure 5.6), and thus deconstructing the raw data, the cyclic component is revealed. This cyclic component has an identifiable period, amplitude, and phase which satisfies the remaining three constants. For Jupiter this method will produce a reasonable reconstruction of the data. For comparison I performed an unconstrained optimization using a cost function consisting of the root-sum-square of the difference between the function and the data. The results were similar however the optimization predictably had a lower error over long periods and resulted in less than 4° of error over most of a 3000 day span (Figure 5.7).







When you examine the error between the model and the observed data (Figure 5.8) it appears harmonic. The model is essentially a Fourier series reconstruction (5-3). You are trying to reproduce a signal using intermittent sampling and a series of frequencies and amplitudes. In the model the amplitudes are associated with the radius of the epicycles/orbits and the frequency is tied to the rotation rate. As with any Fourier series reconstruction, if the input is not a perfect waveform the result will become increasing accurate with additional terms. Using a Fast Fourier Transform (FFT) a frequency and amplitude chart (Figure 5.9) can be produced. This analysis demonstrates that while the majority of the orbital pattern is contained in localized frequencies it is not truly a harmonic pattern. Historically, the geocentric model was refined with additional epicycles added in the centuries after its initial formulation.

$$F(x) = \frac{1}{2}A_o + \sum_{n=1}^{N} A_n \cdot \sin\left(\frac{2\pi nx}{P} + \varphi_n\right)$$
(5-3)





This analysis has shown that the geocentric model was an effective way of rationalizing the observed data of planetary position. Even in its simpler forms the errors were low considering that the measurements were taken by eye. The higher-order models developed a millennium later were very capable of producing useful predictions. However, because at their core they relied on the incorrect assumption, the planets move in a perfect harmony of circles, they would never produce a truly accurate mode (Johnson, 1949).

5.5 Quasi Clean Room Design

Clean room design is a process developed for the legal reverse engineering of protected intellectual property. This process was initially developed in the 1980's to produce IBM compatible software for home computers. It consists of three roles: the inspectors, the legal team, and the engineers. First, the inspectors examine the original product thoroughly documenting everything about its function. They use this inspection to create a series of specifications that encompass the function and characteristics of the design. Next the legal team examines the specifications and removes any protected intellectual property. Finally the engineers take the specifications and create a "clean" design.



Since much of the device is lost, no one is entirely sure about the exact design of the original Antikythera mechanism. This makes a true replica impossible. Additionally, without access to the original, it can be used as a case study of applying my methodology to a clean-room type design process. Using research performed by others (Jian-Liang Lin, 2016) a series of specifications and requirements for the device were created. While full recreations do exist, only research on the mechanics and function of the device were used, not completed designs. With this speculative gear-train in place, the next step was to create a "clean" design that would follow those specifications without being contaminated by others specific design choices. Since the inspector and engineer roles were performed by the same person this would not be a legally clean design, but since there are no actually intellectual property issues it allowed me to better understand the application of my reverse-engineering process.

5.6 **Design Principles**

Using the high-level specifications developed earlier as a guide, a series of standard design choices were necessary that would be modified to meet the goals of the individual sub-assemblies. There were two major categories, the supporting structures and the mechanical movement.

Due to the wide array of needed sizes a flexible gear design was necessary. A constant pitch involute profile was chosen for all of the gears to aid in manufacturability. Involute profiles are the most common modern gear tooth profile and are designed for efficient power transfer and smooth meshing (Buckingham, 1988). Unfortunately, choosing fixed pitch constrains the center-to-center distance of the gears in a way the original was not. In some situations this causes problems for the mechanism that had to be addressed. In order match a commonly available involute gear cutter (32 pitch), and to improve visibility of the components, the recreation is approximately 50% larger than the original. This mean adjusting the distances of certain features to maintain the correct kinematics. In order to match the original, and to benefit from its machinability, the gears should be brass. A standard 1/8th inch thickness was used for all the gears. This was a balance between saving material, minimizing deflection, and overall mechanism thickness. A bolt on boss was developed to ensure alignment of the gears to their shafts (Figure 5.11). This feature also provided room for a set screw to allow the gears to be adjusted during final assembly and calibration but still reversibly attached to the shafts. Cut-outs in the gear's web were added for the larger gears. This honored the appearance of the original mechanism and reduced overall weight. Some of the gears were too small to allow a boss to be bolted on. In these cases, lantern gears with pressed in pins were used. Their length allowed integral set screws and maintains alignment (Figure 5.12). Unfortunately, since pins of small diameter are only manufactured in certain diameters, the circular pins of the lantern gears do not perfectly mesh with the involute spur gears. This results in sliding, efficiency losses, and rougher overall movement. Some gears required the ability to rotate freely from the shaft that held them. In these cases a small ball-bearing can be directly pressed into the web of the gear. The mechanism makes use of pin and sliders at several locations. This movement was achieved with a dowel pin pressed into the gear's web and a corresponding slot cut into its mating gear's web.





The next major choice was how to support the mechanism. Solid steel shafts supported by bearings pressed into acrylic plate frames wherever possible meets the necessary requirements. Since the recreation is for display purposes, the acrylic allows observation of the internal gears and movement. This layout is similar to the design of clocks and has several advantages. Gear to gear distances and alignment are constrained by the holes drilled in the plates so as long as the plates are sufficiently rigid the gears will maintain their position. The plates are held apart by a series of nested columns so the mechanism can be assembled in stages (Figure 5.13). The shafts are prevented from sliding axially by attaching small adjustable collars at both ends. Lastly, like the hands on a clock, the dials on the front face require independent movement along a common axis. By using precision ground brass telescoping tube stock, the interior tubes are allowed to rotate smoothly while still being supported by the other tubes.



5.7 Calendar and Eclipse Prediction

The back face of the Antikythera mechanism had a series of dials (Figure 5.14). The large dial was associated with the Metonic calendar. This was a 6940 day, about 19 years, cycle used in astronomy at that time. This dial would indicate the date that the machine is currently set to. The calendar was written on a spiral groove around the dial with each month end-to-end. The dial arm had a slider that would follow in the groove. Therefore the position of the slider would indicate both the date and year of the Metonic cycle. Next to the Metonic dial is the Olympiad dial. It rotated once every 4 years to indicate the date and location of scheduled PanHellenic Games, such as the Olympics in Olympia. Also by the Metonic dial was the Callippic dial. A Callippic cycle is 76 years consisting of 4 full Metonic. The combination of the Metonic and Callippic dials would provide accurate representation of the

date for a 76 year span. The second large dial was the Saros cycle. This dial also used a spiral groove and sliding pointer. The Saros cycle consists of approximately 6585 days, about 18 years, and identifies the relative position of the sun to the moon. Next to the Soros dial was the Exeligmos dial. An Exeligmos cycle is about three Soros cycles, 54 years 33 days. These two dials were used for eclipse prediction since every cycle the sun and moon will return to same position. Based on an observed eclipse you can predict when a similar one will occur. However, due to the complexities of actual planetary motion, the location and duration of the eclipse shifts from cycle to cycle and will drift by about 8 hours every Soros cycle.



Source: (Freeth, Jones, Steele, & Bitsakis, 2008) (Freeth, et al., 2006)

From a design perspective the calendar eclipse dials are fairly straightforward. First the gear reduction/multiplier ratios must be as accurate as possible. This is complicated since you cannot have half a gear-tooth. Also, assuming a constant pitch, the size of the gear must increase with more teeth. Therefore you must use a series of integer teeth values to achieve the correct ratio while minimizing the total size of the gears. The next consideration necessary was how to space the gears without interfering with its movement but still allow the dials to be located. Using the center-to-center distances calculated from the gears, a schematic to layout and connect them in a compact manner was created (Figure 5.15).



5.8 Moon

The front face of the Antikythera mechanism has a series of dials indicating the position of the visible planets, the moon (Luna), the sun (Sol) and the current date of the year. Like the hour, minute, and second hands on a clock these dials all share a common axis and rotate at different speeds. This requires a series of nested tubes that run through the centerline of the device. The innermost shaft, and thus the longest shaft, is stationary and it is used for an end cap. The next larger shaft is used to indicate the ecliptic position of Earth's moon known as Luna.

Even in Greek times the orbit of the moon was known to be slightly irregular. This, called lunar anomaly, is the result of the moon's orbit being an ellipse instead of a circle. There is physical evidence (Figure 5.16) that suggests the Antikythera mechanism used a pin and slot mechanism to simulate this motion. Two gears with their axis of rotation slightly displaced have a pin and slot connecting them. Kinematically, the ratio between the shaft offset distance and the distance from the shaft to the pin determine the motion. This pin and slot mechanism creates a variable gear ratio that changes during the course of the motion. This makes the output gears speed sinusoidal, turn slower during part of its rotation and faster for the rest but averaging to the same speed as the input.



Figure 5.16: Colorized CT scan of Antikythera Fragment with pin and slot overlay Source: (Freeth, et al., 2006)

The CT scans of the original show that the shaft offset was created by having a large shaft that reduced in diameter with a new centerline. To reduce manufacturing complexity, the single shaft was changed into two smaller standard sized shafts held together with a fixed eccentric coupling (Figure 5.17). For compactness, this subassembly was nested inside the ring of the primary driving gear for the calendar sub-assembly. This necessitated that the bearing in the calendar gear being enlarged to allow the moon's output shaft to pass through it.

I designed an inter-stage support plate to provide a mounting point for the stationary central shaft of the Antikythera. Additionally, without this plate to provide a second mounting point, several of the shafts would be have to be cantilevered to avoid interference with other gears. A set of small gears is used to offset the location of the moon sub-assembly output shaft and change its direction so that it aligns with the central shaft.



Figure 5.17: Calendar and Moon Assembly

5.9 Inner Planets

The next dials on the front face are Mercury and Venus. These two together are the inner planets. Since both of these planets are in smaller orbits of the sun than Earth their relative motion shares common characteristics. Primarily, their apparent motion is easier to model and therefore the mechanical analog in the Antikythera is simpler.

To simulate inner planet motion there are five major components: The input gear, the stationary gear, the drive gear, the pin holder, and the slider (Figure 5.18). The input gear is what is adjusted by the user drives the motion. The input gear is one of the six largest gears of the Antikythera. This size is necessary to get the ratios correct for the outer planets discussed later. For the inner planets the input gear directly carries a shaft for each of the Mercury and Venus. Passing through the input gear is a stationary shaft that supports the stationary gears. A drive gear meshes with the stationary gear and is held by the shaft in the input gear. In this configuration the rotation of the drive gears are not driven by the connected spur gear but by the rotation of the input gear supporting them. This is a similar motion to the rotation of a planet-carrier of a modern planetary gear-set. For the inner planets there is also a pin and slot mechanism but with a different function. You can imagine the location of the drive gear's shaft on the input gear as the Deferent and the pin for the slider is the Epicycle. The ratio of sizes of the Deferent and Epicycle are such that the pin cannot be pressed into the drive gear and thus requires a pin holder of the correct radius. The stationary gear provides the rotation rate of the Epicycle and the input gear drives the Deferent. Finally the slider rides the pin and simply points in the direction of the planet. This is a very direct and intuitive mechanical example of the geocentric model.



Unfortunately, the next stage up, the sun dial, needs power from the input gear. At this location in the mechanism the outermost shaft is Venus' and it covers the input gear's shaft. A raised platform connected to the input gear's perimeter ring is added to carry the rotation of the input gear up to the next assembly. This platform also serves to provide support to the cantilevered inner planet assembly adding stability.

5.10 Solar Anomaly

The next two dials on the front face are the Date and Sun (Sol) dials. Like the moon, the sun does not track perfectly with a circular model of the solar system. Therefore during the course of a year the sun will gain and lose ground on the ecliptic relative to the stars. Again this is the result of earth's orbital eccentricity. Therefore, a similar mechanism is used to accommodate the difference in the calendar date and the sun's ecliptic angle (Figure 5.19) as was used for the moon. This mechanism is attached to the underside of the upper plate of the inner planet assembly.



5.11 Outer Planets

The next three dials on the front face are the Mars, Jupiter, and Saturn. Collectively they are the outer planets. With larger orbits of the sun than Earth their relative motion again shares common characteristics. Unfortunately, they are more difficult to model than the inner planets. In addition to the Deferent and Epicycle they each have an anomaly as well. In this design, the slider is another gear mounted on common offset shaft with the epicycle gear. This gear then interfaces with another gear on the central axis to produce the sinusoidal anomaly. Additionally, the large deferent gear does not rotate once per year and therefore has to connect to the input gear through a shaft and pair of gears. Each of the three outer planets has its own stage with a nearly identical mechanism. Each stage has varying gear sizes, shaft sizes, pin locations and eccentric shaft offsets to produce the correct movement.



5.12 Fabrication

After designing a complete assembly of the Antikythera Mechanism recreation in SolidWorks we had to physically build a prototype. In addition to being a goal for the project, producing a physical mechanism helps identify issues that a 3D solid model can obscure. Achieving proper movement of clockwork mechanisms is non-trivial and requires the careful manufacturing of components and frames (Smith, 1991). Opportunities to improve the design's manufacturability were identified during fabrication that may have otherwise been missed.

There are 64 gears totaling 4215 individual teeth or pins in the design we developed. In these quantities it was prudent to develop processes to manufacture the gears efficiently. It was quickly identified that we wanted to use CNC machines as much as possible to semiautomate the machining. To simplify the manufacturing we used checklist approach to each gear with a series of standardized features. Each gear could have a specific number of teeth, center hole, mounting pattern, cutout pattern, pin location, and/or slot (Figure 5.21).



To ensure each step was accomplished, and that no gears were missed, a checklist for each step of manufacturing the gears was used. The first step of the spur-gear fabrication was to cut out the blanks out of brass sheet. To minimize waste and accomplish the task quickly we used a water jet cutter to cut the shapes (Figure 5.22). To provide a safety factor the blanks were cut slightly oversized. The next feature was drilling the center hole. At this point the blanks were round so a lathe with a fixed tail-stock was used to drill the holes while ensuring they were appropriately centered and aligned.



Next using the drilled hole as the datum to ensure centering, the blanks were mounted to a tooling plate on a Bridgeport CNC mill. To simplify the G-Code programming for the CNC mill, the number of mounting hole-patterns and cutout styles was reduced to four of each. Following the checklist, the appropriate programs were executed to drill the hole-pattern and mill the cutouts (Figure 5.23 & Figure 5.24). Next, if needed, the hole for a pressed pin was drilled or a slot milled. The last operation on the Bridgeport was to tap the holes when needed.



Figure 5.23: Spur Gears prior to machining



Figure 5.24: Hole pattern and Cutouts being added to a gear blank (left to right)

The last major operation on the gear was cutting the teeth (Figure 5.25). Since we chose to design the recreation to use a standard involute gear profile we were able to use a set of off-the-shelf gear cutters. An adaptable fixture was designed and built to hold the gears in our Haas CNC mill's 4th rotary axis (Figure 5.26). This fixture would sandwich the gear between two ground-flat steel plate rings. A dowel pin of the appropriate size was used to center the gear blank in the fixture then it was bolted to the fixture to prevent inadvertent rotation or movement during machining. We wrote a parameter-based G-Code to incrementally rotate the 4th axis and cut the teeth into the gear. The parameter-based code allowed a single generic program to be used for each gear while only changing the tool offsets and key values such as number of teeth. After cutting the teeth was complete pins or bearing were pressed into the gears.



Figure 5.25: Teeth being cut into gear blank



Fabrication of the lantern gears followed a similar checklist. First the blanks were drilled and turned appropriately on a lathe from a piece of brass rod stock. They were then transferred to the Haas CNC mill to drill the pin holes. After the holes were drilled, it was moved back to the lathe where the grooves in the side were cut. Next the set screw hole on the side was drilled and tapped using the Haas CNC. Care was taken to ensure that the set-screw hole was out-of-phase from the pin holes so that the set-screw could be tightened between the pins. The set screw was added and pins were then pressed through the holes to complete the assembly (Figure 5.27).



Due to their size being in excess of the working area of our CNC machines the frame plates were initially cut to size using a CO2 laser. The laser was a good choice because it provides an excellent surface finish and accurate cuts. After this they were mounted in the CNC mill to have the pocketed bearing holes drilled. To avoid scratching the surface during machining, plastic was used on the mounting clamps and flood coolant was used to clear away chips (Figure 5.28).



Figure 5.28: Picture of back frame being machined on Hass CNC Mill

The remaining hardware, such as the shaft collars and mounting bosses were produced using design checklists or individual drawings depending on quantity. For example only one of each of the sliders for the inner planets was needed so they were programmed and CNC machined individually. However, dozens of the shaft collars were required so fixturing was built they were drilled and tapped in sets of five. The University of Idaho's CNC Haas Lathe has a single tool post. Therefore the support columns were designed specifically to allow CNC turning with a single tool. This primarily restricted the slope of the angles. Additionally, due to the high length to diameter ratio, a live center was used to support the back-end of the column during machining.

5.13 Assembly

Care was taken during the design process to ensure that every component of the machine could be reversibly assembled in order. Using the SolidWorks assembly as a guide the mechanism was built up in stages starting at the rear face-plate (Figure 5.29). Once a stage was loosely assembled, with the gears meshed, the set screws were tightened to fix the gears to the shafts and the collars fixed in place. The next plate would be added and the assembly would progress forwards (Figure 5.30). The shafts and frames from previous stages would support the gears of the next stage as it was assembled.



Figure 5.29: Picture of first plate being assembled



5.14 Results

After the prototype assembly was completed (Figure 5.31) several issues were obvious. Firstly, the stationary shafts pressed into the acrylic plates would slip and rotate. This prevents the correct movement of the machine. In future assembly these shafts can be glued in place. Based on the assembly process fixing the stationary shafts to the plates will not prevent assembly or calibration. The next issue was crazing of the acrylic components. Shortly after machining, cracks started to form in the acrylic components. These cracks grew significantly in the days following the assembly and resulted in the inner-planet support platform breaking entirely. The cause was identified as the flood coolant on our CNC machine being chemically incompatible with the acrylic. The replacement part with the same features didn't crack at all. Since the device will be on display, I suggest the acrylic components be rebuilt without using coolant to eliminate the cracks. Lastly, the three drive shafts carrying torque from the input gear to the radius gears of the outer-planets was incorrectly positioned. On the Mars subassembly the gears impact the shaft and prevent rotation. This interference was not identified in the SolidWorks model (Figure 5.32) because it only occurs on particular dates during Mars' orbit. This could be addressed by building an offset coupling into the shaft.



Figure 5.31: Picture of completed assembly



While the first prototype is not fully functional it shows the validity of the design and requires only minor adjustments. Using my methodology, with clean room design principles, we were able to make a valid representation of a 2000 year old machine. The implementation of modern manufacturing techniques allowed a small team to build the entire mechanism in a matter of weeks when the original likely took many man-years to complete. Overall, this case study shows the flexibility of the methodology and its application in cases with restricted knowledge of the original regardless if the cause is legal or practical.

Chapter 6: Discussion and Conclusions

6.1 Summary of Results

The overall goal of using diverse case studies to examine the effectiveness of my methodology was achieved. The methodology provided a framework with which to apply engineering effort. During the course of these projects continuous progress was made throughout. Unlike the common analysis paralysis associated with engineering projects we were never stuck not knowing what to do next. The individual project goals were achieved to various levels of success. However, with them, like any project, future work could improve results further. On the hydro-turbine we methodically dissected the drawings and extracted the information required to produce 3D models then reapplied that knowledge to the new small-scale design. During the Stirling-engine project we had an extensive pool of experience of both the machine's design and manufacturing. Using this knowledge we were able to custom tailor the new design drawings to the target undergraduate audience. Lastly, while the Antikythera mechanism has puzzled and inspired for decades it is noteworthy that only a handful of non-hypothetical designs exist. A functioning recreation consists of a lot more than an analytical study of the kinematics and gear-chains. The prototype we constructed is by itself a success of design, and the minor issues are readily solvable.

6.2 **Reverse Engineering and Philosophy**

At its core, reverse engineering is attempting to "reinvent the wheel". It is a careful balance between knowledge and speculation, creativity and analysis. As with the Stirling engine case study, if enough is known about the original design then the reverse engineering process is almost trivial. However, if very little is known, such as the Antikythera mechanism, the line between reverse engineering and purely creative design is blurred. While the first category is the focus of most research in the field, I feel the middle ground is where my contribution lays. In this region you have to be guided as much by what-if as what-is. This is a philosophical approach that is often unfamiliar to many hard-science technical professionals. Due to this, a generic methodology, such as mine, that guides reasoning instead of forcing a prescriptive procedure can be of great use. I believe that this framework can be used to help resolve some of the reverse engineering problems that have previously been mysteriously difficult.

References

- Allen, N. B. (2002). *The Stirling Engine Project Fabrication and Experiments for Sophomore Laboratory*. Moscow: University of Idaho.
- ASTRO Machine Works Inc. (2018). *REVERSE ENGINEERING PARTS: THE BASICS*. Retrieved 6 9, 2018, from ASTRO Machine Works Inc: https://astromachineworks.com/services/reverse-engineering/
- Bailey, C. M. (2018). MANUFACTURING COMPLEX SURFACES TO RECREATE THE DESIGN INTENT OF LEGACY ARTIFACTS. Moscow: University of Idaho.
- Bereau of Economic Analysis. (2016). *National Datea: Residential Fixed Assets*. U.S. Department of Commerce.
- Buckingham, E. (1988). *Analytical Mechanics of Gears*. Mineola N.Y.: Dover Publications, Inc.
- Buonamici, F., & Carfagni, M. (2016). Reverse Engineering of Mechanical Parts: A Brief Overview of Existing Approaches and Possible New Strategies. *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference.*
- ETERNOO Machinery Co., LTD. (2018). *Francis turbines*. Retrieved from ETERNOO Machinery Co., LTD the expert in Hydropower: http://www.eternoohydro.com/turbines/francis-turbines.html
- Freeth, T., Bitsakis, Y., Moussas, X., Seiradakis, J. H., Tselikas, A., Mangou, H., et al. (2006). Decoding the Antikythera Mechanism: Investigation of an Ancient. *Nature*, 444, 587-591.
- Freeth, T., Jones, A., Steele, J. M., & Bitsakis, Y. (2008). Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism. *Nature*, 454, 614 617.
- Groundspeak, Inc. (2018). Noxon Rapids Dam Noxon, MT. Retrieved from Waymarking.com : http://www.waymarking.com/gallery/image.aspx?f=1&guid=77f7ec47-2c71-436aab26-62dc02b4f8c2
- Jet Propulsion Laboratory California Institute of Technology. (2018). *HORIZONS Web-Interface*. Retrieved from Jet Propulsion Laboratory California Institute of Technology: https://ssd.jpl.nasa.gov/horizons.cgi
- Jian-Liang Lin, H.-S. Y. (2016). *Decoding the Mechanisms of Antikythera Astronomical Device*. Springer Science + Business Media.

Johnson, M. (1949). Art and Scientific Thought. New York: Columbia University Press.

- Kolko, J. (2010). Abductive Thinking and Sensemaking: The Drivers of Design Synthesis. *Design Issues*, 26(1), 15-28.
- Otto, K. N., & Wood, K. L. (1998). Product Evolution: A reverse Engineering And Redesign Methodology. *Research in Engineering Design*, 226-243.
- Pearson Education. (2014). Statistical Process Control. (pp. S6-12). Prentice Hall.
- Rakić'. (2018). *Renaissance violin*. Retrieved from Rakić' Violins: http://www.violinsrakic.co.rs/htm/eng/renaissance.html#a
- Schroeder, D. V. (2011). Astronomy Before Copernicus. Retrieved from Weber State University Department of Physics: http://physics.weber.edu/schroeder/ua/BeforeCopernicus.html
- Skinner Inc. (2018). *Modern Violin*. Retrieved from Skinner Auctioneers and Appraisers: https://www.skinnerinc.com/auctions/2595B/lots/85
- Smith, W. R. (1991). *Clockmaking and Modelmaking Tools and Techniques*. Powell, Tennessee.
- Tandler, W. (2008). The GD&T Encoding Process—Final Steps. Quality Digest, 1-3.
- The Antikythera Mechanism Research Project. (2018). *The Antikythera Mechanism Research Project*. Retrieved from http://antikythera-mechanism.gr/
- The Metropolitan Museum of Art. (2018). "*The Francesca*" Violin. Retrieved from The Metropolitan Museum of Art: https://www.metmuseum.org/toah/works-of-art/34.86.2/
- Wagstaff Inc. (2017). Noxon Rapids Hydro Turbine Drawing Package. Spokane.
- Walton, V. (1987). *Economic Lives of Machinery and Equipment*. Sacramento: California State Board of Equalization.
- Yan, H.-S., & Lin, J.-L. (2013). Reconstruction synthesis of the lost interior mechanism for the solar anomaly motion of the Antikythera mechanism. *Mechanism and Machine Theory*, 70, 354-371.
Appendix A: Noxon Rapids Reference Drawings

The following is a selection of the drawings provided by Wagstaff Inc. These drawings were used in the digital recreation of the Noxon rapids hydro turbines.

Summary of Drawings Selected	
Drawing Number	Description
02-502-455	Stay Ring: Sections & Details
02-502-386	Head Cover
02-502-387	Head Cover: Sections
02-502-388	Head Cover: Sections
02-502-437	Spiral Case
02-502-361	Runner
02-502-362	Runner: Sections & Details
02-502-363	Runner: Sections & Details
02-301-992	Upper Wear Ring
02-301-994	Runner Shrink Band
02-401-813	Runner Cover
02-401-859	Runner Cone































Appendix B: Noxon Rapids Recreated Drawings

The following is the drawings produced from the 3D model of the Noxon rapids turbine (produced by the "Intolerables" senior design team).

Summary of Drawings	
Description	
Full Assembly	
Exploded View	
Stay Rings & Vanes	
Runner Section	
Runner Section	
Head Cover	













Appendix C: 2016 Stirling Engine Drawings

The following appendix is the old (2016) drawings from the Stirling engine case study. The original (2002) drawings can be found in Nathaniel Allen's thesis: "The Stirling Engine Project - Fabrication and Experiments for Sophomore Laboratory"

Summary of Drawings	
Description	
Base Plate	
Bearing Plate	
Bearing Plate	
Bent Connector Lever	
Connector Lever	
Connector Link	
Crank Web	
Cylinder Plate	
Flywheel Hub	
Flywheel Shaft	
Gudgeon Block	
Lever Shaft	
Power Connector Lever	
Power Cylinder	
Power Piston	
Displacement Piston	
Displacement Piston Cap	
Displacement Piston Front Cap	
Displacement Piston Rod	
Heat Exchanger	
Guide Bushing	
Displacer Cylinder	
Displacer Cylinder Plug	
Steel Flywheel	

NOTE: Scale no longer as indicated on drawings.
















































Appendix D: Updated Stirling Engine Drawings

The following is the revised new drawings from the Stirling Engine case study.

Summary of Drawings						
Drawing Number	Description					
AA-01	Full Assembly					
SA-A	Heat Exchanger Assembly					
A-01	Heat Exchanger					
A-02	Displacer Cylinder					
A-03	Displacer Cylinder Plug					
SA-B	Displacer Assembly					
B-01	Displacement Piston					
B-02	Displacement Piston Cap, Rear					
B-03	Displacement Piston Cap, Front					
B-04	Displacement Piston Rod					
SA-C	Piston Assembly					
C-01	Gudgeon Block					
C-02	Power Piston					
C-03	Power Connector Lever					
SA-D	Bulkhead Assembly					
D-01	Cylinder Plate					
D-02	Guide Bushing					
D-03	Power Cylinder					
SA-E	Bearing Assembly					
E-01	Bearing Plate, Left					
E-02	Bearing Plate, Right					
E-03	Bearing Place Spacer					
SA-F	Timing Linkage Assembly					
F-01	Connector Lever					
F-02	Lever Shaft					
F-03	S Linkage					
F-04	Connector Link					
SA-G	Flywheel Assembly					
G-01	Flywheel					
G-02	Flywheel Hub					
G-03	Flywheel Shaft					
SA-H	Crankweb Assembly					
H-01	Crankweb					
SA-I	Base Assembly					
I-01	Base Plate					
J-01	Burner Cap					

NOTE: Scale no longer as indicated on drawings.













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Appendix E: Antikythera Math Model Programs

The following is the MATLAB codes for the geocentric math model of planetary motions. These codes are used to find the coefficients that best fit the model to the observed data, perform a FFT analysis on the harmonic component, and produce a series of plots.

```
%% This program will convert the data from ecliptic angles to a single
continuous function and perform FFT analysis
clc
clear all
%% Import data from Excel should be Date(n) and Angle (n) of equal lengths
Name='Jupiter'
RawDate=xlsread('Thesis Data.xlsx',Name,'B:B');
RawAngle=xlsread('Thesis Data.xlsx',Name,'D:D');
%% Trim and adjust length of dataset
totaldays=27759; %one Callippic Cycle
everynumdays=14; %reduce data set to one point every X days
for i=1:totaldays/everynumdays
Date(i) = RawDate(1+(i-1) * everynumdays);
Angle(i) = RawAngle(1+(i-1)*everynumdays);
end
%% Combine data into continuous function (since data of angles reset at
360°)
cyclenum = 0
for i=1:(length(Date)-1)
   x(i) = Date(i) - Date(1);
   y(i) = (360 \times cyclenum) + (Angle(i));
   if (Angle(i+1)) - (Angle(i)) <-100
   cyclenum=cyclenum+1;
   end
   if (Angle(i+1)) - (Angle(i)) >100
   cyclenum=cyclenum-1;
   end
end
% figure
% plot(x,y) % plot the continuous function
y r=y*(pi()/180);
%% linearize data and plot
for i=1:length(y) % remove starting offset
    y2(i) = (y(i) - y(1));
end
%plot(x,y2)
[pks,pkloc] = findpeaks(y2)% find the peaks of the function
lastpk = pkloc(length(pkloc)) % index location of last peak
firstpk = pkloc(1) % index location of first peak
```

```
slope=(y2(lastpk)-y2(firstpk))/(lastpk-firstpk) %slope of line between
peaks
for i=1:length(y) % remove linear component (assuming matched phases at
peaks)
     y3(i)=y2(i)-(slope)*i;
end
figure % plot the linearized function (harmonic component)
plot(x,y3)
startdays=0;
stopdays=365*10;
xlim([startdays stopdays])
titlename=strcat('Harmonic component of', {' '}, Name);
title(titlename);
xlabel('Time (Days)')
grid on
ylabel('Amplitude (Degrees)')
filename=strcat('Harmonic', Name, '.png');
print('-dpng', filename);
%% Perform FFT analysis on harmonic component
% make data even (required)
if mod(length(y3), 2) == 0
v4=v3;
else
y4=y3(1:length(y3)-1);
end
N=length(y4)
X hat=fft(y4);
X=X hat(1:(N/2)+1); %keep only unique values
fs=(365/everynumdays); %sample freq
fn=fs/2; %nyquest freq
f=linspace(0, fn, (N/2)+1); %create frequency bins
for i1=1:length(f)
    period(i1)=1/f(i1);
end
% find amplitudes
A(1)=(1/N)*real(X(1)); % take real part to ensure no roundoff in imag part
A(2:(N/2)) = (2/N) * abs(X(2:(N/2)));
A((N/2)+1) = (1/N) * real(X((N/2)+1));
% find phases
phi(1)=0;
phi(1:(N/2)+1)=angle(X(1:(N/2)+1));
phi((N/2)+1)=0;
phi deg=phi*180/pi;
% Visualization of FFT
% Plot amplitude and phase diagrams
figure
subplot(2,1,1); bar((f),A,'k');
axis([0,2,0,inf])
titlename=strcat('Amplitude and Phase Spectrum of', {' '}, Name, '
(Linearized)');
```

```
title(titlename);
ylabel('Amplitude');
% set(gca,'Xdir','reverse','Xscale','log')
subplot(2,1,2); bar((f),(180/pi)*phi,'k');
axis([0,2,0,inf])
ylabel('Phase (Degrees)');
xlabel('Frequency (cycles/year)');
filename=strcat('FFT',Name,'.png');
print('-dpng',filename);
```

```
%% This program will optimize the epicycle ratios to predict the position
of a major planet
% Governing equation
8
theta=((((speed1*(pi/180))/size1)*time)+phase1)+(((atan(size2/size1))*(180/
pi))*(sin(((speed2/size2)*time)+phase2)*(180/pi)))
% Symbolic form
% theta r=((v 1/r 1)*t)+p 1+atan(r 2/r 1)*((sin(v 2/r 2)*t)+p 2)
% Simplified form
% theta r alt=(((C1)*t)+p1)+(C2*(sin(((C3)*t)+p2)));
% setup
t=x;
syms C1 C2 C3 p1 p2 t real
% Initial Guesses for optimization
C 1=0.3667;C 2=0.2743;C 3=0.0150;p 1=.44;p 2=4;
guess alt=[C 1;C 2;C 3;p 1;p 2]; %initial guesses
calibration alt=[C1;C2;C3;p1;p2]; %calibration variables
%% Create functions
theta r alt=(((C1)*t)+p1)+(C2*(sin(((C3)*t)+p2)));
matlabFunction(theta r alt, 'File', 'F theta r alt', 'vars', {[C1, C2, C3, p1,
p2, t]});
% import data for x and y using earlier program
% produce a matrix of errors abs(guess-measured)
for n=1:length(x)
error(n)=abs((F theta r alt([calibration alt',x(n)]))-y r(n));
end
% RSS of errors (square term-by-term, sum, root)
cost alt=[t]*0; %initialize cost vector (symbolic matrix)
for m=1:length(error)
    cost alt=cost alt+((error(m))^2);
end
cost alt=cost alt^.5;
% create cost function (scalar)
```

matlabFunction(cost alt,'File','cost0 alt','vars',{[C1, C2, C3, p1, p2]'});

```
costO alt(guess alt) %used for testing evaluate the initial cost (at the
quess)
%% Optimize using cost function and calibration variables
options3=optimoptions('fminunc','Display','iter') %setup optimization
opt3=fminunc(@cost0 alt, quess alt, options3) %run optimization
cost0 alt(opt3) %final cost value for comparison
%% This program will plot the optimized data and show the error
for i=1:length(x)
y cal alt r(i)=F theta r alt([opt3',x(i)]); %radians
y cal alt deg(i)=(F theta r alt([opt3',x(i)]))*180/pi; %degrees
end
startdays=0;
stopdays=365*15;
figure
hold on
plot(x,y,'-c','LineWidth',4)
plot(x,y cal alt deg,':k','LineWidth',2)
xlim([startdays stopdays])
titlename=strcat('lst-order Geocentric Model of', {' '}, Name);
title(titlename);
legend('Observed Data', 'Optimized Model')
xlabel('Time (Days)')
ylabel('Ecliptic angle (Degrees)')
grid on
filename=strcat('Optimized', Name, '.png');
print('-dpng', filename);
hold off
for n=1:length(x)
error r(n)=F theta r alt([opt3',x(n)])-y r(n); %error in radians
error deg(n)=(y cal alt deg(n)-y(n)); %error in degrees
end
8
figure
plot(x, error deg, '-k')
titlename=strcat('1st-order Geocentric Model Error of', {' '}, Name);
title(titlename);
xlabel('Time (Days)')
grid on
ylabel('Error (Degrees)')
xlim([startdays stopdays])
filename=strcat('Error', Name, '.png');
print('-dpng', filename);
```

Appendix F: Antikythera Drawings

The following is the drawings of the prototype Antikythera Mechanism.

NOTE: Scale no longer as indicated on drawings.

Summary of Drawings
Description
Full Assembly
Back Plate
Back Plate 2
Interstage
Sun Support
Middle Plates
Front Plate
Planet Dials
Lantern Gears
3A-E Spur Gears
3A-F Spur Gears
3A-G Spur Gears
3A-H Spur Gears
3A-NA Spur Gears
3BT-F Spur Gears
NA-H Spur Gears
#50 Ring Gear
Spur Gear Boss
Lunar Shaft Offset
Sun Shaft Offset
Mars Shaft Offset
Jupiter Shaft Offset
Saturn Shaft Offset
25B Pin Holder
26B Pin Holder
Pin Slot
2-5 Shaft Support
2-5 Bearing Ring
608 Shaft Support
608 Bearing Ring
Shaft Collar

Number of Teeth (A) (B) (B)	48 0.126	50 0.126	50 0.157	50 0.313	53 0.126	53 0.126	53 0.126	54 0.126	54 0.251	55 0.313	57 0.126	57 0.126	60 0.126	60 0.126	60 0.126	60 0.126	60 0.219	61 0.188	64 0.126	64 0.157		Antikythera Mechanism		UNIVERSITY OF IDAHO	N/A Various	SCALE 2:1 SHEEF: 1 OF 1
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