Does an Observer's Content Knowledge Influence the Feedback Offered About Mathematics Lessons?

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AUTHORIZATION TO SUBMIT DISSERTATION

This dissertation of Duane C. Peck, submitted for the degree of Doctorate of Philosophy with a Major in Education and titled "**Does an Observer's Content Knowledge Influence the Feedback Offered About Mathematics Lessons?**" has been reviewed in final form. Permission, as indicated by the signatures and dates below, is now granted to submit final copies to the College of Graduate Studies for approval.

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ABSTRACT

The purpose of this study was two-fold. First, feedback from 3 different groups of observers, mathematics content specialists, content specialists in areas other than mathematics, and building principals, was analyzed using an inductive approach to identify themes within the feedback. Second, differences in the feedback offered by participants of the 3 groups were analyzed to determine whether there is a relationship between content knowledge and the feedback offered following an observation of mathematics instruction.

The analysis first identified six forms of feedback, three of which were used for further analysis. These three forms were queries, recommendations, and value statements. Further analysis of these three feedback forms revealed six feedback focus themes which described the content of the feedback given. These focus themes were analyzed qualitatively and quantitatively to identify differences in the quantity and focus of the feedback given by each of the three groups. The results identified a significant difference in the focus of feedback offered by mathematics content specialists as compared to the other study participants without a specific mathematics content background in the focus areas of conceptual understanding and connections, and mathematics content, suggesting there is a relationship between the content knowledge of an observer and the focus of feedback they offer following a mathematics instructional segment.

The importance of the relationship between content knowledge and observation feedback was then discussed within the context of current recommended mathematics instructional practices and current practices of supervision of instruction. The observer's level of content knowledge plays a role in their ability to provide feedback with the potential to improve the conceptual nature of a lesson, as well as to identify strengths and weaknesses in the mathematical content at the center of the instructional segment. The study concludes with recommendations for improving feedback following mathematics instruction as well as suggestions for further feedback research.

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DEDICATION

To the lights of my life, my dear wife and companion Suzanne,
and the three most amazing daughters in the world,
Adell, Malia, and Lauren.

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CHAPTER 1: INTRODUCTION

How does the content knowledge of a supervisor impact the feedback given to a teacher following an observation of an instructional segment? This question has intrigued me for most of my professional career as a teacher, an administrator, a supervisor of student teachers, a researcher, and as an instructional coach of mathematics teachers. This study is part of my own personal journey towards answering this question as well as learning how possible answers might impact my own coaching and evaluatory practices. There are two guiding educational principles for me as I approach this topic. The first principle is to help students maximize their potential to understand, reason about, and communicate using mathematics. Second, I believe we can best help students maximize their potential by helping those who teach these students to understand, reason about, and communicate using mathematics themselves, and then take these skills and knowledge and effectively apply them in their own teaching practices.

My personal journey began as a teacher in a large middle school in the western United States. I taught both mathematics and United States history to approximately 150 seventh and eighth grade students. The school in which I taught had three administrators with different backgrounds and teaching styles. As teachers we were each assigned to one of the three administrators for evaluation. Although the district had a standard evaluation form, I soon learned that each administrator had a unique approach to using the form, and looked for different instructional practices.

I was assigned to one of the two assistant principals during my first year of teaching. My evaluator was a dynamic and passionate history teacher. It was not a surprise that all of his observations took place during my history classes. Although I do not remember the exact content of our discussion following my first observation, I do clearly remember how I felt. My evaluator was quick to point out some of the strong areas of my practice. However, for each of these points he also identified specific weaknesses he saw. I understood that he wanted to help me become the best teacher I could be; however, each time I heard a positive comment followed by the word “but,” my heart sank a little lower, and I soon felt overwhelmed.

After years of practice I have come to realize this feeling is not uncommon among first year teachers. It bothered me enough however that I approached the building principal to help me gain some perspective. I explained how I was feeling, and he nodded his head in a knowing kind of way. It was at this point that he set me on the path of this dissertation. He suggested to me that since my evaluator taught history as well, he was likely to be much more critical and specific in regards to the feedback he gave me. His passion and knowledge for the subject would surface in the amount and types of feedback he would give. While he might be very general in a mathematics setting, he would be much more focused in a history setting.

This idea that content knowledge and focus could have an impact on feedback given during an evaluation session caused me some concern and reflection. My principal helped me to understand that such was the nature of observation and evaluation, and that I would need to learn to live with and make the most of those evaluation sessions if I were to be successful in education. I soon wondered if other teachers had had similar experiences. I asked my mentor teachers if they had had similar experiences. Both assured me that they had, and that they too felt more comfortable having certain administrators as evaluators.

Several years later my family and I found ourselves in another state and school district. I had grown up in this district, and because it was small and rural I knew most of the staff, students, and families in the area. I taught at the junior high and high school, teaching mathematics, science, history, and technology. There was one administrator over the high school and one administrator over the middle school. They split the evaluation responsibilities of both schools. I was soon faced with a stark contrast. One of the principals had taught for less than two years, none of which were in a mathematics class. However, mathematics was the class he chose to observe. While I appreciated that he provided some useful instructional advice, I found his content suggestions unhelpful at best. Without going into detail, I approached a few of the senior faculty members for advice on how to interpret and apply the feedback. Their responses added to my growing concern about the feedback I received, and I wondered if evaluation would help me be a better teacher, or whether it was just a formality that had to be observed.

The next year I was evaluated by the other principal. Both principals used the same form, but looked for very different instructional practices. Their feedback was very different, and the situation left me wondering how evaluations could actually impact instructional practice in a continuous and positive manner if they were not consistent. To add to my growing concern, I began a Masters program in educational administration during this time. Supervision of instruction was one of the key elements of the content and curriculum of this program (Ubben & Hughes, 1997). Part of the program involved participation in an internship program in which I conducted observations and provided feedback to the teachers I observed. I soon found that: 1) I was giving different feedback than either of the two principals, and that 2) my own comfort in providing feedback increased as I observed content similar to my own specialty areas. While I did find many basic instructional principles were the same across all of the content areas I observed, I was able to interact with the content in the mathematics and history classrooms more deeply than some of the other content areas.

During these same years my thoughts about feedback continued to develop. I found myself very frustrated with two polar issues. First, I had one large mathematics class, in which about one-third of my students were on individualized education plans (IEP) and receiving special services. The numbers of these students were high enough that I had two aids in class for the specific purpose of helping just the students on IEPs. The aids and I spent hours in collaboration trying to find ways to help those students. We asked colleagues and administrators and did not seem to be able to come to any solid conclusions about what would help these students the most. All of these students showed growth in their test scores and had some success with mathematics, but as a team we were still concerned that many of the students lacked the necessary depth of knowledge and confidence in their mathematics skills to succeed in the mathematics courses they would encounter in high school.

I experienced the second issue while teaching a trigonometry class. Although nearly all of my students had a history of very good grades in mathematics, they were seriously challenged by the idea that there is often more than one way to approach problem solving, especially when working with trigonometric identities. Many of my students became frustrated that I wasn’t giving them a step by step algorithm or process to solve problems like they had experienced in other mathematics courses. I tried to help them understand that there wasn’t an algorithm for everything. I could give them characteristics to look for, but in the end, they would need to explore and try some techniques on their own to identify connections and relationships. If one relationship was not useful, they might need to try another.

The feedback I received from my principal was that I should teach the students the procedures they would need and stop trying to help them understand why the procedures worked. While this may have seemed like a logical suggestion to him based on his own experience and content knowledge, his lack of a mathematics background led him to provide feedback that was unhelpful at best. I found myself in a situation where the feedback I was given was in direct contradiction to the nature of the mathematics I was being asked to teach. I was left to question how I could possibly meet such diametrically opposed mandates while maintaining the integrity of the subject as well as the focus on helping students develop a deep understanding of trigonometry and its applications.

In both of these cases the feedback I received seemed to encourage teaching students procedures, having them practice these, and to discourage teaching the "why" of mathematics. The feedback also encouraged teaching via direct instruction and teaching students only what they needed to know to get a good grade on procedural tests. Needless to say my frustration with my inability to reach all of the students was matched only by my frustration with feedback that didn't seem to help me improve my own instructional practice. It would be a number of years before I would understand why these instructional suggestions and practices did not yield the results I was looking for.

My Introduction to Mathematics Reform

My personal journey in mathematics education took a decided turn in 2008 when I was hired to work with the Initiative for Developing Mathematical Thinking (IDMT). During the first few weeks of my work with the IDMT I discovered one of the deficiencies that led to many of the instructional issues that had always frustrated me as a teacher of mathematics, namely the overall lack of American adult mathematical understanding. Ball, Hill, and Bass (2005) eloquently described the circular issue of mathematics instruction in the United States:

> That the quality of mathematics teaching depends on teachers' knowledge of the content should not be a surprise. Equally unsurprising is that many U.S. teachers lack sound mathematical understanding and skill. This is to be expected because most teachers – like most other adults in this country – are graduates of the very system that we seek to improve. Their own opportunities to learn mathematics have been uneven, and often inadequate, just like those of their non-teaching peers. Studies over the past 15 years consistently reveal that the mathematical knowledge of many teachers is dismayingly thin. Invisible in this research, however, is the fact that the mathematical knowledge of most adult Americans is as weak, and often weaker. We are simply failing to reach reasonable standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers. This is a big problem, and a challenge to our desire to improve. (p. 14)

Over the next six years I was exposed to instructional practices and ideas that transformed my own thinking about 1) how mathematics should be taught, and 2) how these ideas and instructional practices might be used to help teachers make changes in their own personal knowledge and understanding of mathematics, as well as in their individual instructional practices.

Several of these ideas had their roots in Realistic Mathematics Education (RME) as proposed by the Dutch mathematician Hans Freudenthal in the 1970s. According to Freudenthal (1991), mathematical understanding begins with "common sense," another way of describing a student's own informal reasoning, or thinking about a meaningful context or situation. Given such a meaningful context, students would be given the opportunity to connect their own prior knowledge to new concepts, representations, and skills (Webb, Van der Kooij, & Geist, 2011). As students begin to make connections they would be introduced to mathematical models in a process of guided reinvention that would lead them to produce a higher level of understanding, a process known as mathematizing (Freudenthal, 1991). Gravemeijer and van Galen (2003) explained that guided reinvention involves a process by which students develop and use informal strategies as they solve problems. Students are then given the opportunity to compare and contrast those strategies leading them to develop more formal and efficient strategies and algorithms.

Treffers (1987) further explored the concept of mathematizing, and distinguished between two types, horizontal and vertical. Horizontal mathematizing describes the process in which students utilize mathematical tools in an effort to make sense of contextual situations, including those found in real-life settings. At this level, the focus is on solving the problem itself rather than taking the mathematics to a higher level. Rasmussen, Zandieh, King, and Teppo (2005) explained that horizontal mathematizing “refers to formulating a problem situation in such a way that it is amenable to further mathematical analysis” (p. 54). Vertical mathematizing describes the reorganization of the process within the system of mathematics. Brendefur, Thiede, Strother, Bunning, and Peck (2013), explained that vertical mathematizing involves

> taking the mathematical matter to a higher level, and is evident when students no longer adhere to the isolated attributes of specific problem solving contexts but instead view their solution processes and representations as objects of mathematical

> examination – a process of reification that places the focus not on the problem at hand but mathematics in general. (p. 65)

Examples of this reorganization might include the discovery of connections within the system of mathematics, as well as finding simplifications that might lead to computational or algorithmic shortcuts. The ideas of guided reinvention and mathematizing resonated with me as an instructor of mathematics. I was intrigued by the thought that students come to teachers not as blank slates, but as people with experiences and ideas of their own, and that teachers can use these ideas and experiences to guide students to make connections with the meaning of mathematics within the context of their own lives.

However, as guided reinvention would suggest, the process is more than taking students through the development of symbols and algorithms. It is about providing meaning and understanding along the way in a manner that the learner can make sense of within the context of their own life and experience. For example, young learners do not have the experience with symbols that adults have. Children primarily have experience with the concrete, informal world around them. The didactical construct of progressive formalization is at the heart of RME (Webb et al., 2011), and describes the process in which initial "instructional sequences are conceived as 'learning lines' in which problem contexts are used as starting points to elicit students' informal reasoning" (p. 48). Students are then guided through increasingly formal strategies and visual models in order to understand and make sense of more abstract, symbolic representations and processes that relate to the current topic.

Bruner (1964) described three such modes of representations – namely enactive, iconic, and symbolic. The enactive mode of representation centers on acting out the situation in some manner such as counting out objects or dividing a cake. The iconic mode of representation focuses more on mental models, diagrams, or other visual representations. The symbolic mode of representation utilizes symbols such as numbers, letters, operational and grouping symbols, or other notation devices that support efficiency, but which require an understanding of meaning in order to accurately reflect the nature of the mathematical communication being addressed. For example, a student might physically cut an apple in half (enactive mode) and refer to one of the two pieces as "one-half" of the apple. This process could then be repeated by drawing a circle to represent the apple (iconic mode), which the student would then partition in two equal pieces, again referring to one of the pieces as "one-half" of the apple. Finally, a student may write "½" on a piece of paper (symbolic mode) and name the symbol as "one-half" of the apple.

I quickly came to the realization that mathematics instruction was much more than teaching students how to solve a problem using a set of steps as I had been taught. This procedural instruction helped me and my students understand how to solve a particular problem. It did not help us understand why such a procedure worked or when a particular procedure should be applied.

My views on mathematics instruction expanded once again as I was exposed to the notion of different kinds of knowledge associated with mathematics instruction. Schulman (1986) described two such types of knowledge, content knowledge and pedagogical content knowledge. Content knowledge refers to the "amount and organization of knowledge per se in the mind of the teacher" (p. 9) as well as to "understanding the structures of the subject matter" (p. 9). Pedagogical content knowledge refers to an understanding of "ways of representing and formulating the subject that make it comprehensible to others" (p. 9). This second type of knowledge is critical in helping teachers identify difficult components of the content, why that component is difficult, an understanding of the misconceptions students may hold in regards to what is being taught, and the formulation of strategies useful in overcoming the difficulties and misconceptions related to that specific content component.

Since Schulman’s work in 1986, other authors have refined and expanded the study of knowledge for the teaching of mathematics. Ball and her colleagues conducted much of the early work in refining ideas related to the importance of both content knowledge and pedagogical content knowledge in teaching mathematics for understanding (Ball, 1990; Ball & Bass, 2000). Their work, in addition to the work of others such as Thompson and Thompson (1996), led to the definition and further study of mathematical knowledge for teaching (MKT). Beginning in the early 1990s, Ball and her colleagues (Ball, 1993; Ball et al., 2005; Ball & McDiarmid, 1990) addressed the question “what do teachers do in teaching mathematics, and in what ways does what they do demand mathematical reasoning, insight, understanding, and skill?” (Ball et al., 2005, p. 17). Silverman and Thompson (2008) extended this work by developing a potential framework for identifying a teacher’s MKT, and using that identification to further develop a teacher’s MKT. The identification of aspects of MKT has led to additional research on the implications of this type of knowledge and how the possession and use of MKT may relate to the improvement of instructional practices and student achievement. Brendefur et al. (2013) found that targeted professional development could improve teachers’ content knowledge as well as help them improve their instructional practice by focusing on teaching for understanding. Ball et al. (2005) developed an instrument for identifying elements of MKT, and found a positive relationship between MKT and student achievement.

My Personal Transformation

My own thinking about mathematics transformed and regenerated as I began to practice guided reinvention and mathematizing for myself. I began to identify and make connections between concepts and processes that I had previously understood as a set of steps I had been taught to follow. As I formed more and more connections, I began to identify misconceptions I had been taught and even passed on to my own students. For example, when following the steps of the subtraction algorithm I had been taught that "borrowing" was required because a large number cannot be taken from a smaller number. This misconception created a few difficulties for me as I was introduced to the concept of negative numbers. I have since identified a number of misconceptions relating to procedural instruction that we teach students in order to justify the procedure itself.

Although the misconceptions I had were problematic at best, perhaps the greater limitations came in my lack of flexible thinking. For example, previous to my employment with the IDMT I would have solved 1007 – 998 by using the traditional algorithm without giving a thought to why I used that method, or if it was the best way. To me it was the only way. Now looking at the numbers it is easy to see that there is a difference of 9 between the two. It does not require a particular process. I can count up, count back, or simply compensate by adding 2 to both values, or by adding ten and subtracting 1.

Comprehending the role of connections in understanding and using mathematics led me to another shift in thinking – about the roles of both teacher and student. Hiebert et al. (1997) explained that a teacher is responsible for selecting tasks based on mathematical goals, sharing essential information, and establishing an effective classroom culture. Students are responsible to actively engage in solving tasks, choose and share their methods for doing so, construct meaning for tools, as well as communicating with others about mathematical ideas. The traditional world of teacher-dominated mathematics instruction fell away and was replaced by a vision of a collaborative community of learners where knowledge and ideas are valued, shared, critiqued, and built on over time. This was the type of learning environment I had craved as a teacher, but did not know how to build. Now I have a chance to share this vision of mathematics education with others and help them find a path to its implementation.

My Goal – Transforming Mathematics Instruction

Since 2008 I have been able to share what I have learned with K-8 teachers of mathematics as a researcher and instructional coach. I have also had the opportunity to work with preservice teachers as an instructor of elementary mathematics methods courses and as a university liaison for student teachers. Most of my time has been spent in elementary and middle school classrooms working to support the improvement of mathematics instruction with the goal of increasing student achievement. As part of my job I have spent many hours observing mathematics instruction and providing specific feedback to teachers on how to strengthen their own instruction in mathematics.

During this time I have noticed several patterns in teacher behavior and knowledge. Some teachers seem poised to focus on teaching for understanding almost immediately, and engage in study and discussion with colleagues readily to find what works best for them and their students. Other teachers want to increase their instructional effectiveness, but find their own lack of content knowledge, MKT, or a fear of change a hindrance to the process. These teachers require time, support, and patience in their instructional and content development. A third group of teachers lack the knowledge of mathematics, the knowledge of classroom management, and/or the aptitude to be a successful teacher. Fortunately, this group has been very small in my experience. A final group of teachers causes me the most concern, and may be referred to as resistant. Although a minority, teachers in this group resist most change and may exhibit characteristics such as: preparing special lessons for observations that are not consistent with their normal instructional pattern, undermining the efforts of colleagues to make instructional changes, isolating themselves from their instructional teams to avoid collaboration, and/or continuing to teach mathematical content as they always have regardless of their knowledge of best practices or the consequences of ineffective practices for students in their class.

Herein was the dilemma I now faced as a mathematics coach and instructional researcher. As can be seen from the discussion thus far, much of the research has focused on the mathematical content knowledge that teachers need for teaching. However, who would be responsible to identify the specific mathematical knowledge for teaching needed? Who could provide or make available the professional development and support needed for teachers of mathematics to develop deep knowledge of these ideas? Who would ensure the knowledge and skills gained from the professional development were implemented in a way that would positively impact student achievement, thereby breaking the cycle of weak mathematical knowledge described by Ball and her associates? While specialists and instructional coaches could help identify weaknesses and provide professional development and support, it became clear to me that in American schools, the building principal is the focal point for instructional improvement. The building principal, or his or her designee, has the authority to evaluate teachers and support ongoing teacher plans for instructional improvement.

I had now come full circle in my own thoughts and questions regarding supervision of instruction. Researchers have been focused on what teachers need to know, but what specialized knowledge, if any, do principals and other observers and evaluators of mathematics instruction need in order to help facilitate the instructional changes suggested by so many researchers today? Does the principal's content knowledge make a difference? Can an observation instrument focused on conceptual teaching mitigate the need for specialized content knowledge? Do observers without specialized content knowledge adequately identify misconceptions, structural connections, as well as appropriate instructional strategies to support students as they navigate the mathematical landscape?

Research Questions

Questions such as these have led to the focus and specific research questions to be addressed by this study.

1) What themes emerge from feedback given to teachers of mathematics by observers with different content and focus backgrounds?
2) Does an observer's educational background influence the content of feedback that is given?

CHAPTER 2: LITERATURE REVIEW

There is evidence that a school principal plays a key influential role when it comes to teaching and learning (Leithwood, Louis, Anderson, & Wahlstrom, 2004; Schoen, 2010). Ubben and Hughes (1997) explained that one of the critical responsibilities of an instructional leader, such as a school principal, is to provide feedback to teachers as a result of formal observations in order to evaluate their instructional performance and to help them set goals for future instructional growth. The purpose of this chapter is to review literature related to feedback and its attributes to set the stage for this study. First, feedback is defined and research findings related to effective feedback attributes are identified and discussed. Second, the case for using mathematics as a specific content focus for feedback is presented. Finally, the gap in research that is the foundation of this study is identified.

Definition of Feedback

Adler and Towne (1990) defined feedback and its role in the communication process as “the discernible response of a receiver to a sender’s message” (p. 12). The authors pointed out such responses may be verbal, nonverbal, or even written. Some examples of verbal feedback include responses in a normal conversation, requests for opinions, and responses to existing situations. Examples of nonverbal feedback include facial responses such as smiles or grimaces, blushing, yawning during a conversation, or even lack of attention. Written feedback can include such common activities as responding in writing to a friend or answering questions in written forms on exams (Adler & Towne, 1990). As described, feedback is a natural component of the communication process.

Within education there are a number of areas where feedback is provided. Some of these include teacher observations and evaluations, peer coaching, and teachers providing comments on student work and progress. Hattie and Timperley (2007) used the existence of a variety of feedback sources in their definition of feedback: "information provided by an agent regarding aspects of one's performance or understanding" (p. 81). The agent in this definition could be a parent or teacher, a book, a peer, or even a principal, and the actual feedback "is a 'consequence' of performance" (p. 81).

Hattie and Timperley's (2007) definition of feedback was used for the purposes of this study with the understanding that "aspects of one's performance or understanding" consists of four elements as described by Thurlings, Vermeulen, Kreijns, Bastiaens, and Stijnen (2012)

1) data on the actual performance of the learners
2) data on the standard of the performance
3) a mechanism for comparing the actual performance and the standard performance
4) a mechanism that can be used to close the gap between the actual and standard performance. (p. 194-195)

Teachers, preservice teachers, and students are at various times all involved in both teaching and learning. There are times when students may play the role of teacher as they present and discuss their ideas with each other. There are times when a teacher may play the role of a learner as they participate in professional development and other professional growth opportunities. There are times when a preservice teacher will engage in both teaching and learning opportunities. Individuals may play a specific role in the educational setting; however, the practices of teaching and learning are not mutually exclusive based on role.

The four elements of feedback as described by Thurlings et al. (2012) still apply to teachers, preservice teachers, and students, regardless of their role in the educational setting. Research on feedback includes studies conducted with teachers, preservice teachers, and students. Although the roles themselves may be different, in each case there is an agent providing feedback to someone regarding their performance (Hattie & Timperley, 2007). It is this feature that makes the following research applicable to this study regardless of educational role.

Attributes of Effective Feedback

Van Houten (1980) organized the attributes of feedback into three categories: the nature of feedback, the temporal dimensions of the feedback, and the role of the person who delivers the feedback. The nature of feedback includes the content of the feedback delivered, as well as the medium of delivery. The temporal dimensions of feedback relate to frequency and timing. Finally, the role of the person providing the feedback may be considered, be it a university liaison, building administrator, or peer coach (Scheeler, Ruhl, & McAfee, 2004; Van Houten, 1980).

Nature of Feedback – Message Content

Scheeler et al. (2004) defined feedback content as what is communicated. They then organized content into five nonexclusive categories: positive feedback, corrective feedback, noncorrective feedback, specific feedback, and general feedback. In addition to these categories Thurlings et al. (2012) identified goal-directed feedback and person directed feedback as additional feedback content areas. A number of studies have been conducted to determine what feedback content is the most effective in increasing teacher behaviors that improve the effectiveness of instruction, and in decreasing teacher behaviors that detract from the effectiveness of instruction.

Feedback with positive content. Cossairt, Hall, and Hopkins (1973) conducted research designed to study factors that would increase teacher praise for appropriate student attending behavior. They found that feedback with positive content contributed to an increase in desired teacher behaviors. Subjects in their study included three elementary teachers and twelve of their students.

Three conditions were implemented and studied to determine their effectiveness in increasing teacher praise when students attended to their instructions. Condition 1 consisted of a set of instructions that included an explanation of why positive teacher interaction may increase student attentive behaviors, instructions for teachers to give praise when the specific behaviors were demonstrated by students, and a written reminder that teachers should increase their praise. For condition 2, the researchers provided the teachers with specific feedback relating the number of intervals during which the students visibly demonstrated they were paying attention and the number of intervals of teacher praise for that behavior. For condition 3, positive social praise was given to the teacher when they provided feedback and praise to students who were attentive.

Based on the results of their observations and analysis, the researchers found that only condition 3 resulted in a sustained increase of positive teacher praise by all three teachers for appropriate student attending behaviors. Condition 3 also resulted in an increase in student attending behaviors in all three classes. These results led the researchers to determine that social praise, or positive feedback content is a necessary ingredient in changing teacher behaviors.

Feedback with specific content. Englert and Sugai (1983) reported results of a research study they conducted involving 20 preservice special education teachers involved in a practicum setting. The observations for these studies were conducted in 20 special education classrooms at the elementary level. The researchers concluded that specific observation systems and feedback were more effective than non-specific observation systems and feedback in increasing desired teaching behaviors.

In their study, peer observers for the treatment group were given well-defined observation systems for the collection of data, while peer observers for the control group created their own systems as they were not given a well-defined system by the researchers. Observers for both groups focused their observations on behavior management and direct instruction techniques. Peer observers provided feedback to the student teachers following their observations.

Although Englert and Sugai (1983) did not find a significant difference related to controlling student behavior between the treatment and control groups, they did find a significant increase in the treatment group's ability to maintain a higher level of pupil accuracy on academic learning trials during direct instruction when compared with the control group's results, $F(1,36) = 3.92$, $p < .05$. Based on the results of their analysis, the researchers concluded that specific feedback led to an increased ability in student teachers to bridge the gap between instruction and monitoring the outcomes of that instruction.

Another study related to specific feedback was conducted by Sharpe, Lounsbery, and Bahls (1997). This study involved 4 preservice teachers involved in a practice-teaching setting for physical education. The researchers found that specific feedback developed from sequential behavior analysis led to a rapid and reliable improvement in successful teacher behaviors related to teaching physical education.

Sharpe et al. (1997) explained that sequential behavior analysis attempts to relate teacher and student behaviors in various contexts. In this study, the researchers were interested in increasing the preservice teachers' ability to help students increase their skills and on-task behaviors in physical education by identifying opportunities for appropriate action. Two different comparison conditions were used. The first condition involved the use of a specific observation and protocol during observations, followed by opportunities for the university supervisor to provide specific feedback related to teacher and student behavior patterns and the relationship between the two. Based on the feedback, preservice teachers would then set goals to improve their practice-teaching performance. During the second condition, preservice teachers received general qualitative feedback based on a 15-item Likert scale.

Sharpe et al. (1997) found that once specific feedback was provided to the preservice teachers, they shifted their teaching focus from organization to instruction, which altered their interactions with the students. In comparison, the researchers found the effectiveness of qualitative notes was minimal in improving teacher practice. The researchers concluded that the specific feedback provided through the use of sequential behavior analysis helped the preservice teachers attend to instructional practices that supported student acquisition of skills in a physical education class.

Feedback with Corrective Content. Hao (1991) compared the impact of corrective and non-corrective feedback on undesirable verbal teaching behaviors of 92 preservice teachers. An example of an undesirable verbal teaching behavior was an overuse of the word “okay” during instructional segments. She found statistically significant differences that led her to conclude that corrective feedback is preferred over non-corrective feedback.

The participants in Hao’s (1991) study were divided into three groups. The first group received corrective feedback regarding undesirable verbal behaviors following a videotaped microteaching lesson. The second group received non-corrective feedback following their lessons, while the control group received general feedback only. Hao then conducted both a quantitative and qualitative review of the changes in verbal behaviors exhibited by the preservice teachers over multiple microteaching lessons.

An analysis of the results found that preservice teachers who received corrective feedback modified undesirable verbal behaviors more effectively than preservice teachers who received non-corrective feedback or general feedback that was neither corrective nor non-corrective. Corrective feedback also led preservice teachers who received it to employ a greater variety of reinforcements than the other two groups. These results led Hao (1991) to recommend that instructors employ corrective feedback strategies whenever possible in helping preservice teachers change undesirable verbal behaviors.

Feedback with Task-Oriented Content. Butler and Neuman (1995) conducted research regarding help-seeking strategies in students. They explain that help-seeking is an adaptive strategy to help students cope with difficulty and develop mastery. It occurs when children are unable to meet the demands of the task or activity in which they are engaged, and when they prefer hints for help instead of solutions. The researchers found that task oriented goals and feedback allowed help-seeking behaviors to be more adaptive than ego or person-related goals and feedback.

Butler and Neuman (1995) conducted a study involving 159 children in grades 2 through 6. Two different sets of goal instructions were provided to students: set one focused on elements related to the task (e. g., this is an interesting game that helps students learn to solve puzzles), and set two focused on elements related to ego (e. g., children who can solve puzzles are smart). Children were then given 6 puzzles and were allowed to ask for hints and help if needed. Results showed that there was a significant main effect for goal condition,

$F(1,147) = 243$, $p < .001$. Butler and Neuman (1995) also found this difference was not dependent on age. Not only did students engage in help seeking strategies more often when given the task related goals, the researchers found a significant effect of goal condition by initial performance, $F(2, 147) = 5.75$, $p < .01$. Butler and Neuman (1995) concluded that task-focused goals and feedback promoted help seeking more than ego- or person-related goals and feedback.

As previously mentioned, while there are differences between teachers, student teachers, and students, the Butler and Neuman (1995) research is applicable to this study for two reasons. First, there is an agent in both settings providing the feedback (Hattie & Timperley, 2007), and second, both teacher and student may fill the role of learner, the student acting in the role of learner in a classroom setting, and the teacher acting in the role of learner during a discussion following an observation.

Temporal Dimensions of Feedback

Van Houten (1980) explained that temporal dimensions of feedback relate to frequency and timing. There is evidence that immediate feedback has a greater impact on behavioral change than delayed feedback. As part of her dissertation, Coulter (1997) reviewed the impact of immediate feedback on the error-correcting and point-awarding behaviors of six teachers and one teacher's aide. Error-correcting refers to the teacher behavior of correcting students when they make an error. Point awarding refers to the teacher behavior of adjusting points awarded based on student errors.

Coulter (1997) established three comparison tests – immediate feedback, after-class feedback, and no feedback regarding error-correcting and point awarding behaviors. Based on the subjects' mean performance data, Coulter found that subjects receiving immediate, in-class feedback acquired the desired skill faster and with a higher level of acquisition than subjects who received after-class feedback or no feedback at all. These results led her to conclude that immediate feedback has a greater impact on changing or developing teaching behaviors than delayed feedback.

Additional evidence of the efficacy of immediate feedback was described in a study by O'Reilly, Renzaglia, and Lee (1994). They also compared the results and effectiveness of immediate and delayed feedback. Their study focused on two preservice teachers participating in a practicum experience in a classroom for students with severe disabilities. The researchers were interested to determine if different temporal aspects of feedback would have different effects on teacher behaviors of appropriate use of positive consequences and instructional prompts.

For the purposes of their study, the authors employed two different temporal dimensions of feedback. The first was immediate, in-class feedback. In this case the supervisor would interrupt instruction to point out needed modifications and to provide corrective counsel as needed. The authors used delayed feedback for the second dimension. In this case feedback was provided no earlier than one day following the observation, and no later than three days following the observation. Based on an analysis of data collected during the observations, the researchers found that immediate feedback was more effective in helping the two subjects acquire the desired instructional behaviors than delayed feedback (O'Reilly et al., 1994).

Who Delivers the Feedback

The final dimension of feedback described by Van Houten (1980) relates to the person who provides the feedback. Interestingly enough, there is not clear evidence in the literature that the role of the individual has a significant impact on changing teacher behaviors. For example, Pierce and Miller (1994) conducted research to compare the effectiveness of traditional university supervision to peer-coaching for increasing desired teacher behaviors and decreasing undesired teaching behaviors.

The subjects in their study included 29 students who were juniors majoring in special education. Each of the students participated in both a practicum and a seminar as part of the class for this study. The students were divided into two groups. One group was given the same coaching training as the university supervisor that would observe all students in both groups. The second group did not receive the training, nor were they informed of the difference between the groups.

Both groups of students showed an increase in desired teaching behaviors as well as a decrease in undesired behaviors. Pierce and Miller (1994) utilized a MANOVA to review differences for significance. The results indicated a significant difference between baseline and treatment, $F(1,26) = 12.26$, $p = .002$; however, there was no significant difference between the treatment and control groups. The authors concluded that traditional university supervision and peer-coaching procedures were equally effective in influencing the desired behavior modifications.

The literature to this point has demonstrated that feedback that is positive, specific, corrective, and task-oriented has been shown to have a positive impact on behavioral change in teachers, preservice teachers, and students. In addition, feedback that is immediate rather than delayed also has a positive impact on the acquisition of new teaching behaviors as well as the modification of existing teaching behaviors. However, the literature is nearly silent regarding feedback that focuses the specific subject matter content of the feedback being given. Taken in this light, it would seem that the previously specified characteristics of feedback are sufficient to consider when giving behavioral feedback regardless of content area. However, is behavior modification, either teacher or student, sufficient to generate a high level of student achievement?

Importance of Feedback in the Educational Setting

Hattie and Timperley (2007) pointed out that feedback is provided after some event, such as initial instruction, has already taken place. When effective feedback is combined with effective instruction, it can powerfully enhance student learning. However, the authors caution, feedback "too rarely occurs, and needs to be more fully researched by qualitatively and quantitatively investigating how feedback works in the classroom and learning process" (p. 104).

If feedback from teachers is as critical a component for improving student learning, as Hattie and Timperley (2007) suggest, we might conclude that teachers would also benefit from feedback regarding their teaching practice. Knowing that certain characteristics of feedback have been shown to have a positive impact on teaching behaviors, one would well address the question of who will provide feedback to teachers in order to help them improve their instructional and feedback crafts. The most likely candidate in our current educational system would be the individual responsible for evaluation, the school principal.

Several studies that reviewed the literature on the relationship between the building principal and student achievement found that the influence of principals on student achievement is primarily indirect, but may be measured (Hallinger, Bickman, & Davis, 1996; Hallinger & Heck, 1996). Leithwood et al. (2004) estimated the total direct and indirect effects of leadership accounted for about a quarter of total school effects on student outcomes.

However, individuals observing the same events are unlikely to provide the same feedback about the event. Perspectives and cultural values possessed by observers will influence and impact how they interpret classroom events, instructional processes, and thus the feedback they give (Knight, 2007; Peel & Shortland, 2004; Shortland, 2010). These values and perceptions frame our "reality," according to Shortland (2010), and may limit an observer's ability to provide accurate feedback that is free from bias and misunderstandings based on perception and background. One may also question whether misunderstandings may be brought on by the content knowledge and background of the observer. Such a possibility exists as we bring to bear past experiences to make sense of what is being observed (Tubbs, 2000).

A Theoretical Case for a Focus on Mathematics Instruction

As can be seen from the previous discussion, much time and effort has been expended to review characteristics of feedback that contribute to a positive and productive improvement in education. There is an underlying assumption in the work reviewed that content neutral feedback is sufficient in order to generate the instructional improvement needed to increase student achievement in any content area because such feedback impacts teacher and student behaviors. However, is there more to knowledge acquisition than behaviors? The following section will review practices and insights into mathematics education that point to the need for more than just a behavioral approach to instruction and instructional improvement.

Behavior and Cognition – Two Approaches at the Heart of Reform

In 1989 the National Council of Teachers of Mathematics (NCTM) released *Curriculum and Evaluation Standards for School Mathematics* and called for reform in the field of mathematics education. Prior to this time, mathematics education was steeped in the philosophies of behaviorist psychology. Battista (1994) observed that under this prevailing theory, the focus of the teacher was on training students (e. g., influencing observable student behaviors) rather than on generating student understanding (education). Students were shown algorithms and processes to solve problems, and then practiced those procedures in a rote manner to generate proficiency. Battista (1994) stated, "Views of school mathematics and school learning were thus mutually reinforcing: school mathematics was seen as a set of computational skills; mathematics learning was seen as progressing through carefully scripted schedules of skill acquisition" (p. 463).

The behaviorist approach to mathematics education focuses primarily on teaching students to follow a set of prescribed steps in a prescribed manner to reach a solution. Direct instruction is the primary instructional conduit for this type of knowledge delivery. Confrey (1990) outlined three key assumptions related to direct instruction. First, student products are relatively short and not process-oriented. Student understanding is primarily measured from homework assignments and tests. Second, for instruction, teachers need only follow their plans and routines with periodic checks for student understanding. Third, the teacher is responsible for determining whether a student has reached an acceptable level of understanding.

The behaviorist approach does not take into account knowledge or abilities students already possess, nor does it address ways to connect new ideas and skills to what students already know. Instead, “the exclusively behavioral characterization of desirable learning outcomes leads educators to rely on the teaching of discrete, disconnected skills in mathematics, rather than on developing meaningful patterns, principles, and insights” (Goldin, 1990, p. 36).

The National Research Council (1989) used a constructivist approach to explain why a purely behavioral approach to mathematics education was problematic. The council stated,

> Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created... Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn. (p. 6)

Carpenter (1986) explained that children possess the natural ability to solve problems prior to being taught algorithmic procedures such as addition and subtraction. He stated,

> The problem-solving analysis that children naturally apply to simple word problems reflects a better model of problem-solving than many of the superficial tricks for solving word problems that are often taught. Solution procedures appear to be linked to conceptual knowledge. They are based on reliable, accurate representations of the problems. (p. 114)

It is clear from these observations that students and their personal knowledge and experience play a role in their learning of mathematics.

A philosophical shift began to take place in the 1960’s and 70’s from behaviorism to structuralism, which attempts to resolve the mind into structural elements, and cognitivism, the study of mental processes. The result was a fundamental change from studying only observable behavior, to reasoning about how the mind learns, stores, and accesses stored information. This change in fundamental psychological thought led to additional study regarding problem solving, concept formation, and connections between cognitive structures, or the basic mental processes people use to make sense of information, and behavior (Noddings, 1990). Constructivism became one form of this cognitive approach. Constructivism holds that all knowledge is constructed. Cognitive structures may be innate (Chomsky, 1968, 1971) or are results of developmental construction (Piaget, 1953, 1970, 1971). Although the conceptual views of many constructivists differ, Noddings (1990) described areas in which they generally agree. Some of these areas include:1) all knowledge, including mathematical knowledge is constructed; 2) existing knowledge and cognitive structures are activated in the process of learning; and 3) cognitive structures are continually developing. The idea that learning is a constructive process is one of the foundational principles of this study.

Conceptual and Procedural Approaches to Mathematics Education

Two types of knowledge connect with cognitive and behaviorist philosophies. Conceptual knowledge connects with cognitive philosophies while procedural knowledge connects with behaviorist philosophies. Although there is a lack of consensus regarding the specific definitions and boundaries between the two types of knowledge (Hiebert & Wearne, 1986), there are aspects of each generally agreed upon that will allow for a working understanding of each.

Conceptual knowledge is most often characterized as containing numerous and rich relationships (Hiebert & Lefevre, 1986; Hiebert & Wearne, 1986). Hiebert and Lefevre (1986) stated that “a unit of conceptual knowledge cannot be an isolated piece of information; by definition it is part of conceptual knowledge only if the holder recognizes its relationship to other pieces of information" (p. 4). In effect, the construction of conceptual knowledge constitutes what could be described as a "connected web of knowledge" (p. 3).

Hiebert and Lefevre (1986) described the two ways for conceptual knowledge to be constructed. The first way is to link two pieces of information already stored in memory. The second way is to link a piece of information already stored in memory with a newly learned piece of knowledge. Similar relationships and connections are developed or constructed between the pieces of knowledge or information in these two ways of linking.

Procedural knowledge on the other hand is characterized by the absence of embedded relationships (Hiebert & Wearne, 1986). Hiebert and Lefevre (1986) described procedural knowledge as having two components: 1) a language or symbolic representation system, and 2) the rules or algorithms used for mathematical manipulation. The language or symbolic representation component is also referred to as the "form" of mathematics (Byers & Erlwanger, 1984). Hiebert and Lefevre (1986) provided the following example of form. A student may well recognize that $3.5 \div \square = 2.71$ as an acceptable form while $3.5 \div = \square 2.71$ is not an acceptable syntactic form of the equation. Although the student recognizes the appropriateness or lack of appropriateness of the syntactic use of the symbols, form in and of itself does not imply knowledge of the meaning of the equation.

The second component of procedural knowledge is the system of rules and algorithms used for mathematical manipulation (Anderson, 1983; Newell & Simon, 1972). Such a system requires an input, followed by sequenced operations to be performed on that input in order to reach the answer or goal state. The sequenced step-by-step nature of this process requires little relational knowledge, only the order in which the steps are executed.

Connections Between Conceptual and Procedural Knowledge

At a cursory glance it may appear that conceptual and procedural knowledge are separate, unique, and possibly opposed to each other. However, Hiebert and Lefevre (1986) claimed that fundamental relationships between conceptual and procedural knowledge must exist in order for a student to fully develop a competent understanding of mathematical knowledge. A weakness or lack in either area lessens the richness of mathematical knowledge. Silver (1986) further contended it is the relationships between conceptual and procedural understanding that are of primary importance when engaging in nontrivial tasks such as problem solving.

The relationship between conceptual and procedural understanding is a critical component of this study. In order to comment on and support the development of instructional skills involving both conceptual and procedural understanding of mathematics, an observer would need background knowledge in the mathematical content that makes up those concepts and procedures. Without this knowledge, it may be difficult at best to identify the connections and interrelations between the two. What then are some of those connections?

Ways Conceptual Knowledge Supports Procedural Knowledge. Hiebert and Lefevre (1986) identified three key ways that procedural knowledge benefits from connections with conceptual knowledge: developing meaning for symbols, recalling procedures, and using procedures. Students develop meaning for symbols as they encounter concrete experiences in life related to those symbols. For example, as students begin to put groups together, and are then introduced to the "+" symbol in a connected way, they develop a conceptual connection between the action of joining groups and the meaning of the symbolic notation.

Procedures are easier to recall when students connect them to the conceptual meanings of the situation or task they are working with. The authors also maintained that creating connections between procedures and their purposes makes it less likely that such knowledge will deteriorate. For example, understanding that the purpose of “lining up the decimal” is to add like units makes it easier to remember the rule as compared with memorization of the rule without an understanding of what the rule means.

Finally, conceptual understanding supports the use of procedures by assisting individuals in developing mental representations of problems, monitoring the selection and execution of procedures, and promoting the selection of appropriate and efficient procedures in new situations. Mental representations allow a person to reason directly about the problem rather than about the meanings of the symbols being used. Conceptual understanding of a procedure allows one to anticipate the consequences or outcomes of a particular process or procedure, which aids in the selection of appropriate processes. For example, a person is better able to evaluate the reasonableness of an answer when they understand the problem and process conceptually as compared to following a set of steps leading to a result that is unconnected to the problem they are trying to solve (Hiebert & Wearne, 1986).

How Procedural Knowledge Supports Conceptual Knowledge. The connection between procedural and conceptual knowledge works both ways (Hiebert & Lefevre, 1986; Silver, 1986). The development and use of conceptual knowledge can also be enhanced by a connection made to procedural knowledge in three ways: symbols enhance conceptual knowledge, procedures themselves apply concepts to solve problems, and procedures promote conceptual knowledge.

Hiebert and Lefevre (1986) stated, “The formal language system of mathematics provides a powerful tool for dealing with complex ideas” (p. 15). Symbols support the enhancement of conceptual understanding in problem solving as they can be used to represent complex concepts. Used in this way, symbols help to organize information to make problem solving more efficient. Additionally the use of symbols as a formal language system has led to the development and expanded use of key mathematical concepts.

In addition to use in a formal language system, symbols are used in routines and procedures. These routine procedures are developed through sometimes difficult and labor-intensive explorations (Anderson, 1983). As new problems arise and are solved repeatedly, patterns and processes emerge that can be applied as new routines or procedures to these new situations to simplify and develop greater efficiency in the process to reach a solution. These new routines and procedures may then be used to reduce the amount of effort needed to solve problems, while allowing cognitive effort to be applied to developing deeper or new connections and conceptual understanding.

Finally, new procedures can trigger the development of new conceptual understanding in similar ways as notation systems. Hiebert and Lefevre (1986) stressed that the development of deeper understanding requires both conceptual and procedural understanding, and that one does not always come from, nor is built on the other. There is a relationship between the two, and both parts are important for the acquisition of mathematical understanding.

As stated previously, one of the purposes of this study is to determine if content knowledge has an impact on the feedback provided to teachers regarding the effectiveness of their mathematics instruction. The shift from primarily procedural knowledge to the inclusion and use of conceptual knowledge in teaching mathematics has been discussed. It would be unlikely that an evaluator can support the teacher in developing students' conceptual understanding of mathematics without a background in the content of mathematics. The next sections will discuss evidence that suggests that the development of conceptual understanding of mathematics has an impact on student learning.

Results of Reform Instruction in Mathematics

The National Council of Teachers of Mathematics called for a shift in mathematics education away from a discrete, hierarchical, sequential, and fixed methodology to one in which learners are supported in constructing their own individual learning through interactions with mathematical concepts, other people, and their environment (National Council of Teachers of Mathematics, 1989, 2000, 2014). In essence, their recommendations make use of both conceptual and procedural knowledge, as well as the relationships between the two approaches to develop student understanding. This shift in instructional focus has been the heart of the reform mathematics movement.

Boaler and Staples (2008) conducted a five-year longitudinal study that compared the mathematics learning of 700 diverse students at three secondary schools. Two of the schools provided students with a choice between a traditional sequence of courses taught in the traditional fixed methodology described by Draper (2002), or an integrated sequence of courses where students worked in a more open applied curriculum. The third school employed a reform approach to mathematics instruction. Teachers at this school rarely lectured. Rather they provided students with tasks and activities to be worked on and discussed in groups. In addition, teachers worked collaboratively, sharing instructional strategies and curriculum ideas and resources.

Boaler and Staples (2008) utilized both quantitative and qualitative approaches to analyze their data. Their analysis showed that in year three, students at school 3 outperformed other students, but not significantly. Teachers in school 3 were also highly successful in reducing the achievement gap between different ethnic groups beginning in year one of the study. By year three, nearly all of the achievement differences had disappeared except for the consistently high performance of Asian students. The achievement differences between ethnic groups at the other two schools remained throughout the duration of the study.

Boaler and Staples (2008) also reported significant differences in student affect towards mathematics beginning in year two, and continuing throughout the remainder of the study. For example, in year four, the researchers interviewed 105 students, most of whom were seniors. They found students from school 3 to be significantly more interested in mathematics ($\lambda^2 = 12.806$, df = 2, p = 0.002, n=67). In addition, they found all of the students interviewed from school 3 intended to take additional mathematics courses as compared to 67% of students from the other schools. They also found 39% of students from school 3 intended on pursuing a future in mathematics as compared with 5% from the traditional classes ($\lambda^2 = 18.234$, df = 2, p = 0.000, n=65).

The results of this study highlight some of the key differences in student achievement made possible by utilizing conceptual knowledge and instructional practices that support the development conceptual knowledge in connection with procedural knowledge. While the results are impressive and desirable, we must recognize that moving to such instructional methods requires a great deal of change from teachers' current instructional practices. Who would oversee this change, and are these individuals adequately prepared to recognize and address instructional practices that support or hinder student conceptual learning of mathematics? The next section will review some of the evolution of instructional supervision in an attempt to determine if changes in instructional supervision align with changes in instructional practice.

Behavioral and Cognitive Approaches to Supervision of Instruction

Behaviorism and cognitive psychology have also played a significant role in the development of supervision of instruction (Nelson & Sassi, 2000). Historically, supervisory practice has focused on pedagogical processes independently of the content being addressed (Glanz, 1998; Haggerson, 1991; Holland, 1998; Nelson & Sassi, 2000; Tracy, 1998). The focus of supervision was to identify teacher behaviors that were assumed to correspond to effective teaching. These teaching behaviors applied to teaching in general and were equally applicable to all subjects (Nelson & Sassi, 2000).

In the late 1980s, cognition began to influence supervision. This influence led to a focus on teachers' cognitive activity, not just their instructional behaviors (Nelson & Sassi, 2000). Although the change in focus led to the practice of teacher reflection and the idea that teachers could be viewed as professionals responsible for their own learning (Darling-Hammond & Sclan, 1992), that focus was still on teacher behaviors, not on content ideas.

The shift to a cognitive view of teaching and pedagogy (Garmston, Lipton, & Kaiser, 1998; Sergiovanni & Starratt, 1998) influenced supervision in the 1990s to begin to focus on student thinking as a major component of instruction and learning in education (Tracy, 1998). It was during this time that researchers began suggesting that content-specific supervision be studied and examined (Nolan & Francis, 1992). This study is a response to the call for content-specific research related to supervision of instruction.

Connections Between Content and Supervision

Previous sections of this chapter have illustrated the evolution of beliefs about effective mathematics instruction from a focus primarily on behaviors and procedures, to the inclusion of conceptual understanding in order to create a deeper understanding and meaning of mathematics. The development of supervision models has followed a similar course, beginning with a focus on teacher behaviors, followed later by the inclusion of teacher thinking, albeit still focused on behaviors. Unfortunately, one significant difference remains – supervision of instruction is still primarily focused on observation and evaluation of pedagogical processes and behaviors without consideration of the content of teaching, a fact lamented by Cook (1998). Nelson and Sassi (2000) argued,

> in order to understand classrooms that are functioning to help students construct subject-matter knowledge, knowledge of pedagogical processes and content knowledge must be fused. Those who would supervise in such classrooms need to attend to both. (p. 558)

Nelson and Sassi (2000) conducted a study of a year-long professional development seminar for eighteen school and district administrators that focused on classroom observation and teacher supervision in elementary mathematics. As part of their study the participants viewed a video depicting a standards-based mathematical lesson two different times during the year. Standards-based mathematical lessons focus on group work, collaboration, and sharing in an attempt to help all children develop proficiency in a content area, such as mathematics, as defined by a set of standards. After the first viewing of the video, the researchers listened to participants' reactions to the lesson segment. They found participants' comments described structural components, such as transitions of the classroom, but not aspects of student thinking, or the teacher behaviors that elicited that thinking.

Administrators were again shown the video eight months later, after they had participated in professional development focused on helping them identify different aspects of a mathematics classroom. Specifically, the professional development was aimed at four aspects of a mathematics lesson: 1) actions or behaviors that count as mathematical knowledge, 2) how students learn mathematics, 3) elements related to student engagement, and 4) the nature of teaching mathematics, or in other words, what aspects of mathematics instruction are recognized as effective.

Nelson and Sassi (2000) found differences in administrators' comments related to their conception of mathematical knowledge. After the first viewing, administrators focused their attention on whether the students had learned the material or not. In other words, a teacher's responsibility was to determine how many students understood the content and how many did not. After participating in the seminar and viewing the video a second time, administrators' comments reflected the notion that they recognized student learning of mathematics as a process, not a dichotomous comparison of students who have the knowledge as compared to those that do not. A teacher's role was now seen as a process of helping students develop mathematical understanding over time.

Administrators' views on how mathematics is learned also changed during the professional development. After the first video, administrators had commented on structural components of the lesson such as direct instruction of computational skills and generic pedagogical strategies. Following the professional development, administrators did not even mention traditional teacher behaviors such as how the teacher handled transitions, pacing, or wait time. Rather they "were looking directly at the mathematical thinking that was happening in this particular class and using what they saw there as the basis for asking questions or drawing conclusions about the nature and the quality of the instruction" (Nelson & Sassi, 2000, p. 571).

Student engagement was also seen differently following the professional development. Following the first video the administrators often commented on how many students appeared passive because they weren't always interacting in a highly visible manner, a characteristic the administrators attributed to standards-based instruction. Following the second viewing of the video, administrators recognized the difference between passiveness, or just being there, and attentiveness, or active student thinking and listening. Administrators also demonstrated a different appreciation for student articulation of ideas. They noticed how students would struggle to form and describe their thoughts, and how they persevered through the process to make these ideas clear to themselves and the rest of the class.

Finally, the administrators viewed the nature of teaching differently. Following the first video, administrators characterized the teacher as being new to this instructional process. The administrators tended to classify teacher behaviors as unfocused and scattered. Interestingly enough, following the professional development these same administrators recognized and categorized this teacher as a very strong teacher capable of purposeful facilitation as well as direct instruction. More importantly, administrators recognized the teacher understood when and how to make the transition between the two.

Nelson and Sassi (2000) concluded that administrators' understanding of mathematics instruction influenced both their ideas of mathematics education reform and their ideas of how to support it. They also concluded that administrators

> had to see that when the students' mathematical ideas are at the center of attention … the teacher's pedagogical process (the process of helping students develop their subject-matter thinking) is inextricably interwoven with her assessment of the content of the thinking itself. To understand what was going on in the classroom and make valid judgments about the quality of instruction, administrators had to attend to both. (p. 575)

Content knowledge then is as important to the identification of quality mathematics instruction and teacher effectiveness as is an understanding of general pedagogy.

Conclusion

A review of the current literature regarding observation and feedback supports the importance of both in improving the quality of instruction (Colvin, Flannery, Sugai, & Monegan, 2009; Fuchs & Fuchs, 1993; Scheeler et al., 2004) as well as improving student achievement (Colvin et al., 2009; Fuchs & Fuchs, 1993; Thurlings et al., 2012) . Although the importance of observation and feedback are well documented, and although the current literature describes the influence of an observer's background and cultural value systems (Shortland, 2010) on the development and delivery of feedback, the current literature has not yet defined how an observer's content experience and background impact the content or the consistency of the feedback provided to educators following an observation.

The review of current literature also detailed some of the unique characteristics of mathematics instruction, specifically the comparison and relationship between conceptual and procedural knowledge. Although there is evidence (Boaler & Staples, 2008) suggesting that the conceptual component of mathematics education is a vital component in conjunction with (not to the exclusion of) procedural knowledge, many teachers continue to pursue traditional, procedural instruction as their primary pedagogy. Why is this? Davis (1986) suggests that

> theory influences practice to a very considerable extent, and many of the shortcomings of typical school mathematics programs are closely related to reliance on weak conceptualizations of what mathematical knowledge really is and what is involved in acquiring such knowledge. (p. 298)

If theory influences practice to a very considerable extent, as Davis suggests, it is reasonable to assume that knowledge of the theory and structure of mathematics is invaluable to instructional leaders in assisting teachers in making the instructional changes necessary to increase and deepen student learning of mathematics, an idea supported by Nelson and Sassi (2000). However, if that content knowledge is unavailable to a person providing feedback, will that person be able to offer feedback that has the efficacy and ability to foster the change required? If not, what knowledge is needed, and who needs it?

This study has investigated these and other related issues by addressing the following questions:

1) What themes emerge from feedback given to teachers of mathematics by observers with different content and focus backgrounds?
2) Does an observer’s educational background influence the content of feedback that is given?

CHAPTER 3: METHODOLOGY

The purpose of this study is to examine the written feedback given to elementary teachers by three different groups of observers following two different observations of mathematics instruction. The feedback was analyzed to identify themes within the feedback provided, and determine whether there were similarities and differences between the observer groups in the themes of feedback they provided. The results and conclusions of this study will (a) be an additional support to the current body of observation and feedback literature, and (b) provide a critical lens from which to begin a discussion on how content knowledge and experience impact the content of the feedback provided to educators.

In order to support the identification of themes within raw text an inductive inquiry model (Thomas, 2006) was employed to provide a framework to allow the themes within the feedback to emerge during the comparison and analysis of the feedback. According to Thomas, the inductive analysis model supports the following purposes:

> 1. to condense extensive and varied raw text data into a brief, summary format;
>
> 2. to establish clear links between the research objectives and the summary findings derived from the raw data and to ensure that these links are both transparent (able to be demonstrated to others) and defensible (justifiable given the objectives of the research); and
>
> 3. to develop a model or theory about the underlying structure of experiences or processes that are evident in the text data. (pg. 238)

A comparison of the nature and purpose of this study with the structure of an inductive inquiry analysis suggests such a methodology is appropriate for this study in the following ways. First, 30 different written instructional observations were collected as part of the study.

These observations and comments varied from participant to participant. The content of the observations needed to be condensed into summary findings. Second, the summary findings needed to be linked to research objectives and the available literature base on mathematics education in order to be defensible. Third, the underlying structure of comments was analyzed in such a manner as to develop a model or theory of that structure. These three criteria match the purposes of an inductive inquiry model.

Setting

This study took place in the northwestern region of the United States beginning in the fall of 2013. Due to distance between participant sites, information dissemination and data collection occurred electronically via Internet resources. Video segments of classroom instruction in mathematics were used for the purposes of the observation. Using videotaped instructional segments allowed all participants to view the same instructional episodes and classroom interactions allowing for the needed element of consistency across participants.

The video selection was made from two different sources. The first source was the public YouTube site, http://www.youtube.com. A search was conducted for elementary mathematics instructional segments between 15 and 45 minutes in length that focused on a mathematics lesson. The video chosen from this collection, labeled Video 1, exemplified the more traditional or procedural approach to teaching mathematics described in Chapter 2 of this study.

Video 2, was filmed during the spring of 2013 as part of two studies occurring in two school districts in the Northwestern United States. The video segments captured during these studies featured 30 to 45 minute instructional sessions in elementary mathematics classrooms occurring in grade levels one through five. For the purposes of this study one teacher at each grade level (grades K through 5) was selected for videotaping purposes, with each teacher being videotaped three times. All videos were scored using the full version of the Developing Mathematical Thinking instruction observation protocol (Brendefur, Strother, & Peck, 2010a). Video 2 (Teacher 4, third grade, second segment) was selected for this study as an example of effective use of both conceptual and procedural elements of instruction as evidenced by the high score of 98 out of a possible 100 points. Table 1 presents the evaluation scores of both Video 1 and the video collection from which Video 2 was selected.

Table 1
Observation Scores for Video 1 and Possible Video 2 Segments

Teacher	Grade level	Video Segment	Observation Score
Video 1			
Teacher 1	Third Grade	1	25
Options for Video 2			
Teacher 1	Kindergarten	1	45
		2	58
		3	62
Teacher 2	First Grade	1	81
		2	86
		3	88
Teacher 3	Second Grade	1	69
		2	76
		3	73
Teacher 4	Third Grade	1	95
		2	98
		3	96
Teacher 5	Fourth Grade	1	75
		2	89
		3	86
Teacher 6	Fifth Grade	1	90
		2	93
		3	95

Participants

The purpose of this study was to compare the content of feedback provided to elementary mathematics teachers following an observation by three different types of observers. As a commonality, all participants for this study have had previous experience in conducting classroom observations and providing feedback to other educators. These observations included a variety of curriculum areas including mathematics.

The first group of participants consisted of mathematics content and pedagogy specialists who provide professional development in mathematics instruction as part of the Initiative for Developing Mathematical Thinking (IDMT). This group will be referred to as “mathematics content specialists.” These participants have all been mathematics educators at either the elementary or secondary level, and specialize in generating mathematics professional development designed to enhance teacher content and pedagogical content knowledge focused on increasing student achievement. These individuals regularly participate in observing mathematics instructional segments and provide specific feedback to educators in ways to enhance their instructional effectiveness. They also participate in study and research activities aimed at supporting the development of conceptual understanding in mathematics and in using that understanding to support procedural knowledge and practices for both teachers and students.

Group 2 participants consisted of content specialists at the university level that support the teacher education program by monitoring and supporting student teachers, but have a content background specialty in an area other than mathematics, such as language arts, social studies, and special education. This group will be referred to as “other content specialists.” These individuals had some exposure to current mathematics professional development through interdepartmental communications and were aware of the current professional development initiatives in mathematics; however, they had not participated in specific professional development focused on content or pedagogical content knowledge within the field of mathematics. Other content specialists possessed a strong background in instructional improvement which includes general instructional observation and feedback sessions with current and future educators.

Group 3 participants consisted of current elementary school principals and will be identified as such throughout the remainder of this paper. Invitations to participate in this group were sent to principals of schools who had a working relationship with the IDMT. These participants had experience in conducting formative as well as evaluative observations. Such observations served the dual purposes of instructional improvement and providing data about teacher performance that supported the retention or dismissal of an educator. School principals had some exposure to mathematics professional development, and had completed at least one professional development course focused on improving mathematics instruction through one of the projects previously mentioned. Table 2 lists the educational experience of the participants, and Table 3 lists the number of college mathematics courses, including professional development courses in mathematics completed by each of the participants.

Other content specialists and building principals were selected in the following manner. First, an invitation was sent out to the various individuals in each group by e-mail. The e-mail included an attachment containing a consent form and a set of instructions should the individual agree to participate. The consent form and instructions may be found in Appendix A. The first five individuals to agree to participate from each group served as the participants for the study. Each participant was given a survey and asked to provide the following demographic data: 1) years teaching in grades K – 12, 2) years in administration, 3) years teaching in higher education, 4) total years in education, 5) content area specialty, 6) number of mathematics courses as an undergraduate, 7) number of post-graduate mathematics courses, 8) number of years of experience in observing inservice teachers, and 9) a description of post-graduate mathematics professional development taken. The survey may be found in Appendix B.

Table 2
Participant Educational Experience

Group	Participant	Years Teaching	Years as Administrator	Years in Higher Ed	Years in Education	Years Observing
1						
	1	6 – 10	0 – 5	0 – 5	16 – 20	0 – 5
	2	0 – 5	0 – 5	6 – 10	11 – 15	6 – 10
	3	6 – 10	0 – 5	0 – 5	6 – 10	0 – 5
	4	6 – 10	0 – 5	0 – 5	11 – 15	6 – 10
	5	0 – 5	0 – 5	6 – 10	11 – 15	6 – 10
2						
	1	6 – 10	0 – 5	0 – 5	16 – 20	6 – 10
	2	0 – 5	0 – 5	16 – 20	16 – 20	16 – 20
	3	16 – 20	0 – 5	0 – 5	21 – 25	0 – 5
	4	0 – 5	0 – 5	0 – 5	11 – 15	0 – 5
	5	16 – 20	0 – 5	6 – 10	16 – 20	6 – 10
3						
	1	11 – 15	0 – 5	0 – 5	16 – 20	0 – 5
	2	0 – 5	0 – 5	0 – 5	21 – 25	0 – 5
	3	6 – 10	6 – 10	0 – 5	16 – 20	6 – 10
	4	6 – 10	11 – 15	0 – 5	16 – 20	0 – 5
	5	11 – 15	0 – 5	0 – 5	11 – 15	6 – 10

Table 3
Mathematics Courses Completed by Participants

Group	Participant	Undergraduate Mathematics Courses	Graduate or Continuing Mathematics Courses
1			
	1	7 – 10	4 – 6
	2	4 – 6	4 – 6
	3	7 – 10	11 +
	4	7 – 10	7 – 10
	5	0 – 3	11 +
2			
	1	0 – 3	0 – 3
	2	4 – 6	4 – 6
	3	0 – 3	0 – 3
	4	0 – 3	0 – 3
	5	4 – 6	0 – 3
3			
	1	0 – 3	7 – 10
	2	0 – 3	0 – 3
	3	4 – 6	4 – 6
	4	7 – 10	0 – 3
	5	0 – 3	4 – 6

Instruments

Two related observation instruments were used to observe the video segments. The first instrument was the full version of the Developing Mathematical Thinking for Instruction observation protocol (Brendefur et al., 2010a), and the second was the simplified version of the Developing Mathematical Thinking for Instruction observation protocol (Brendefur, Strother, & Peck, 2010b). See Appendix C for the full protocol and Appendix D for the simplified protocol. The full version of the protocol was used as previously mentioned to select the video segments for the study, and the simplified version was used by participants of the study as they observed the video segments and prepared written feedback for the teacher in each video segment. Both instruments were built on the Developing Mathematical Thinking (DMT) instructional principles: 1) taking students' ideas seriously, 2) pressing students conceptually, 3) encouraging multiple mathematical models, 4) addressing students' misconceptions, and 5) focusing on the structure of mathematics.

The full version of the protocol was developed for the purpose of identifying a teacher's use of elements of the five DMT instructional principles during instruction in a mathematics classroom, and measuring change in the use of these instructional elements over time. This instrument is by nature and design complex, and requires significant training and practice to ensure validity and inter-rater reliability. The effective use of the instrument also requires both significant mathematical content knowledge as well as pedagogical content knowledge of mathematics. Table 4 displays the 2010 reliability measures for the full version of the protocol when used by trained observers.

The first section of the protocol focuses on the DMT instructional element of taking students' ideas seriously. The protocol indicators describe the level to which instructional activities relate to students' experiences and responses, as well as to their lives, experiences, and cultural backgrounds, as opposed to activities that come from a textbook or resources that make no connections to students. This section also describes the level to which students are allowed to participate in the selection of problem solving strategies, whether or not student ideas are valued by the teacher, and whether the teacher is able to recognize which student ideas may be used in a generalized fashion.

Table 4
DMT Protocol Scoring Reliability Measures

Protocol Section	Number of Items	Number of Valid Cases	Cronbach's Alpha
Taking Student Ideas Seriously	4	55	.953
Pressing Students Conceptually	4	55	.936
Encouraging Multiple Strategies	4	55	.922
Addressing Misconceptions	4	55	.899
Focus on the Structure of Mathematics	4	55	.912

The second section of the protocol addresses whether or not the teacher is able to help students develop a deeper cognitive understanding of mathematical concepts and processes. The protocol does this by focusing on the level of justification required of students as they explain their thought processes, whether students are able to describe the strengths and/or limitations of their own thought processes as well as the thought processes of other students in the classroom, and the presence or absence of progressive formalization as illustrated by models beginning at a concrete or informal level, moving through more abstract and formal models and thought processes. In other words this section reviews both the teacher's and students' use of reasoning and communication that lead to a deeper understanding of mathematics and formal thought processes.

The third section of the protocol addresses the level to which student strategies are reviewed, compared, and encouraged as part of helping students develop a deeper conceptual understanding. This is accomplished by reviewing the level to which the teacher encourages or inhibits the use of student-developed strategies, the level to which models suggested or used by a teacher to model student thinking actually match the thought processes of the student, the level to which the model used matches the context of the task given to students, as well as the use of enactive, iconic, and symbolic representations, and the connections between them. In general this area provides a measurement of the degree to which student thoughts, mathematical processes, and models are treated with fidelity, and then connected in a progressive manner to develop increasingly more formal and abstract thought.

As students are allowed to actively participate in the generation of mathematical ideas, teachers have the opportunity to expose thought processes that are limiting or may contain misconceptions that will hinder student progress. The fourth section of the protocol addresses the level to which teacher-driven instruction is free from misconceptions, the level to which student misconceptions are used as part of the learning process for both individuals and groups of students, and whether or not the manner in which misconceptions are addressed supports or hinders students' feelings of self-efficacy.

The final section of the protocol addresses the levels to which instructional processes utilize and support students' understanding of the fundamental structure of mathematics. Indicators for this section include recognition and development of connections between various mathematical topics and concepts, an understanding that correctness resides in the mathematics versus the teacher or textbook, the evidence of student understanding over time (mathematical residue), as well as opportunities for students to generalize both procedural and conceptual understanding. In other words, this section evaluates the level at which instruction utilizes these structural principles in supporting students in making connections within and between mathematical topics, maintaining that knowledge over time, and then applying and modifying this knowledge to find a solution to a previously unseen problem or process.

Over time, the full version of the DMT observation protocol has been refined and simplified in order to reduce the amount of training required for effective use, so it could be used by instructional coaches to provide ongoing instructional support including feedback framed around those practices shown to increase student achievement in mathematics. These refinements led to the simplified version of the DMT observation protocol. Although the two instruments are related in theory and structure, they differ in complexity and functional purpose. At this time only trained individuals have utilized these protocols as tools for instructional feedback and improvement. As the purpose of this study is to compare feedback responses of observers of differing levels of mathematical content and pedagogical content knowledge, the simplified protocol was selected for use by the three groups of participants while observing the classroom vignettes. This form was selected for this study because it could be used without training that may contaminate the data by focusing participant’s attention on specific instructional processes and ignoring others they may have focused on without training.

Procedure

During the observation phase of the study, participants viewed Video 1 first and prepared feedback comments for that instructional segment. The process was then repeated for Video 2. Participants were asked to use the simplified DMT protocol for these observations. Although each participant received the protocol, they were free to use it as they saw fit. They were also allowed to start, stop, or review sections of each video at their discretion. Following the observation of each video, participants provided written feedback to the teacher for the purpose of increasing instructional effectiveness and student achievement based on the observation.

Data were collected via a Qualtrics form where participants copied their written comments for the teacher in the video and reported the requested demographic information. In addition to the five component areas on the DMT protocol, an additional field was added in the Qualtrics collection where participants provided feedback to the teacher on items that were not specifically addressed by the protocol, such as classroom management. Care was taken to ensure that none of the participants had supervisory responsibilities over the teachers in either of the video segments. Both the observation and data collection phases took place between December 2013 and May 2014.

Limitations of the Study

There are a number of limitations that must be kept in mind when reviewing the analysis and findings of this study. First, no follow up discussion was conducted with any of the participants. Although this was a purposeful element of the design of this study, it limited my ability to receive clarification from each participant on the intentions of each comment. Some comments were very brief, with little context, and therefore left open to interpretation.

It is possible that my own interpretation of the comments may have limited or defined certain themes in ways that were not intended by the participant.

Second, feedback generally includes a conference between the observer and the teacher for the purposes of discussion and clarification. This study was intentionally focused on the preconference data for the purposes of illuminating the viewpoint of the observer only. Some may argue that ignoring the discussion between the observer and teacher limits the raw data in such a way that it does not meet the definition of feedback. However, allowing such a discussion adds the possibility that an observer's feedback may be influenced by the perspective of the teacher being observed, which may obscure the very differences this study is trying to identify.

Third, the sample size for this study is relatively small: $N = 15$. With only 15 participants, care must be taken not to generalize the findings of this study to a larger population. This limitation is revisited prior to the recommendations section of this study in Chapter 5.

Finally, there is a possibility that my own biases, experiences, and language may have influenced the neutrality of the inductive process. Identification of emerging themes may have been influenced by personal experience and my ability to communicate what I saw. It was important for me as a researcher to be open and honest with myself and others about this limitation prior to describing the coding process and the themes that emerged from it.

Analysis

One purpose of this study was to compare feedback results from three types of observers that interact with mathematics teachers. As previously described, there are varying group characteristics that may impact the content of the feedback provided by members of each of these groups. An inductive analysis approach was utilized to allow themes from the feedback provided by the participants to emerge from the study rather than constraining them from the beginning. Thomas (2006) described inductive analysis as “approaches that primarily use detailed readings of raw data to derive concepts, themes, or a model through interpretations made from the raw data by an evaluator or researcher” (pg. 238).

The first phase of analysis began in August 2014. The feedback comments were blinded to prevent the identification of the participant or group the feedback came from. The data were then read to get a general sense of the participants’ comments. No codes were created or attached to feedback during this initial reading.

In a second read, the entire data set was once again reviewed. At this point themes connected to the DMT observation tool began to surface. This made sense as the observation tool was based on the five DMT themes previously discussed. According to Thomas (2006), a researcher should attempt to generate between three and eight themes. This further solidified the idea of using the five DMT themes to generate an initial set of codes.

The initial coding process began following the second reading, using the DMT ideas as section headers for the 5 DMT themes displayed in Table 4. For example, one participant’s comment, “different work was shown on board” was coded as Taking Student Ideas Seriously as the students were able to choose and share their methods.

While the process of linking the codes to the specific DMT categories initially appeared straightforward, difficulties soon arose which required me to rethink my original approach. The first issue arose as feedback from different categories began to overlap, or were placed by observers in categories in which they did not align. A second issue appeared when participants gave feedback that was not directly related to one of the DMT components.

Finally, and perhaps most importantly, the use of the predetermined observation themes seemed to constrain and limit the possibility of other themes rising to the surface during the analysis process. The purpose of the inductive approach is to allow themes to surface without first constraining them (Thomas, 2006). Although the themes were used after reading the data first, using the predetermined framework made it difficult to address the three previous issues in a satisfactory manner. It was evident to me at this point that I needed to interact with the raw data in a less constrained manner.

I started again with a general reading of all of the data without generating codes. Another set of themes began to surface. A second reading was completed a week later, and although no codes were assigned, the new codes were kept in mind to see if similar issues would arise as they did during the first coding process. The issues with the first coding attempt did not arise and so the second coding process began. Table 5 provides a description and examples of the codes that were used to complete the second attempt.

The initial coding phase was completed early in March 2015. In order to review the comments for coherence they were compiled in separate documents by code. The blinded comments were ordered in a random fashion. I then read through the comments to see if they matched the assigned code. Any comments that did not align with other comments in that code for one reason or another were reviewed and recoded as necessary.

During this phase I was struck with the variety of comments assigned to each of the six coding categories. As I read through the comments made in each category I found additional themes were emerging. I decided at this point to undertake a second coding process in an attempt to identify specific focus themes that existed within the initially identified codes of Query, Recommendations, and Value. I focused my attention on these three sets of codes for two reasons. First, these codes were the most clearly defined and most recognizable. Second, and perhaps more importantly, these codes represented comments most likely to be passed on to teachers in an actual conference setting, rather than notes made for the observer's use only.

Table 5
Primary Analysis Codes – 2nd Attempt

Code	Informal Description	Comment Samples
Conclusion	Arrive at an opinion through reasoning	Activities engage students in the concept of probability. The context (of clothes/outfits) fits students' lives. You really listened to your students' ideas and responded with an inquiry stance.
Description	A written account of an event	Student groups brought up the topic of fractions when identifying the left-over blocks when solving a specific word problem. Lots of giggling/laughing from members in the audience during the volunteer's role
Evidence	Signs or indications in support of something	Encouraged prior learning application: "Let's use that knowledge and bring it back to our boxes…" In last few minutes, finally identified and used word connections between "chance" and "probability" – i.e. "What is the chance that you will get… then your probability is…"
Query	Asking politely for additional information	How could you incorporate higher order questioning beyond the question/one word answer format? How do you use this information to adjust the current lesson or drive future lessons?
Recommendation	A suggested course of action by one in an authoritative position	The students' models could have been connected more explicitly. For example, the ratio table and the arrays could be linked for more students in order to extend their thinking. Consider exploring student responses further during the class discussion when incorrect answers are expressed during questioning.
Value	Identifying the worth, importance or usefulness of something	Visuals on board are good. Overall sense – excellent lesson focused on students' mathematical thinking and perseverance.

In a separate document beginning with the Query code, I randomized all of the comments and read them without developing any codes. During a second reading, comments were grouped and color-coded based on the content focus of the comment. Once they were grouped, a set of focus codes was developed to capture the common idea within the comments. As with the initial set of codes, all focus comments were then grouped together by code and reviewed as a group for coherence. Any codes that seemed to deviate from the other codes were reviewed and recoded as needed. During this second coding process, few if any codes needed recoding. However, I split the assessment code into two, more specific codes. The first code focused on using assessing student understanding. The second assessment code addressed using assessment to inform instructional practices. Table 6 displays the focus codes developed from the second reading of the Query codes.

Table 6
Query Focus Codes and Examples

Code	Informal Description	Comment Samples
Assessment of Student Understanding	Assessment for the purpose of measuring student understanding	Who did not benefit as much out of the lesson? How do you know? How do you determine their depth of understanding?
Assessment to Inform Instruction	Using assessment to modify or plan instruction	Where will you go next based on what you heard and saw today? How could you informally/formally assess students of their learning to drive your instruction for the next lesson or series of lessons?
Conceptual / Connections	Making or developing connections or relationships	At what point in the lesson did you sense that students were being pressed to think about the mathematical concepts in probability in ways that will be useful in later grades? Explain how you moved your students' understandings forward either in

		representational models or abilities to generalize or make connections with other math concepts.
Mathematics Content	Related specifically to the structure of mathematics	Probability is based on meaningfully understanding ratio concept – was this previously addressed? The use of ‘part of a set’ for fraction understanding in 3rd grade is not in the CCSS. What was the rationale for going this direction?
Instructional Practices	Elements of instructional practice	How could you incorporate higher order questioning beyond the question/one word answer format? How could students participate in the design/modeling of the lesson more than simply putting on the clothing?
Management / Class Culture	Elements of classroom management and culture	How to you establish a classroom culture that allows students to be incorrect and to feel comfortable sharing their incorrect thinking? The students appeared to participate at a high level. Was this because they were examining their own thinking?

I used the same coding process to conduct a second reading of the Recommendations and Value codes as well. I found the same focus codes emerging from this second reading process. I also used the same coherence process to finalize the focus codes for the initial Recommendations and Value codes. Table 7 provides examples of comments in each of the focus codes within the Recommendations code while Table 8 provides examples of focus codes within the Value code.

Table 7
Recommendations Focus Codes and Examples

Code	Examples
Assessment of Student Understanding	… but I would love to see individual work and formative assessment based on it. Reflection at the end of the lesson gives you a way to informally assess students of their "walk away" of the lesson.
Assessment to Inform Instruction	(No comments for this category.)
Conceptual Understanding and Connections	A possible aspect to improve on would be to be even stronger in making connections between models for more students. This was an aspect of some of the interaction but could have been given more emphasis. Structurally I think the connection to multiplication could have been more meaningful all the way around.
Content	When dealing with fractional remainders, be cautious that you are essentially treating fractions as "set" models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts. Perhaps this would have been better suited to discuss the connection between combinations and multiplications rather than spending time on probability.
Instructional Practices	Might be good to label pants and shirts on the table to assist students. Teacher provides justification for students' answers here. I suggest allowing students to do so.
Classroom Management and Culture	(No comments for this category)

A final check for focus code coherence was performed once all of the comments in the Form codes had been coded with Focus codes. This step involved randomly grouping all comments by Focus code without consideration for the Form code. Grouping comments in this fashion allowed me to read through all comments using a particular code to determine whether they referred to the same idea and code construct. Any codes that were questionable were flagged for further review. This final review completed the coding process.

Table 8
Value Focus Codes and Examples

Code	Examples
Assessment of Student Understanding	Now we will "make questions" – show work. I love this idea… I'm not sure how it happened for individual students.
Assessment to Inform Instruction	I think this represents a strong representation of student thinking as the driver for education. The kids clearly enjoy this class - probably for far better reasons than presented in the first video.
Conceptual Understanding and Connections	I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue.
	Models used are appropriate for the task.
Content	"9 out of 3" makes no sense may be problematic
	In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.
Instructional Practices	The teacher did an excellent job of having students discuss their ideas, both correct and incorrect.
	Effective modeling with the Smart Board.
Classroom Management and Culture	Extremely well-managed lesson with students standing, sharing, and enjoying the idea of trying on clothes.
	If this is what your classroom looks like on a typical day for math instruction your students will be well prepared for later grades and highly successful at meeting grade level expectations.

Testing Coding Reliability – Second Reader

Once the coding process was completed I asked a colleague to code a random selection of codes as a measure of reliability. The colleague in question has had experience as an elementary teacher, building administrator, district curriculum director, and state mathematics coordinator. I used the following process to generate the random list of codes. A minimum of 10% of the comments in each Focus code category was randomly selected using a random number generator. Although 10% was set as the initial threshold, a minimum of 5 comments from each category was selected randomly to make up the sample set of 30 comments for the second reader test. The formula found in Equation 1 (Miles & Huberman, 1994) was used to determine the level of reliability of the coding.

Equation 1.

$$Reliability = \frac{\#\ agreements}{\#\ agreements + \#\ disagreements}$$

Following the coding check, the second reader and I reviewed the differences between our coding. During the conversation we noted several factors that played a role in these differences. The first factor was the amount of context that was included with each comment in question. It was noted that too much or too little context had an impact on the variability of code assignment. The second factor emerged for comments that contained elements of more than one coding area. For example, the comment, “What will you do next? Will you focus on division as an operation, fraction concept, or area?” contained elements of both content and connections. Table 9 shows the results of the reliability calculations for both the Form and Focus comments coded by both myself and the second reader.

As the overall number of these differences was small, and the overall reliability as well as the reliability value for both primary and secondary codes was above the .8 threshold, I determined that the definitions and coding of the comments were satisfactorily consistent and reliable. This determination allowed me to proceed with the data analysis phase of the study.

Table 9
Reliability Values for Coding Reliability

Code	Reliability
Overall	.95
Primary Code	.97
Query	1.00
Recommendation	.83
Value	1.00
Secondary Code	.93
Assessment of Student Understanding	1.00
Assessment to Inform Instruction	1.00
Conceptual / Connections	.8
Mathematics Content	1.00
Instructional Practices	1.00
Management / Class Culture	1.00

Test for Statistical Significance

The final analysis of the data was a test for statistical significance between the frequencies of each type of form and focus of the feedback provided by three participant groups. For the purposes of this analysis, I used the Chi-square test with $p < .05$ to determine whether there was evidence of a relationship between the groups and 1) the form of the feedback, and 2) the focus of the feedback given by the participants in each group (Field, 2013).

For part 1, the null hypothesis (H_0) was there is no evidence of a relationship between the participants' group and the forms of feedback they provided. The alternative hypothesis (H_1) was there is evidence of a relationship between the participants' group and the forms of feedback they provided. For part 2, the null hypothesis (H_0) was there is no evidence of a relationship between the participants' group and the focus of the feedback they provided. The alternative hypothesis (H_1) was there is evidence of a relationship between the participants' groups and the focus of the feedback they provided.

Cramer's V was used to determine the effect size of the results of the Chi-square test. Based on guidelines suggested by Cohen (1988), an effect size of 0.1 was considered to be a small effect size, 0.3 was considered a medium effect size, and 0.5 is considered a large effect size.

In chapter 4 I identify and discuss the themes that emerged from the coding of the raw data. Examples of raw text relating to each of the themes is identified, and links or relationships between the various themes will be explored. These links were "based on commonalities in meanings between categories or assumed causal relationships" (Thomas, 2006, p. 240). Also included in Chapter 4 is a comparison of the appearance and prevalence of these themes among and between the participants of the difference groups. The purpose of this analysis was to describe the similarities or differences in the themes of feedback provided based on the characteristics of these groups.

CHAPTER 4: FINDINGS

In this chapter I report the findings of the data analysis conducted for this study in order to address the following research questions:

1) What themes emerge from feedback given to teachers of mathematics by observers with different content and focus backgrounds?
2) Does an observer's educational background influence the content of feedback that is given?

Emerging Feedback Themes

As part of the discussion that follows I review the themes that emerged from the readings and coding processes described in Chapter 3. Each theme is defined and possible links between the themes are explored in order to address the first research question. These themes are then compared by group to address the second research question.

Forms of Feedback

As described in Chapter 3, six primary codes emerged from the comments provided by each participant. Upon examination, these codes suggest different forms in which feedback might be given. These forms are listed and described in Table 10.

Of these six forms, three were identified as being the most meaningful for this study: Query, Recommendations, and Value. These forms pressed for some form of additional thought, modifications to instructional practice, or consideration of the importance of the instructional element being commented on. They also represented the form of comment that might be made to a teacher during a post-observation conference, which is a specific purpose of this study.

Table 10
Feedback Forms

Forms	Definition	Examples
Conclusion	A comment describing a decision, an opinion, or a judgment reached based on a review of evidence. Such a decision may be based on deductive or inductive reasoning.	Direct connections were made to other strategies and this was encouraged by the teacher.
Description	A comment reviewing identified elements of an instructional segment. Such comments are made from the point of view of the observer.	When a student gave an incorrect answer, the question was asked again.
Evidence	A comment identifying elements such as written work, verbal statements, or other elements leading to one or more additional feedback comments.	The teacher points this out at one point, asking the students not to copy their friends.
Query	A comment in the form of a question or request for additional thought or information posed to the teacher by the observer.	What about probability did you want your students to understand?
Recommendation	A comment providing a specific suggestion given to the teacher by the observer suggesting a course of action for future planning or instructional episodes.	Discuss enactive – iconic – symbolic representations and what aspects the task addressed or didn't address.
Value	A comment made by the observer highlighting the importance or worth of an observed behavior or practice. The comment may be either positive or negative.	In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.

Form theme: Query. The form query was one of the easiest to recognize and distinguish from other themes as most queries were structured in the form of a question. The length of each comment ranged from a single word, such as “Benefits?” to several sentences, such as: “Individually, who had a break through today? Who was left behind? How do you know?” The content of such requests often differed, however. Here, the content of a query will be described in a general sense. For example, queries often asked the teacher to reflect on a particular aspect of the instructional segment and to provide a response to such reflection. These aspects often referred to teacher-controlled elements such as instructional components and classroom culture. Other queries asked teachers to reflect on aspects of student involvement as well as elements of mathematical content in general. Some of the observed elements of the instructional episodes in each video might be noticed by observers regardless of content background, while others require specific mathematical content knowledge to identify and comment on. This difference is discussed further in Chapter 5.

The following query comments provide a general sense of the topics of feedback belonging to this theme. First, there were comments that referred to **instructional practices and decisions**. These components included learning objectives and targets, instructional decisions, and other aspects of instruction that the teacher has control over. Some examples were:

- ”How do you know what concepts to focus on and which you will de-emphasize in the moment?”
- “Was it the intent of the teacher to go this direction or did they decide to go this way based on what they saw? Was a clear goal set for the lesson?”

Other queries asked the teacher to reflect on aspects of the **classroom culture**. Culture may include management practices, behavioral expectations, and other elements of a classroom that have an impact on students' desire to contribute and learn in a meaningful manner. Some of these examples included:

- "How do you establish a classroom culture that allows students to be incorrect and to feel comfortable sharing their incorrect thinking?"
- "The students appeared to participate at a high level. Was this because they were examining their own thinking?"

Some of the queries related directly to **student engagement** in the instructional process. These comments focus explicitly on student interactions with the content and their own learning. For example, one participant asked,

- "What visual (iconic) models did students use?"

Another participant asked,

- "How could students participate in the design/modeling of the lesson more than simply putting on the clothing?"

Other comments belonging to the query form focused on the **content and structure of mathematics**. These comments focused on connections between concepts both within and without mathematics. Examples of this type of comment included:

- "The use of 'part of a set' for fraction understanding in 3rd grade is not in the CCSS. What was the rationale for going this direction?"
- "Probability is based on meaningfully understanding ratio concept – was this previously addressed?"

In these cases the teacher would need to reflect on their knowledge of the structure of mathematics in order to frame a response to the query.

A specific limitation of this study must be addressed at this point. Only the initial feedback was reviewed. It may be argued that not all queries require an immediate response from the teacher; rather their purpose is reflection on practice. One could ask if there is a substantial difference between a query that required an immediate response and one that did not. The feedback itself does not appear to differentiate between the two, nor does the difference impact the primary purpose of this study which is to review the themes of the feedback given following an observation in order to determine whether or not there are differences based on the content knowledge background of the observer. The primary code and definition of the theme seems to cover both cases adequately.

Form theme: Recommendations. The second theme to be used as part of the analysis was labeled "recommendations." Recommendations included suggested courses of action relating to such areas as instructional practice, work with students, and suggestions related to the content and structure of mathematics. Recommendations may be structured as reinforcement of appropriate instructional practices, enhancements, or corrections (Scheeler et al., 2004)**.** As with Query the specific content of the feedback comments had a range of subject matter and will be addressed in the similarities section of this chapter.

Participants gave quite a few recommendations based on **instructional practices**. Examples of these kinds of comments included:

- "A possible aspect to improve on would be to be even stronger in making connections between models for more students. This was an aspect of some of the interaction but could have been given more emphasis."
- "Could potentially adjust task to allow students to explore individually or in small groups prior to whole group discussion of methods or once a few outfits had been

enacted – ask students to complete the table prior to completing the enactive representations and have them compare."

- "In my limited knowledge of progressive formalization, I'm not sure that this is sufficient, but my intuition tells me that contribution of student ideas in developing the concept would have been a more solid instructional approach."

The first two comments are specific to mathematics instruction, while the third could be seen as a more general recommendation. In each of these situations the observer made some kind of recommendation or suggestion to strengthen the instructional practices and processes found within the instructional segment.

Student engagement was the subject of other comments coded as recommendations. Examples of this type of recommendation included the following:

- "Students need multiple chances to speak to the group and each other in a group or partnership."
- "I suggest spending a little more time on this so that students are not just passively responding, but pushed to self-evaluate their learning for a real purpose."

In these cases the observer focused his or her comments on how students themselves can play a more integrated role in their own learning.

Another topic of recommendations was the **content and structure of mathematics**, which can be seen in the following examples:

- "When dealing with fractional remainders, be cautious that you are essentially treating fractions as "set" models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts.

- “Perhaps this would have been better suited to discuss the connection between combinations and multiplications rather than spending time on probability.”

In each of these recommendations we see an observer requesting the teacher to consider aspects of the structure of mathematics as a content area.

Note that in each of the previously described cases labeled as recommendations, there was not a specific request for reflection, nor was there a requirement of a response from the teacher being observed. It is likely that these types of comments would be the subject of further conversations following subsequent observations and/or discussions.

Form theme: Value. The third form being used for analysis was labeled “value.” The value form highlighted specific comments from observers that gave a strong indication of whether or not the practice being identified was appropriate or not from the perspective of the observer. These comments could be either positive or negative in nature. The value theme did not require a recommendation to be made; rather it typically involved a simple indication of the relative value of the practice overall. Comments in this category covered various topics ranging from instruction and management to content. These additional categories will be further described in the next section. For now, examples of various topics will be provided to convey a general idea of the scope and range for this theme.

Some participants placed value on various **instructional components and practices**. Some examples of these kinds of comments include:

- “This is a strength of this lesson.”
- “Visuals on board are good.”
- “Overall sense – excellent lesson focused on students’ mathematical thinking and perseverance.”

- "It is important for students to try something they think "may" be correct before being told (or shown) what is correct."

Each of these comments provides an indication to the teacher of the value the observer places on some aspect of their instructional process.

Participants also identified areas of **management and classroom culture** they felt important enough to comment on. The following comments exemplify this aspect of the value theme:

- "There were important transition and management strategies employed (3, 2, 1, 0) and individual, small group, whole group scaffolding."
- "Positive classroom climate and good sense of humor."

These examples provide fairly straight-forward examples of value. However, some examples required more inference. For example, participants made comments such as:

- "If this is what your classroom looks like on a typical day for math instruction your students will be well prepared for later grades and highly successful at meeting grade level expectations."
- "Learning environment is conducive to students taking risks and questioning each other."

In each of these cases value is implied in the statement made. In the first statement value was placed on the typical day of mathematics instruction and its positive impact on student achievement. The second statement suggests that the observer believes there is value in a learning environment in which students feel safe in sharing their own ideas and critiquing the ideas of others. In both cases the value of the classroom culture may be implied from the comment made.

Similarities between the forms. The three themes previously described differed in form and purpose, making them distinguishable in most cases from each other. However, as previously noted the forms themselves did not address the content or focus of the feedback comments, nor the similarities that exist between the forms. Content differences within each form were more apparent when multiple comments were chained together within a theme. For example, one participant commented,

> "How could students participate in the design/modeling of the lesson more than simply putting on the clothing? How else might students interact with conditional probability or concepts of combinatorics?"

The two comments chained together appear to address two different ideas. The first comment asks the teacher to reflect on how he might increase the amount of student participation in the activity. The second comment asks the teacher to consider ways to build on the student interaction with this lesson, and connect them to future lessons and topics. Both comments belong to the Query theme, however, the content of the two comments suggests different purposes.

There were also relationships between the three form themes suggesting similarities in content between the themes. For example, one participant commented,

> "How do you monitor the methods students are using to problem solve? How do you determine their depth of understanding? Consider asking questions that press students to explain their thinking."

The first comments clearly fit within the Query form as the observer asked the teacher to consider and respond to concepts of assessment. The third comment also refers to assessment; however, in this case the comment belongs to the Recommendation form. Although the comments belong to two different forms, their content suggests a common element of assessment of student thinking.

Focus Themes of Feedback

While the three forms described provide one lens to compare and contrast the comments made by the three groups, the content of the focus codes suggested a more rich and robust comparison with which to compare the feedback comments. As described in Chapter 3, the focus on the content of the comments within each theme led to the development of another set of codes. These codes captured the content and similarities within the forms, and led to the identification of a set of themes which shall be referred to as focus themes. Table 11 below identifies and defines the six focus themes.

Table 11
Feedback Focus

Theme	Definition	Example
Assessment of Student Understanding	A comment that addresses how and/or whether assessment was used as a tool to understand or measure student understanding.	Reflection at the end of the lesson gives you a way to informally assess students of their “walk away” of the lesson.
Assessment Used to Inform Instruction	A comment that addresses how and/or whether a teacher uses assessment to influence current or future instructional practice.	Where will you go next based on what you heard and saw today?
Classroom Management and Culture	A comment addressing classroom management practices, including those that refer specifically to the culture and environment of the classroom.	Extremely well-managed lesson with students standing, sharing, and enjoying the idea of trying on clothes.
Conceptual Understanding and Connections	Comments that address the idea of making connections or relationships between ideas or concepts. This may occur with knowledge that already exists, or between knowledge that already exists and new knowledge that is being or will be presented.	What are some related math topics to those you were addressing?

Instructional Practices	Comments addressing procedural elements of instructional practice controlled by the teacher, e.g. task selection, lesson sequencing, questioning strategies, use of tools, algorithmic steps, or lesson activities.	Was it the intent of the teacher to go this direction or did they decide to go this way based on what they saw? Was a clear goal set for the lesson?
Mathematics Content	A comment addressing the structure or components of mathematics or a particular area of mathematics such as probability.	In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.

The purpose of the second coding section of Chapter 3 was the development of each content focus code. As such, the focus codes were developed in isolation within each initial code and form theme. The purpose of the following sections is to frame the focus themes that arose from the content focus coding as similarities within and between the three form themes, making it possible to use the focus themes as a lens through which comparisons between participant groups can be made.

Focus theme: Assessment of student understanding. Assessment was divided into two categories based on the purpose for assessment. The first of these was assessment for the purposes of approximating the level of student understanding of the content being discussed. Although all of the comments in this theme address the idea of assessing student understanding, participants addressed the idea in a variety of ways. Some of the comments were general in nature while others addressed more specific items related to content and procedures. Table 12 provides examples of such comments in each of the three form themes.

Table 12

Assessment of Student Understanding Examples by Form

Form	Comment Examples
Query	• What students benefited from the lesson? How do you know? • What are the ideas of probability your students struggle with? • How do you assess student thinking? • What did you learn about your students' understandings of the concept and their developmental use of strategies today?
Recommendation	• Reflection at the end of the lesson gives you a way to informally assess students of their "walk away" of the lesson. • I would love to see individual work and formative assessment based on it.
Value	• Now we will "make questions" – show work. I love this idea… I'm not sure how it happened for individual students.

Focus theme: Assessment used to inform instruction. Participants also addressed assessment as a method for informing instruction and planning. This theme differed from the first in that teachers were asked to consider how the knowledge gained through assessment could be applied to their own practice, both current and future. The application to the practice aspect of this focus theme differentiates it from the previous focus of assessment of student understanding. Table 13 below provides examples of this focus theme across the three form themes where they existed.

Although all but one form, recommendation, had examples of both types of assessment themes present, I observed that the majority of assessment comments were queries. There were 25 queries related to assessment as compared to 4 assessment comments that expressed recommendations or values. While this difference may be partially explained by observers limiting the number of actual recommendations given to teachers as part of an observation, I found the small number of assessment comments in the value focus to be of interest.

Table 13

Assessment Used to Inform Instruction by Form

Form	Comment Examples
Query	• How do you informally/formally assess individual students and their current depth of knowledge to press them, academically, during future lessons? • How did you plan to address these before the lesson? How did you address these difficulties during the lesson? • Where will you go next based on what you heard and saw today?
Recommendation	None
Value	• I think this represents a strong representation of student thinking as the driver for education. The kids clearly enjoy this class - probably for far better reasons than presented in the first video.

Focus theme: Classroom management and culture. Although it was not specifically addressed on the DMT observation tool, a number of participants provided specific comments related to classroom management and culture. The fact that it surfaced without prompting speaks to the general perception of its importance. This theme described elements of the classroom not specifically related to instruction. Although the overall number of comments in this theme was small, the comments provided represent a distinct and important facet of a mathematics classroom. Table 14 below displays examples of comments within the focus theme of classroom management and culture. As with previous tables the comments are organized according to the forms of feedback.

Table 14

Classroom Management and Culture by Form

Form	Comment Examples
Query	• How do you establish a classroom culture that allows students to be incorrect and to feel comfortable sharing their incorrect thinking? • The students appeared to participate at a high level. Was this because they were examining their own thinking?
Recommendation	None
Value	• Positive classroom climate and good sense of humor. • There were important transition and management strategies employed (3, 2, 1, 0) and individual, small group, whole group scaffolding • If this is what your classroom looks like on a typical day for math instruction your students will be well prepared for later grades and highly successful at meeting grade level expectations.

Focus theme: Conceptual understanding and connections. As described in Chapter 2 of this study conceptual knowledge describes knowledge that contains numerous and rich relationships in which the holder of knowledge recognizes those relationships (Hiebert & Lefevre, 1986; Hiebert & Wearne, 1986). This theme focused on comments relating to making connections such as content topics, visual models, visual models and their related symbolic expressions, solution strategies, and previous learning. Table 15 below provides examples of these types of comments found within each form.

Table 15
Conceptual Understanding and Connections by Form

Form	Comment Examples
Query	• Where would students typically use these concepts in later math courses as well as outside mathematics classrooms? • What are some related math topics to those you were addressing? • How would you move towards progressive formalization? • How else might students interact with conditional probability or concepts of combinatorics?
Recommendation	• Discuss enactive – iconic – symbolic representations and what aspects the task addressed or didn't address. • When he speaks about not being able to have a probability of 9 out of 3, I would have liked there to be some connection to probability being a percentage or fraction between 0 and 1. • The students' models could have been connected more explicitly. For example, the ratio table and the arrays could be linked for more students in order to extend their thinking. • Structurally I think the connection to multiplication could have been more meaningful all the way around.
Value	• I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue. • You did a great job of letting the students discover the misconceptions.

Focus theme: Instructional practices. The theme instructional practices focused on specific instructional choices and practices employed by the teacher during the instructional segment. Examples of these choices and practices include task selection, lesson sequencing, questioning strategies, use of tools, algorithmic steps, or lesson activities. Since instructional practices relate to nearly every aspect of the instructional process, such as planning, task selection, instructional sequencing, and delivery, it is not surprising a large number of comments surfaced to shape this categorical focus. Table 16 below provides examples of comments from each form coded as instructional practices as a focus theme.

Table 16
Instructional Practices by Form

Form	Comment Examples
Query	• Who did most of the work and thinking in this lesson? • What are the pros and cons to approaching the problem through a whole-class enactive representation? • Was it the intent of the teacher to go this direction or did they decide to go this way based on what they saw? Was a clear goal set for the lesson?
Recommendation	• Teacher provides justification for students' answers here. I suggest allowing students to do so. • I suggest taking more time with this word, to show the morphological relationship to the word probability, which is the focus of the lesson. • Consider asking questions that press students to explain their thinking.
Value	• Students were asked to explain the thinking of their table partners though, which was impressive. • The teacher did an excellent job of having students discuss their ideas, both correct and incorrect. • Visuals on board are good.

Focus theme: Mathematics content. The final focus theme that surfaced focused specifically on the content and topics of mathematics. It is not surprising that some form of feedback would focus on the subject being taught. This theme focused specifically on mathematics topics, although it should be noted that mathematics topics also appeared in some of the other focus themes such as conceptual understanding and connections. In such cases, content was extended to include elements of other themes. Comments in this theme focused more on the mathematics topics themselves. Table 17 below provides examples of comments coded as content within each of the three forms.

Table 17
Mathematics Content by Form

Form	Comment Examples
Query	• The use of 'part of a set' for fraction understanding in 3rd grade is not in the CCSS. What was the rationale for going this direction? • Probability is based on meaningfully understanding ratio concept – was this previously addressed? • What about probability did you want your students to understand?
Recommendation	• Perhaps this would have been better suited to discuss the connection between combinations and multiplications rather than spending time on probability. • When dealing with fractional remainders, be cautious that you are essentially treating fractions as "set" models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts.
Value	• "9 out of 3" makes no sense may be problematic • In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.

Summary of Themes – Research Question 1

Thus far this chapter has identified themes emerging from feedback given to teachers of mathematics by observers with different content and focus backgrounds. Group 1 consisted of mathematics content specialists. Group 2 consisted of instructional specialists in content areas other than mathematics. Group 3 consisted of elementary principals with a variety of content area backgrounds. During the course of the data analysis six forms of feedback were identified: Conclusion, Description, Evidence, Query, Recommendation, and Value. Of these six, three forms – Query, Recommendation, and Value – were selected for examination in this study and were subjected to further analysis. Based on that further analysis, six focus themes emerged from the data: Assessment of Student Understanding, Assessment Used to Inform Instruct, Classroom Management and Culture, Conceptual Understanding and Connections, Instructional Procedures, and Content. With but two exceptions, these six focus themes were found in all three form themes.

Similarities and Differences Between Groups – Research Question 2

The form and focus themes will now be used to address the second research question, "Does an observer's educational background influence the content of feedback that is given?" In order to address this question, participant comments will be reviewed using both form and focus theme lenses. This analysis was accomplished by comparing the number of comments as well as their qualitative similarities and differences.

Analysis of Group Comments by Form

The first comparison consisted of a tabulation of the total number of comments made by each group for each form theme. Table 18 below displays the results of this tabulation. Two similarities immediately surfaced from an observation of the totals. First, all three groups made more comments in the form of a query than as recommendations or value statements. Second, these comments accounted for more than half of the total comments made by each group, suggesting that having teachers reflect on their own instructional practices was an important part of the feedback process for all observers.

The differences between the groups, however, were substantial. First, mathematics content specialists contributed nearly 50% more total comments when compared with other content specialists, and nearly double the number of comments made by building principals. These differences also surfaced in the form of the feedback. In some cases, such as recommendations, the number of comments made by mathematics content specialists was double the number of comments made by other content specialists, and triple the number of comments made by building principals.

Table 18
Number of Form Comments by Group

Group	Query	Recommendations	Value	Totals
1. Mathematics Content Specialists	33	16	15	64
2. Other Content Specialists	25	8	11	44
3. Building Principals	16	5	5	26
Totals	74	29	31	134

A second comparison between groups was made by examining the form of feedback by group and video. Video 1 illustrated mathematics instruction that exemplified a traditional or procedural approach. Video 2 provided an example of mathematics instruction that utilized a more conceptual approach with procedural components and connections. Table 19 below displays the numerical results of this comparison.

Several comparisons can be made at this level. The first comparison focused on the consistency in the number of comments made between video 1 and video 2 for each group. Mathematics content specialists provided nearly identical numbers of query comments for both videos, three times the number of recommendation comments for the teacher in video 1, and twice as many value comments for the teacher in video 2. The feedback comments from other content specialists demonstrated the most consistency between videos in the number of comments provided overall. Other content specialists provided half again as many query comments for the teacher in video 1, but nearly identical numbers for each teacher in the respective themes of recommendation and value. The largest difference was seen with building principals. Only three query comments and no recommendation comments were provided for the teacher in video 2. The number of value comments for each teacher was nearly the same.

Table 19

Number of Form Comments by Video and Group

Video	Group	Query	Recommendations	Value
Traditional Instruction	Mathematics Content Specialists	17	12	5
	Other Content Specialists	15	5	5
	Building Principals	13	5	2
Conceptual Instruction	Mathematics Content Specialists	16	4	10
	Other Content Specialists	10	3	6
	Building Principals	3	0	3

A second comparison involved the number of comments from each group in each form. For video 1, the numbers of comments made as queries were very similar, only differing by four total comments between the minimum and maximum number. However, the numbers were much different for video 2. Mathematics content specialists provided nearly the same number of query comments for the teacher in video 2 as they did for video 1. Other content specialists provided one-third fewer comments for the teacher in video 2, and building principals provided only two comments for the teacher in video 1 and three comments for the teacher in video 2. This suggests that a difference in content knowledge may impact the amount of feedback provided to a teacher based on the nature of the lesson, whether it is more procedural or more conceptual.

The recommendation theme also saw a number of differences between the groups. Mathematics content specialists provided more than double the number of recommendations for the teacher in video 1 as compared to the other two groups. For the teacher in video 2, both mathematics and other content specialists provided a few comments, however building principals provided no recommendations at all. In the value theme, mathematics content specialists provided twice as many comments for the teacher in video 2 as compared to video 1, while other content specialists and building principals provided nearly identical numbers of comments for the teachers in both videos.

Forms – Qualitative Comparison by Video

The frequencies alone suggest a fundamental difference in the formation of feedback comments following each respective observation. This difference would suggest a third, more qualitative comparison be made regarding the differences in the form of feedback. Two comparisons suggested by Thurlings et al. (2012) were applied for this comparison. The first, general comments versus specific, applied to all three forms. The second type, positive comments versus negative, applied to the recommendation and value forms. For the first category, comments were examined to see whether they applied to instruction in general, or specifically to mathematics instruction and/or content.

Video 1: Procedural instruction – general versus specific feedback. For mathematics content specialists, 12 of the 17 query comments were related specifically to mathematics instruction or content. Examples of these kinds of comments include:

- At what point in the lesson did you sense that students were being pressed to think about the mathematical concepts in probability in ways that will be useful in later grades?
- What are the ideas of probability your students struggle with?

- Probability is based on meaningfully understanding ratio concept – was this previously addressed?
- How else might students interact with conditional probability or concepts of combinatorics?

More general comments from this group included examples such as:

- How could students participate in the design/modeling of the lesson more than simply putting on the clothing?
- What made you choose to approach (or set up) the task the way you did?

For other content specialists, 7 of the 15 comments had elements of mathematics content or instruction. For example:

- How are statements of probability related to other math concepts such as fractions, how are they different?
- How is probability helpful, what real problems does it help us solve.
- What about probability did you want your students to understand?

General comments from this group included:

- Who did most of the work and thinking in this lesson?
- What modes of formative assessment did you use?

For building principals, 4 out of the 13 query comments had some connection to mathematics content or instruction. They were:

- What would you say to a student who responded, "There are 9 possibilities every time I choose 3 outfits."
- Does each shirt/pants have an equal likelihood of being chosen? Is each article equally represented?

- How could you formal tag, in a symbolic way, the proportion of outfits to the total available?
- There are multiple ways to demonstrate all of the combinations possible with the shirts and pants along with the ratio-type table you modeled. How could we give kids the chance to think about those ways and then share?

More general comments from this group included:

- How could you incorporate higher order questioning beyond the question/one word answer format?
- How could you informally/formally assess students of their learning to drive your instruction for the next lesson or series of lessons?
- How do you monitor the methods students are using to problem solve?

After analyzing query comments from video 1, there is evidence that differences exist in the feedback provided by each of the three groups both numerically and qualitatively. When the query comments were compared considering the specificity of the mathematics focus, mathematics content specialists asked teachers questions directly connected to mathematics nearly twice as often as the next closest group, other content specialists. When compared with building principals, the difference was more than three times as large. There was also a qualitative difference in the depth of general questions posed by participants of different groups. Mathematics content specialists and building principals asked questions that required reflection and analysis. Other content specialists tended to pose questions requiring little if any analysis.

Upon review of the recommendations made by mathematics content specialists, 7 of the 12 comments related specifically to mathematics instruction and content. Some examples of these types of comments include the following:

- Here's an example: "Design a card game for 3 players in which each play has an equal chance of winning. Now, let's design a variation of that game in which one player has better chance to win but that seems fair to the players."
- When he speaks about not being able to have a probability of 9 out of 3, I would have liked there to be some connection to probability being a percentage or fraction between 0 and 1.
- Perhaps this would have been better suited to discuss the connection between combinations and multiplications rather than spending time on probability.

The remaining 5 comments were of a more general nature and included the following examples:

- Consider exploring student responses further during the class discussion when incorrect answers are expressed during questioning.
- Could potentially adjust task to allow students to explore individually or in small groups prior to whole group discussion of methods or once a few outfits had been enacted – ask students to complete the table prior to completing the enactive representations and have them compare.

Other content specialists contributed five recommendations for the teacher in video 1. Of these five comments only one pertained specifically to the content area of mathematics:

- I suggest more student input and rationalization to come to the conclusion that 9:3 is impossible.

Examples of more general comments for this group include the following:

- Teacher provides justification for students' answers here. I suggest allowing students to do so.
- I suggest spending a little more time on this so that students are not just passively responding, but pushed to self-evaluate their learning for a real purpose.

Of the five recommendations provided by building principals only one was specifically addressed the mathematics:

- Extension: How would the probability change if you wear a shirt and/or pants one time and put it in the laundry and don't wear it again?

More general comments included the following feedback:

- Reflection at the end of the lesson gives you a way to informally assess students of their "walk away" of the lesson.
- Consider asking questions that press students to explain their thinking.
- Students need multiple chances to speak to the group and each other in a group or partnership.

The differences in the specificity of the recommendations suggested by the different groups are interesting. Over half of the recommendations made by mathematics content specialists related specifically in some way to the content of mathematics. Only one comment from each of the other groups addressed mathematics content specifically. Most comments made by other content specialists and building principals were not connected specifically to the content of mathematics. As with queries there was a distinct difference in the feedback provided by the three groups related to the specific nature of the feedback provided.

There were fewer value comments than the other two forms reviewed, however there was an identifiable difference here as well in relation to the specificity of the comments provided. Of the five value comments made by mathematics content specialists, two of the comments related specifically to mathematics content and instruction:

- “9 out of 3” makes no sense may be problematic.
- In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.

The three remaining comments related to the role the students played in the lesson based on instructional decisions made by the teacher.

- It is important for students to try something they think “may” be correct before being told (or shown) what is correct.
- I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue.
- Beyond this, I would say, because all models and thinking were the teachers, that based on the descriptions of this category he failed miserably here.

Of the five value comments made by other content specialists, none were specifically related to mathematics. The comments were also short and succinct.

- Visuals on board are good.

- Now we will "make questions" – show work. I love this idea… I'm not sure how it happened for individual students.
- Model is appropriate.
- Extremely well-managed lesson with students standing, sharing, and enjoying the idea of trying on clothes.
- Effective use of math journals as resources for future problem solving.

Building principals contributed two value statements, both of which applied generally:

- The more opportunities we can give students to write, discuss, and justify their thinking the better.
- Positive classroom climate and good sense of humor.

As with the forms query and recommendations, there was an identifiable difference in the specificity of the content of the comments made to the teacher in video one by the participants of different groups. Mathematics content specialists provided more feedback specifically related to mathematics content and instruction than the other two groups.

Video 1: Procedural instruction – positive versus negative feedback. The second comparison suggested by Thurlings et al. (2012), positive versus negative feedback, was used as a second comparison for the forms recommendations and value. For the purpose of this comparison I viewed positive feedback as an affirmation by the observer that the teacher was addressing the topic appropriately, and the instructional approach taken was appropriate for the content and students in the class. Any positive recommendation or value statement was a suggestion for increasing the effectiveness of an already effective instructional segment. A negative feedback comment took issue with the correctness of the instruction or instructional practice, and identified specific errors made by the teacher or suggested changes to be made to approach the content in a structurally correct manner. The two forms will be discussed together for this comparison.

As previously discussed, mathematics content specialists offered 17 comments identified as recommendations or value to the teacher in video 1. Of these 17 comments, eight were characterized as positive while nine of them were characterized as negative. Examples of the positive comments include the following:

- Could potentially adjust task to allow students to explore individually or in small groups prior to whole group discussion of methods or once a few outfits had been enacted – ask students to complete the table prior to completing the enactive representations and have them compare.
- Might be good to label pants and shirts on the table to assist students.
- Could potentially adjust task to allow students to explore individually or in small groups prior to whole group discussion of methods or once a few outfits had been enacted – ask students to complete the table prior to completing the enactive representations and have them compare.

In contrast, the following examples were identified as negative feedback, two of which referenced specific mathematical content:

- “9 out of 3” makes no sense may be problematic
- I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue.

- Beyond this, I would say, because all models and thinking were the teachers, that based on the descriptions of this category he failed miserably here.
- In a critique of the information presented, I had some difficulty with his overall description of probability and use of theoretical assumptions as a dictate on what would occur in reality. I think he rushed to some generalizations that can cause misconceptions in the future.

I noticed that nearly every comment made by mathematics content specialists left the teacher with something to consider regardless of whether the comment was positive or negative. At times this made the distinction between positive and negative comments more difficult, a situation that was not the case with the other two groups as shall be seen in the next comparisons.

In contrast to mathematics content specialists, other content specialists made ten total comments identified as recommendations or value statements. Of these ten comments, seven were identified as positive while the remaining three were characterized as negative, only one of which addressed mathematics content specifically. Examples of the positive comments included:

- Visuals on board are good.
- Model is appropriate.
- Extremely well-managed lesson with students standing, sharing, and enjoying the idea of trying on clothes.
- Effective use of math journals as resources for future problem solving.

The following comments were identified as negative:

- Teacher provides justification for students' answers here. I suggest allowing students to do so.
- I suggest more student input and rationalization to come to the conclusion that 9:3 is impossible.
- In my limited knowledge of progressive formalization, I'm not sure that this is sufficient, but my intuition tells me that contribution of student ideas in developing the concept would have been a more solid instructional approach.

Building principals made seven comments labeled as recommendations or value, none of which addressed mathematics content specifically. Of these comments one comment was characterized as negative: "Students need multiple chances to speak to the group and each other in a group or partnership." The other six comments either directly pointed out a positive aspect of the lesson or suggested a way to enhance the lesson. Examples include:

- Positive classroom climate and good sense of humor.
- I wonder if after they caught the "gist" of the pattern the lesson could be adapted to have students pair/share, predict, continue the pattern.

This positive versus negative feedback comparison provided a striking difference between the three groups. Mathematics content specialists identified concerns that related specifically to the way the content itself was addressed by the teacher, and how the approach may lead to student misconceptions as they were introduced to additional topics in the future. Several comments even suggested that the lesson itself, while being fun and entertaining to the students, may not support the students in their acquisition and understanding of fundamental mathematics. None of the participant comments made from the other content specialist group or the building principal group identified issues with the content instruction. This difference and its importance are discussed in Chapter 5.

Video 2: Conceptual instruction – general versus specific feedback. The following section will continue the two comparisons of general versus specific and positive versus negative with regards to the feedback given to the teacher in video 2. This video provided an example of instruction focused on conceptual learning in addition to procedural learning. Mathematics content specialists provided 16 query comments, four of which were identified as being specific to mathematics content and instruction. These comments were:

- What will you do next? Will you focus on division as an operation, fraction concepts, or area?
- How do you know what concepts to focus on and which you will de-emphasize in the moment?
- If students had not gone the direction of writing the remainder of the fraction, what other direction might you have taken the lesson?
- The use of ‘part of a set’ for fraction understanding in 3rd grade is not in the CCSS. What was the rationale for going this direction?

Examples of more general comments made by this group included:

- The students appeared to participate at a high level. Was this because they were examining their own thinking?
- What students benefited from the lesson? How do you know?
- Was it the intent of the teacher to go this direction or did they decide to go this way based on what they saw? Was a clear goal set for the lesson?

Other content specialists made ten comments belonging to queries. Of these queries, two were identified as addressing mathematics specifically:

- Explain how you moved your students' understandings forward either in representational models or abilities to generalize or make connections with other math concepts.
- Context was obscure and problem could have been situated in a stronger context. For example, what was the reason that boxes were being stacked. Why did it matter?

The remainder of the examples applied to instruction more generally. Examples of these comments included the following:

- How could the teacher be sure no students left with misconceptions – that at least were addressed later in another lesson.
- What was your objective? How do you know that all your students met it?
- Individually, who had a break through today? Who was left behind? How do you know?

Building principals made three query comments, only one of which closely tied to mathematics instruction practices:

- How do you press students to have/show equal representation of all modes (e, I, and s)?

The two general comments were:

- How do you informally/formally assess individual students and their current depth of knowledge to press them, academically, during future lessons.

- You seem to informally assess their comments by "working the room" and listening to student discussions. Is this intentional? How do you use this information to adjust the current lesson or drive future lessons?

As with video 1 there were discernible differences in the specificity of the feedback comments given by the three groups of participants. Mathematics content specialists provided more comments specifically related to mathematics than the other two groups combined. There was also a noticeable difference between the numbers of specific comments provided to the teachers of the two videos. A possible significance of this difference will be explored following the positive versus negative comparison that follows later in this section.

The general versus specific analysis of the recommendations form yielded some surprising results. All four recommendations made by mathematics content specialists focused specifically on aspects of mathematics content or instruction. They were:

- The students' models could have been connected more explicitly. For example, the ratio table and the arrays could be linked for more students in order to extend their thinking.
- A possible aspect to improve on would be to be even stronger in making connections between models for more students. This was an aspect of some of the interaction but could have been given more emphasis.
- When dealing with fractional remainders, be cautious that you are essentially treating fractions as "set" models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts.
- This could be a great opportunity to build on what they know. However – if the goal is division, then it might take away from what students get from solving the original

task. Without seeing the whole lesson (or previous/next days) this is hard to determine.

Only three recommendations were offered by other content specialists, only one of which had at least a weak link to mathematics specifically:

- Context was obscure and problem could have been situated in a stronger context. For example, what was the reason that boxes were being stacked.

The other two comments focused on general instructional practices:

- … but I would love to see individual work and formative assessment based on it.
- Suggest teacher elaborates on this so that student understands why this is important.

None of the building principals provided recommendations to the teacher in video 2.

It would seem that the conceptual nature of the second video segment had an impact on both the number of recommendations made, as well as the specificity of the content of those recommendations, whether the content related to mathematics specifically or to instruction in a more general sense. There are several reasons why this might be. First, an observer lacking an understanding of the content themselves would likely have a limited ability to notice important conceptual moments of instruction and student thinking. Second, if important moments are not noticed, whether they be positive or negative, an observer would be severely limited in their ability to provide recommendations for either instructional improvement or enhancement.

The final general versus mathematics-specific comparison was made between value comments made by the three groups. Upon review of the value comments made by mathematics content specialists, three of the ten were found to be specifically related to mathematics:

- There was an effective use of mathematical models and well-orchestrated student discussion.
- Good focus on identifying the unit fraction and/or the ‘whole’
- How she addressed the issue of the child using the ratio table was interesting; the student was counting by 7s (attempting to) but misrepresents the number of stacks represented by the value below. She asks the student to transfer the ratio table to a picture to better examine the mistake.

All six of the comments made by other content specialists were general in nature. For example:

- Nice opener with ticket from yesterday.
- Overall sense – excellent lesson focused on students’ mathematical thinking and perseverance.
- Learning environment is conducive to students taking risks and questioning each other.

Building principals contributed three value comments. Of these, two had reference to instructional practices related to mathematics:

- Thoughtful and thorough discussion was had by the group regarding how to appropriately label a portion of a stack using a fraction.
- Models used are appropriate for the task.

As previously noted, the majority of the comments focusing on mathematics content and instructional approaches came from mathematics content specialists. Building principals provided several specific value comments while all of the comments from other content specialists were found to have a more general instructional focus. As with the results from video 1, the significance of these findings is discussed in Chapter 5.

Video 2: Conceptual instruction – positive versus negative feedback. As with the analysis process used to compare the feedback given to the teacher of the video 1 segment, the recommendations and value comments for video 2 were identified as either positive or negative in nature. In a review of the recommendations and value statements given by mathematics content specialists, one comment might be best characterized as advisory:

- When dealing with fractional remainders, be cautious that you are essentially treating fractions as “set” models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts.

All of the other 14 comments were positive in nature, although all of the recommendations made specific suggestions as to how to improve the instructional segment. Of the nine comments made by other content specialists, one comment was identified as negative:

- Context was obscure and problem could have been situated in a stronger context. For example, what was the reason that boxes were being stacked.

All of the other comments made by this group were positive yet general in nature as has been previously discussed. In addition, all of the feedback provided by building principals was positive in nature.

Results of video comparison by form. The previous analysis clearly identifies differences in group feedback for each of the teachers in the two videos. However, when the overall results of both videos were compared, other interesting differences surfaced and led to an interesting and important finding. First, there were more specific comments made to the teacher in video 1, especially by mathematics content specialists. The second difference was that a large number of negative comments was given to teacher one as compared to teacher two, again predominantly by mathematics content specialists. The results of this analysis suggest that mathematics content specialists recognized weaknesses and errors in the content component of the lesson that prompted them to provide more mathematically specific feedback in all three forms to teacher one as compared to teacher two. It is more important to note that only mathematics content specialists responded in this way as compared to the other two groups of observers that did so minimally at best.

Nearly all of the recommendations and value statements made to the teacher in video 2 were positive in nature. This perhaps led to more comments being focused on more general aspects of the instructional segment being observed by participants of all three groups. Based on the results of this analysis it appears that the content knowledge of the observer has an impact on the ability of the observer to 1) recognize both major and minor errors in content instruction, and 2) provide specific feedback designed to address areas of weakness in content and instructional practices that may hinder student understanding.

Mathematics content specialists observe the content portion of the classroom as a significant, perhaps even initial element on which to focus and provide feedback on. Other content specialists and building principals that have a content specialty other than mathematics focused their comments on general instructional practices. For mathematics content specialists, once the content portion of the lesson has been solidified, they may then turn their attention to other areas of instructional practice to enhance learning opportunities for students.

Focus Theme Comparison

The final comparison of this study was made by reviewing the feedback of each form using the focus themes identified earlier in this chapter. These themes were: Assessment of Student Understanding, Assessment Used to Inform Instruction, Classroom Management and Culture, Conceptual Understanding and Connections, Instructional Procedures, and Content. The purpose of this section is to determine whether there are identifiable differences in the content of feedback provided by the three groups of participants. Table 20 displays the total number of comments from each focus theme made by participants from each group.

Table 20
Total Number of Focus Theme Comments by Group

Group	Assessment of Student Understanding	Assessment Used to Inform Instruction	Classroom Management and Culture	Conceptual Understanding and Connections	Instructional Practices	Content
1	3	3	3	27	20	8
2	11	3	3	10	14	3
3	5	4	1	3	11	2

Note: Group 1 = mathematics content specialists; Group 2 = other content specialists; Group 3 = building principals.

While there were some similarities, such as those in the Assessment Used to Inform Instruction and the Classroom Management and Culture themes, there were striking differences in the other four themes. The first difference is in the large number of comments made by other content specialists in the Assessment of Student Understanding theme. These participants contributed more than half of the total comments made in this category.

The second difference can be seen in the Content theme. While the total number of comments made by each group was not as different as in other areas, mathematics content specialists made four times as many comments relating to the mathematics content of the two video segments as building principals, and nearly three times as many as other content specialists. While this may not be a surprising result as mathematics content specialists have had the most experience with and knowledge of mathematics content, it does provide evidence that the three groups provided vastly different amounts of feedback related to mathematics content. Those with the greatest content knowledge provided the most feedback focused on mathematics content. This difference is explored in more detail in the section discussing comments by form theme and video number.

The greatest difference between the groups can be seen in the Conceptual Understanding and Connections and Instructional Practices themes. Mathematics content specialists contributed 17 more comments than group two and 24 more than group three. These numbers are interesting when compared with the overall number of comments made by other content specialists and building principals. Building principals contributed 26 comments total while other content specialists contributed 44 comments overall. Considering the importance placed on conceptual understanding and making connections in mathematics (Carpenter, 1986; Davis, 1986; Hiebert et al., 1997; National Council of Teachers of Mathematics, 2014; Vergnaud, 1997), this difference may play a significant role in instructional improvement and student achievement.

It is clear there are numeric and content differences between the participants of the different evaluator groups. In the next level of review the number of comments was further disaggregated by form. Table 21 provides the results of this disaggregation.

A review of the comments at this level further highlighted identifiable differences between the group comments. Mathematics content specialists asked teachers to reflect and respond to prompts regarding conceptual understanding, connections, and instructional practices more frequently than the other groups. Other content specialists and building principals made equal numbers of comments regarding instructional practices and content.

While other content specialists did ask the teachers being observed to respond to more conceptual questions than building principals, it is clear that mathematics content specialists identified opportunities for the teachers in the videos to reflect on both conceptual understanding and instructional practices much more often than the other groups.

Table 21

Number of Focus Theme Comments within Each Form Theme by Group

Theme and Group	Assessment of Student Understanding	Assessment Used to Inform Instruction	Classroom Management and Culture	Conceptual Understanding and Connections	Instructional Practices	Content
Query						
1	3	2	2	13	11	2
2	9	3	0	6	5	2
3	4	4	0	1	5	2
Recommendation						
1	0	0	0	9	3	4
2	1	0	0	1	5	1
3	1	0	0	0	4	0
Value						
1	0	1	1	5	6	2
2	1	0	3	3	4	0
3	0	0	1	2	2	0

Note: Group 1 = mathematics content specialists; Group 2 = other content specialists; Group 3 = building principals.

The participant comments associated with the recommendations theme also revealed identifiable differences between the groups worth further study. Mathematics content specialists clearly focused on the importance of identifying opportunities where the teachers in the two videos might assist students in making important connections within and between mathematics topics. The overall difference is striking. Nine of the ten comments coded as Conceptual Understanding and Connections were made by participants with significant mathematics content and instructional knowledge.

Mathematics content specialists also provided the most recommendations related to content. Four of the five comments in this theme were attributed to this group. Again, as with the Query theme, this result may not be surprising given the background of the participants;

however, the difference between the groups suggests a relationship between the content knowledge of the observer and the ability of that observer to then identify instructional opportunities for teachers to help students build conceptual understanding and depth in the content itself in the moment that instruction is taking place. Within the context of a real-time observation, such moments pass quickly for both the teacher and the observer.

The third difference can be seen in the number of comments made related to instructional practices. Although the differences between the groups were slight, other content specialists and building principals did contribute more comments in the area of Instructional Practices than mathematics content specialists. It can also be seen that other content specialists and building principals made more procedural recommendations to the teachers observed in the videos than all of the other secondary themes combined. This result may suggest that observers without a specific mathematics background may focus their observation and recommendations on general instructional practices rather than on the connections and content of the instructional segment. While such practices are integral to classrooms and instruction, the lack of a tie to conceptual understanding and content limits the power of the given feedback to encourage and support difficult and necessary instructional changes such as those described in Chapter 2 of this study.

Although there were fewer overall comments of value contributed by the study participants, similar trends in focus responses can be identified. Mathematics content specialists contributed as many comments related to the value of observed Instructional Practices as well as instances of Conceptual Understanding and Connections as the other two groups combined. In addition, the only comments related to the value of Content observed were made by mathematics content specialists.

Comparison of Focus by Video

Comparisons have been made thus far on the overall numeric counts of comments made in each of the feedback focus themes, as well as by the form of the feedback. The final comparison will be made at the video level. For the purposes of this study it is important to keep in mind the focus of the two videos. Video 1 was selected based on the procedural focus of the instructional segment, while Video 2 was selected based on the connections between conceptual understanding and procedural knowledge made during the lesson. Table 22 displays the total number of focus comments by group and video.

Table 22
Total Number of Comments by Video and Focus

Video and Group	Assessment of Student Understanding	Assessment Used to Inform Instruction	Classroom Management and Culture	Conceptual Understanding and Connections	Instructional Practices	Content	Total
Video 1							
1	1	1	0	17	10	5	34
2	6	0	1	6	9	3	25
3	5	2	1	1	9	2	20
Video 2							
1	2	2	3	10	10	3	30
2	5	3	2	4	5	0	19
3	0	2	0	2	2	0	6

Note: Group 1 = mathematics content specialists; Group 2 = other content specialists; Group 3 = building principals.

The numerical differences in the feedback provided in the different groups shown in the table are striking in some areas. Beginning with the overall number of comments provided, mathematics content specialists contributed more comments than either of the two other groups for video 1 (34 as compared to 25 and 20 respectively), and more than the other two groups combined for video 2 (30 as compared to 19 and 6 respectively).

In addition to this difference, the number of comments for each focus theme revealed another interesting difference. For video 1, mathematics content specialists contributed more than double the number of comments as the other two groups combined. They also made the focus theme Conceptual Understanding and Connections the topic for half of their feedback comments made. Mathematics content specialists also contributed a large number of comments relating to Instructional Practices, thereby creating a balance between conceptual understanding and procedural practices, all the while keeping a focus on the weakness of the conceptual element of the lesson being presented to students. Other content specialists split their comments between the themes of Assessment of Student Understanding, Conceptual Understanding and Connections, and Instructional Practices. Only 6 of the 25 feedback comments provided the teacher focusing on procedural knowledge insight into helping students develop a conceptual knowledge of probability and making connections within and between probability and other mathematical ideas and topics. Building principals concentrated the majority of their comments in the focus areas of Assessment of Student Understanding and Instructional Procedures. Only one comment was made regarding Conceptual Understanding and Connections, one of the significant limitations of this video segment. Both other content specialists and building principals placed their focus more on practices and procedures than conceptual understanding. By so doing, neither group provided teachers support in ways to help students make connections between conceptual and procedural knowledge.

The distribution of comments for video 2 also revealed a number of interesting patterns. Mathematics content specialists again provided a balance of conceptual and instructional practice comments with ten each. Even though the conceptual piece was stronger in video 2, these participants still made it a point to address the conceptual theme. Also of note, mathematics content specialists made the only three comments coded as content. Other content specialists again split the majority of their comments between the focus themes of

Assessment of Student Understanding, Conceptual Understanding and Connections, and Instructional Practices. Participants from this group did provide balance between conceptual knowledge and instructional practice and procedures, but only at half the rate at which mathematics content specialists made those comments. Building principals only made six comments total, split evenly between Assessment Used to Inform Instruction, Conceptual Understanding and Connections, and Instructional Practices.

These differences raise interesting questions such as: why are there differences, and why those themes? Are the differences related to experience? Are they related to a focus on evaluation rather than instructional improvement, or are the differences related to the content knowledge of the individual providing the feedback? Are there other factors that have not been taken into account? As a final comparison, the content of the feedback for each video is reviewed in an effort to identify differences that may help address the previously mentioned questions. This comparison will focus on the feedback focus themes Conceptual Understanding and Connections, and Instructional Practices. Table 23 displays the conceptual comments made by each group to the teacher in Video 1.

As can be noted from the comments in the table, the feedback provided by mathematics content specialists is more specific, detailed, and contains a level of specific vocabulary related to the mathematical topic of probability not found in the other groups. For example, mathematics content specialists referred to conditional probability, combinatorics, trials, combinations, and experimental design. They also used vocabulary used in mathematics instruction such as residue, formal, informal, enactive, iconic, and symbolic representations. Other content specialists used mathematics vocabulary such as fractions and progressive formalization; however, as one participant noted, sometimes "intuition" has to be relied on when knowledge of topics such as progressive formalization is limited.

Table 23
Conceptual Feedback by Group for Procedural Instruction

Group	Feedback Comments
Mathematics Content Specialists	• Clothes = residue for this lesson? • At what point in the lesson did you sense that students were being pressed to think about the mathematical concepts in probability in ways that will be useful in later grades? • Where would students typically use these concepts in later math courses as well as outside mathematics classrooms? • What are some related math topics to those you were addressing? • Could students have designed their method of running trials in an experiment to determine the possible combinations and then compared their experimental designs? • How else might students interact with conditional probability or concepts of combinatorics? • Could we design a related lesson that places the emphasis on students' own intuitive notions of probability and then builds on this? • Here's an example: "Design a card game for 3 players in which each play has an equal chance of winning. Now, let's design a variation of that game in which one player has better chance to win but that seems fair to the players." • Give an example of when students' ideas progressed from informal to more formal during the lesson. • It is important for students to try something they think "may" be correct before being told (or shown) what is correct. • How would you move towards progressive formalization? • Discuss enactive – iconic – symbolic representations and what aspects the task addressed or didn't address • Next steps based on what was accomplished during this lesson (if you choose to continue with the topic, which should be a big if) • Consider exploring student responses further during the class discussion when incorrect answers are expressed during questioning. • When he speaks about not being able to have a probability of 9 out of 3, I would have liked there to be some connection to probability being a percentage or fraction between 0 and 1. • Structurally I think the connection to multiplication could have been more meaningful all the way around. • I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue.
Other Content Specialists	• Building on prior knowledge of fractions, or of chance? • Model is appropriate • Is there only one way to solve this problem? • How are statement of probability related to other math concepts such as fractions, how are they different? • How is probability helpful, what real problems does it help us solve. • In my limited knowledge of progressive formalization, I'm not sure that this is sufficient, but my intuition tells me that contribution of student ideas in developing the concept would have been a more solid instructional approach.
Building Principals	• Benefits? Restrictions?

Several comments made by mathematics instructional specialists also called into question the appropriateness of the topics as well as the connections that were, or were not, being made. One other content specialist suggested a different instructional approach, but did not question the actual content-related appropriateness of the topic. The single comment made by a building principal was very general in nature, and may or may not lead to a discussion of value or improvement of instruction.

The differences related to vocabulary, specific mathematical connections, as well as the appropriateness of mathematical content and connections appear to be directly related to the content knowledge of the individuals providing the feedback to the teacher in video 1. Conceptual feedback for the teacher in video 2 will now be reviewed to determine whether these or other similarities and differences exist there as well. Table 24 displays the conceptual feedback for video two.

As with the comments made in regards to video 1, the comments made by mathematics content specialists were more specific and content focused than the other two groups. Mathematics content specialists referred to a number of mathematical models used to highlight student thinking, such as the ratio table and grid squares. They also highlighted possible connections with other content topics such as division as an operation, fraction concepts, and area. These participants also highlighted specific ways that this conceptual lesson could be improved, for example "making connections between models for more students" and "the ratio table and the arrays could be linked for more students in order to extend their thinking."

Several participants from the other content specialist group referred to the ideas of "representational models" and "abilities to generalize." One such participant suggested a stronger context might be needed. The two comments made by building principals referred to areas they felt were effective for students in the classroom. No suggestions for improvement were provided by building principals.

Table 24

Conceptual Feedback by Group for Conceptual Instruction

Group	Feedback Comments
Mathematics Content Specialists	• How do you know when to press students (e.g. the student who miscounted the grid squares) and when to simply help them? • What will you do next? Will you focus on division as an operation, fraction concepts, or area? • How do you know what concepts to focus on and which you will de-emphasize in the moment? • The students' models could have been connected more explicitly. For example, the ratio table and the arrays could be linked for more students in order to extend their thinking. • A possible aspect to improve on would be to be even stronger in making connections between models for more students. This was an aspect of some of the interaction but could have been given more emphasis. • If students had not gone the direction of writing the remainder of the fraction, what other direction might you have taken the lesson? • Is it important to have a clear direction in mind when presenting a task to students? • How she addressed the issue of the child using the ratio table was interesting; the student was counting by 7s (attempting to) but misrepresents the number of stacks represented by the value below. She asks the student to transfer the ratio table to a picture to better examine the mistake. • The teacher does a nice job of cultivating a lot of student thinking and argument regarding this problem. The rich discourse creates opportunities for students to think and rethink conclusions they arrived at. • It also provides the teacher a number of good structural components to address along the way – many avenues to big ideas.
Other Content Specialists	• Overall sense – excellent lesson focused on students' mathematical thinking and perseverance. • Explain how you moved your students' understandings forward either in representational models or abilities to generalize or make connections with other math concepts. • You did a great job of letting the students discover the misconceptions. • For example, what was the reason that boxes were being stacked? Why did it matter?
Building Principals	• Thoughtful and thorough discussion was had by the group regarding how to appropriately label a portion of a stack using a fraction. • Models used are appropriate for the task.

It may not be surprising to note the differences in the content of the feedback given in the focus theme Conceptual Understanding and Connections as this category integrates content knowledge of mathematics. What about the more general category of Instructional Practices? Table 25 displays the instructional practice comments made by study participants to the teacher in video 1.

The instructional practice feedback given by the various groups was similar in quite a few cases. All three groups asked the teacher to reflect and think about various ways of helping students learn the material. Some of these instructional practices included asking questions – especially meaningful questions, working in groups, as well as enhancing student participation, communication, and justification. There were two interesting differences between the comments of mathematics content specialists and those of the other observers. First, several mathematics content specialists asked the teacher to reflect and discuss the reasons why the approach they used in the video was selected. These types of questions allow the teacher to consider the connections made with past and future learning. No participant in either of the other two groups requested such action from the teacher. The second difference is seen in the last two comments made to the teacher by mathematics content specialists. Both of these comments suggest an instructional problem that needs to be remedied. Again, no participant in either of the other two groups identified an instructional problem.

Table 25

Instructional Practice Feedback by Group for Procedural Instruction

Group	Feedback Comments
Mathematics Content Specialists	• What visual (iconic) models did students use? • How could students participate in the design/modeling of the lesson more than simply putting on the clothing? • What made you choose to approach (or set up) the task the way you did? • What are the pros and cons to approaching the problem through a whole-class enactive representation? • What made you select the particular problem and topic of probability for your lesson? • Mentioned Xcel sheets – is that what drove topic? • Might be good to label pants and shirts on the table to assist students. • Could potentially adjust task to allow students to explore individually or in small groups prior to whole group discussion of methods or once a few outfits had been enacted – ask students to complete the table prior to completing the enactive representations and have them compare. • I cannot put my finger on what was wrong per se, but I believe he (the teacher) should have expanded his discussion to address some larger issues. • Beyond this, I would say, because all models and thinking were the teachers, that based on the descriptions of this category he failed miserably here.
Other Content Specialists	• Could you have brought more clothing in or used a different variation with manipulatives or something else so they all could try it? • What about academic language like denominator and numerator? Reducing? A definition of probability? • Visuals on board are good. • Who did most of the work and thinking in this lesson? • How does providing them a template solution help students to make their own decisions about problem solving? • Teacher provides justification for students' answers here. I suggest allowing students to do so. • I suggest taking more time with this word, to show the morphological relationship to the word probability, which is the focus of the lesson. • I suggest spending a little more time on this so that students are not just passively responding, but pushed to self-evaluate their learning for a real purpose. • Effective use of math journals as resources for future problem solving
Building Principals	• How could you incorporate higher order questioning beyond the question/one word answer format? • How do you monitor the methods students are using to problem solve? • How could you formal tag, in a symbolic way, the proportion of outfits to the total available? • There are multiple ways to demonstrate all of the combinations possible with the shirts and pants along with the ratio-type table you modeled. How could we give kids the chance to think about those ways and then share? • Consider asking questions that press students to explain their thinking. • I wonder if after they caught the "gist" of the pattern the lesson could be adapted to have students pair/share, predict, continue the pattern. • Extension: How would the probability change if you wear a shirt and/or pants one time and put it in the laundry and don't wear it again? • Students need multiple chances to speak to the group and each other in a group or partnership. • The more opportunities we can give students to write, discuss, and justify their thinking the better.

Table 26
Instructional Practice Feedback by Group for Conceptual Instruction

Group	Feedback Comments
Mathematics Content Specialists	• Where did you find this task (curricular resources) or did you design it yourself? • This is a strength of this lesson. • There was an effective use of mathematical models and well-orchestrated student discussion. • Does the teacher have a clearly identified goal or are they investigating to determine students' current thinking? • What was the goal of the lesson? • Was it the intent of the teacher to go this direction or did they decide to go this way based on what they saw? Was a clear goal set for the lesson? • Was this an introductory lesson on the particular topic where the teacher was trying to ascertain the level of student understanding? • The teacher did an excellent job of having students discuss their ideas, both correct and incorrect. • Good focus on identifying the unit fraction and/or the 'whole' • Students were asked to explain the thinking of their table partners though, which was impressive.
Other Content Specialists	• Nice opener with ticket from yesterday. • Nice questions focused on approaches and responses. • Explain the different strategies your students were using. Were there any solutions or strategies that you did not anticipate? • Suggest teacher elaborates on this so that student understands why this is important. • Context was obscure and problem could have been situated in a stronger context.
Building Principals	• How do you press students to have/show equal representation of all modes (e, I, and s)? • Effective modeling with the Smart Board.

As with the Instructional Practice feedback given to the teacher in video 1, the feedback given to the teacher in video 2 is similar from group to group, focusing mostly on various aspects of instructional practice. There was a notable difference, however. Mathematics content specialists focused a number of their comments on the goal or purpose for the lesson. Although the comment fits within the focus theme of Instructional Practices, the thought invokes a conceptual practice – that of making connections with past lessons and/or student understanding. This difference manifested itself in the feedback given to the teachers of both videos, and was found most strongly in the comments made by mathematics content specialists.

Quantitative Analysis – Determining Significance of Differences

The purpose of this section is to describe the evidence, if there is any, of a relationship between the content knowledge of the participants in each of the three groups, and the form and focus of the feedback they provided following their observation of mathematics instruction. As described in Chapter 3, a Chi-square test was used to determine whether there was evidence of such a relationship.

Analysis 1 – Groups and Feedback Form

The first analysis utilized a Chi-square test of independence to examine a possible relationship between the groups and the form of feedback they provided. The test revealed there was no statistical evidence of a relationship between the groups and feedback form, χ^2 (4, N = 134) 1.250, p = .870.

Analysis 2 – Groups and Feedback Focus

The second analysis also utilized a Chi-square test of independence, this time to examine a possible relationship between the groups and the focus of feedback they provided. As 50% of the cells had an expected count less than 5, the Likelihood Ratio value was reported instead of the Pearson Chi-Square value (Field, 2013). In this case, the results showed evidence of a relationship between the groups and the feedback focus, χ^2 (10, N = 134) 20.900, p=.022. Since $p < .05$ we reject H_0, and accept H_1, that evidence of the relationship exists. Such an analysis also warrants further exploration of the data to identify where those differences might be. Table 27 displays the results of the group by secondary crosstabulation calculations.

Table 27
Group by Feedback Focus Secondary Crosstabulation

		Feedback Focus						
		Assess. to Inform Instruct.	Assess. Of student underst.	Concep. Underst. and Connect.	Content	Mgmt. and Culture	Inst. Pract.	Total
Math Content Specialists	Count	3	3	27	8	3	20	64
	Expected Count	4.8	9.1	19.1	6.2	3.3	21.5	64.0
	% within Group	4.7%	4.7%	42.2%	12.5%	4.7%	31.3%	100.0%
	% within secondary	30.0%	15.8%	67.5%	61.5%	42.9%	44.4%	47.8%
	% of Total	2.2%	2.2%	20.1%	6.0%	2.2%	14.9%	47.8%
Other Content Specialists	Count	3	11	10	3	3	14	44
	Expected Count	3.3	6.2	13.1	4.3	2.3	14.8	44.0
	% within Group	6.8%	25.0%	22.7%	6.8%	6.8%	31.8%	100%
	% within secondary	30%	57.9%	25.0%	23.1%	42.9%	31.1%	32.8%
	% of Total	2.2%	8.2%	7.5%	2.2%	2.2%	10.4%	32.8%
Building Principals	Count	4	5	3	2	1	11	26
	Expected Count	1.9	3.7	7.8	2.5	1.4	8.7	26.0
	% within Group	15.4%	19.2%	11.5%	7.7%	3.8%	42.3%	100%
	% within secondary	40.0%	26.3%	7.5%	15.4%	14.3%	24.4%	19.4%
	% of Total	3.0%	3.7%	2.2%	1.5%	0.7%	8.2%	19.4%
Total	Count	10	19	40	13	7	45	134
	Expected Count	10.0	19.0	40.0	13.0	7.0	45.0	134.0
	% within Group	7.5%	14.2%	29.9%	9.7%	5.25	33.6%	100.0%
	% within secondary	100.0%	100.0%	100.0%	100.0%	100.0%	100.0 %	100.0%
	% of Total	7.5%	14.2%	29.9%	9.7%	5.25	33.6%	100.0%

There are three different focuses that merit attention based on the information in Table 27. In each of these areas one group accounts for more than double the percent within the secondary comparison as compared to either of the other two groups. The first area for consideration is Assessment for Student Understanding. Participants with a content specialty in an area other than mathematics contributed 57.9% of the total compared to 26.3% contributed by building principals, and 15.8% contributed by mathematics content specialists.

The purpose of this study is to examine the connections between observers' mathematics content knowledge and the content of feedback provided. While assessment of student understanding was not as much a focus for mathematics content specialists, the specific content knowledge of the individual may not have been the determining factor as other content specialists represented a variety of content specialties. The other two categories, however, have specific connections to mathematics content.

The second focus area with a large difference between groups is Conceptual Understanding and Connections. Mathematics content specialists accounted for 67.5% of the focus comments in this area as compared to 25.0% from other content specialists, and only 7.5% from building principals. The content-specific knowledge of mathematics content specialists played a significant role in their focusing on the importance of conceptual understanding and connections within the instructional segments they observed.

The third area was in the focus of Content. In this category mathematics content specialists accounted for 61.5% of the comments as compared to 23.1% from other content specialists, and 15.4% from building principals. It is not surprising that content knowledge would play a significant role in the identification of content in feedback comments, and the analysis supports this conclusion.

Based on the statistically significant results of the chi-square test, the results were further analyzed to determine effect size. A Cramer's V score of .274 was reported for this analysis. Cohen (1988) suggested using a value of 0.1 as a small effect size and 0.3 as a medium effect size. This comparison would suggest that the relationship between the groups and the focus of feedback has a small, nearly medium effect size.

Conclusions for Research Question 2

The second purpose of this chapter has been to determine if there is evidence that an observer's content focus influences the feedback that is given. Based on the analysis conducted in the previous sections, it is reasonable to conclude that there are statistically significant differences in the number and focus of the comments made to teachers based on the content specialty of the person giving the content.

In this study, participants in all three groups had educational experience as teachers in their respective content area. All of the participants have had experience in conducting observations and providing feedback to individuals engaged in an instructional capacity. The main difference between the three groups is content background. Mathematics content specialists have had significant experience in the study and practice of mathematical content and instruction. They also have spent time reading and learning about the process of making connections between mathematical ideas and instructional processes involving student thinking. Group 2 participants have significant experience in content areas other than mathematics, yet have significant knowledge in instructional practice. Elementary building principals have significant generalist experience in observing and working with teachers and students at the elementary level.

Differences in form and focus of feedback were found at each stage of the analysis beginning with the forms of Query, Recommendations, and Value. The differences continued to emerge with the analysis of the focus of the feedback given and continued to manifest themselves as the feedback given to the two teachers was compared. The connection to content background differences was seen most strongly in the focus themes of Conceptual

Understanding and Connections and Content. Each of these themes is tied very closely to content knowledge.

With the themes defined and content knowledge identified as a factor in feedback given, I will now turn my attention to discussing possible implications of these results, the development of a framework from which future studies can build, as well as describing several possible research extensions to this study.

CHAPTER 5: DISCUSSION AND RECOMMENDATIONS

Chapter 4 of this study provided an analysis of feedback comments provided by three different groups of observers to two different teachers of mathematics. The analysis identified emerging themes within the feedback comments. These themes described the form the feedback was given in as well as the focus or content of the form based on the mathematical content knowledge of the observer. In this chapter I: 1) review the research questions and results described in Chapter 4, 2) discuss the importance of the focus of feedback, 3) discuss the connection of content knowledge to effective feedback, 4) discuss possible implications of this study to supervision of instruction, 5) provide recommendations for application of the results of this study, and 6) address further research opportunities that now present themselves.

Review of Research Questions

The two research questions addressed by this study were:

1) What themes emerge from feedback given to teachers of mathematics by observers with different content and focus backgrounds?
2) Does an observer’s educational background influence the substance of feedback that is given?

Emergence of Feedback Forms

In addressing the first question two categories of themes emerged from the feedback data provided by three different groups: mathematics content specialists, content specialists in areas other than mathematics, and building administrators. The themes were divided into two categories, form and focus. Forms addressed the manner in which the feedback was used by the observer for their own purposes, or the manner in which feedback was conveyed to the teacher. Forms included Conclusions, Descriptions, Evidence, Queries, Recommendations, and Value. The first three form themes assist in the development of feedback while the second three forms describe ways the feedback can be presented or communicated to the teacher. Focus themes addressed the content of the feedback being given to the teacher. Focus included Assessment of Student Understanding, Assessment to Inform Instruction, Conceptual Understanding and Connections, Mathematics Content, Instructional Practices, and Classroom Management and Culture.

Feedback forms described the manner in which feedback may be delivered to the recipient. For example, an observer may attempt to draw out additional thoughts or ideas, as well as help teachers self-evaluate through the use of queries. An observer may provide more directive feedback through the use of recommendations. Such statements convey direction for the teacher, and outlines for them the important modifications the observer wants them to make, and which will likely be followed up on during future observations. Alternatively, an observer may place emphasis on particular instructional practices observed by placing value on those practices. By placing value on a practice, the observer communicates the importance of a particular practice as they see it based on their own values, and whether that practice needs to be strengthened or removed.

Although using the first three forms for developing feedback and the second three forms for presenting feedback was pragmatic, this study identified an inherent weakness in focusing on feedback only at this procedural level. All six categories are general in nature, and may or may not address specific instructional practices that lead to increased student achievement. As previously addressed in the literature review of this study, mathematics instruction has undergone an evolution from focusing almost exclusively on procedural learning, to incorporating conceptual learning to strengthen and deepen a students' understanding of mathematics. Without additional focus specifically on how mathematics ideas were developed in the lesson, it is unlikely that these themes alone will provide the feedback specificity needed to ensure instructional change. The real power to generate change in mathematics instruction comes in the specific focus of the feedback given, which is related to the content knowledge of the observer.

Emergence of Feedback Focus

The second category of themes, the focus of feedback comments, described the content of the feedback observers provided to the teachers being observed. Six focus themes emerged during the data analysis process: Assessment of Student Understanding, Assessment Used to Inform Instruction, Classroom Management and Culture, Conceptual Understanding and Connections, Instructional Practices, and Mathematics Content. These six focus themes were contained and identified in the six feedback forms. It was at this focus level that statistically significant differences between the different participant groups emerged, most notably in the focus areas of Conceptual Understanding and Mathematics Content. It is at this deeper level of analysis that the influence of the content knowledge of the observer emerges, and where we may now find a more powerful avenue of supporting effective instructional improvement.

Importance of the Focus of Feedback

As described in the literature review of this study, feedback characterized as being effective is specific, detailed, corrective, and positive. Interestingly, these same characteristics exhibit the same limitation as the feedback forms identified in this study; they fail to address the actual content or focus of the feedback being given. Much of the research on feedback thus far has focused on teacher behaviors and general educational practices, thereby treating instruction of all content areas in the same way (Nelson & Sassi, 2000). Without a doubt many instructional practices may be effectively used in multiple content areas, such as classroom management and questioning techniques. However, such general feedback may or may not address the content specific issues limiting student understanding, appreciation, and application of mathematics both within the educational setting and outside of school.

Conceptual Understanding and Content – Critical Focus Components

The use of an inductive inquiry model (Thomas, 2006) allowed the identification of six focus themes within the feedback data collected in this study. Each of these six themes plays a role in the instructional processes of a mathematics classroom. However, as the results discussed in chapter 4 revealed, there is a relationship between the content knowledge of the observer and the ability of the observer to notice and comment on aspects of instruction that relate to each of these themes. Two themes in particular stood out when comparing the group results: 1) Conceptual Understanding and Connections and 2) Content.

It is not surprising that content knowledge would impact an observer's ability to provide specific and helpful feedback on the content and instruction of a lesson segment. This should be expected and accounted for. However, a lack of content knowledge limits an observer's ability to recognize and make use of conceptual knowledge and connections during instruction in order to provide feedback that supports and encourages the further development of teachers' instructional strategies, which in turn directly impacts students' development, understanding, and application of the mathematics being taught.

Such a situation entertains the possibility of several unfortunate consequences. First, a lesson that may be described as very fun, engaging, and effective may in reality not be effectual in increasing student knowledge beyond a superficial level. For example, the instructional segment seen in video 1 was identified by one administrator as having a "Positive classroom climate and good sense of humor." An observer who specializes in a content area other than mathematics commented, "Extremely well-managed lesson with students standing, sharing, and enjoying the idea of trying on clothes." From these comments one would naturally assume that the lesson was effective in developing student knowledge. However, several comments from content area specialists with strong mathematics knowledge cast doubt on whether the lesson was supportive in developing student knowledge. One such observer made the following comment:

> I am struggling to find much to say here which could be considered good. I feel like the acting out of the problem was fun for kids and that they enjoyed the day, but I wonder what they will take forward. There is very little assessment of actual understanding here and I would guess he may feel like it was a strong lesson, but there is really little residue.

This observer recognized the lesson was engaging, but engagement alone does not necessarily translate to long-term student understanding and application. Another mathematics content specialist observed, "Structurally I think the connection to multiplication could have been more meaningful all the way around." This comment suggests that the conceptual foundation of the lesson may have been masked, missing, and/or flawed from the beginning, and would need attention in the future if student understanding were to be affected in a positive manner. In essence, without adequate content knowledge, weak conceptual components may go unnoticed while the outward instructional embellishments are identified as the desirable elements of the lesson. Unfortunately, student understanding is the true casualty in such a situation.

The second unfortunate consequence is exemplified by comments made to the teacher in Video 2. While observers from all groups made comments supporting the overall strength of the lesson, only comments from mathematics content specialists suggested ways that the lesson could be made conceptually stronger. For example, one mathematics content specialist commented: “When dealing with fractional remainders, be cautious that you are essentially treating fractions as ‘set’ models in this case. This is not an error, but set models are sometimes confusing for students just learning fraction concepts.” Another mathematics content specialist commented, “This could be a great opportunity to build on what they know. However – if the goal is division, then it might take away from what students get from solving the original task.” These comments demonstrate the possibility of providing feedback that is designed to make a strong lesson even stronger by providing insights into the connections that might either strengthen or weaken student understanding. It is highly doubtful such observations would be made by an observer lacking a strong foundation in mathematics.

Connecting Content Knowledge to Effective Feedback

Content knowledge is one of three interrelated components within what Stein and Nelson (2003) call Leadership Content Knowledge (LCK). The other two components are views on how mathematics is learned and what high quality instruction should look like. Weinberg (2010/2011) found that, “principals’ LCK greatly influences what they focus on when they observe mathematics classes and what they discuss with teachers in post-observation conferences” (p. 30). She found that the weaker a principal’s LCK the more he or she focused on instructional processes related to executing correct procedures. As a principal's LCK increased, he or she attended more to what students were doing, and finally to what they were thinking. In addition, she found the principal's understanding of mathematics impacted the ability of the principal to both notice and understand the mathematics of students' thinking, as well as to help teachers create lessons designed to develop the students' understanding of mathematics. Weinberg emphasized the importance of content knowledge by recommending that principals engage in ongoing professional development designed to increase both content and pedagogical content knowledge of mathematics. Both of these areas, understanding students' mathematical thinking and developing lessons to increase student understanding of mathematics, might be addressed in the feedback a principal would provide a teacher following an observation when he or she possesses the relevant content knowledge.

In 2011 van Es introduced a framework for learning to notice student mathematical thinking. Her research findings identified four different levels of noticing within two areas: what teachers notice, and how teachers notice. At the less experienced levels of the scale, teachers focus more on classroom environment, behavior, and isolated teaching strategies. As noticing skills improve, teachers begin to focus on students' mathematical thinking and behaviors. Teachers eventually attend to the relationship between particular student thinking and the teaching strategies that might be used to impact student thinking.

There are interrelated aspects of knowledge and skills required for developing and providing effective feedback. In order to be effective, feedback for mathematics instruction must not only be specific (Black & Wiliam, 1998; Mory, 2003), detailed (Scheeler et al., 2004), and provide correction (Scheeler et al., 2004), the specificity, detail, and correction must focus on important content-specific aspects of instruction and student thinking.

Bennett, Amador, and Avila (2015) applied the professional noticing work of Jacobs, Lamb, and Philipp (2010) and van Es's (2011) noticing framework to determine what administrators noticed as they observed instructional segments in classrooms. Jacobs et al. (2010) defined professional noticing as attending to, interpreting, and responding to students based on their thinking. Bennett et al. (2015) contended, "School administrators have the ability to support teachers' instructional practice, however, administrators' ability to notice pivotal moments in students' mathematical thinking greatly influences the quality of support they can provide" (p. 14).

Bennett et al. (2015) found that prior to participating in professional development aimed at increasing noticing skills, principals primarily attended to teacher actions or comments, with little attention provided to student thinking. They also found principals' interpretations of comments and evidence lacked depth. The principals were able to acknowledge items such as teacher identification of different approaches to solving problems, but were unable to extend that interpretation to a determination of whether or not the students actually understood the mathematics behind their solution strategies. The authors additionally found differences in principals' descriptions of an observed instructional segment. Principals in their study focused on environmental evidence and student behaviors tied to classroom culture, items which are "worthy of attention but … of little to no assistance in helping teachers improve their instruction or the mathematical learning experiences of students" (p. 18).

Based on these findings, Bennett et al. (2015) have called for professional development opportunities for principals to help them increase their effectiveness in noticing important instructional components and instances of student thinking. They suggest that in order to notice important components of students' thinking as well as help to improve the instructional practices of teachers, principals must attend to, interpret, and reflect on the most important aspects of mathematics instruction and student thinking, and then construct feedback for teachers that has the form and focus necessary to increase the effectiveness of the instruction being given. A solid understanding of the content of mathematics is a foundational component of noticing and the construction of effective feedback designed to deepen student thinking and increase student achievement in mathematics.

In this study, participants with different content knowledge selected different focus elements within the instructional segment to provide feedback on. For example, all groups provided feedback focused on classroom management and culture, assessment, and other general instructional practices. However, only those with a strong mathematics background commented on important connections within the mathematical content or important corrections that needed to be made to strengthen students' mathematical understanding.

The work completed by Bennett et al. (2015) provides several possible reasons for this difference. First, the observers with little mathematics conceptual knowledge may not have been aware of what to look for. For example, in the second video the teacher approached the remainder of a division problem from a "part of a set" context. Mathematics content specialists in the present study pointed out that set models are sometimes confusing for students being introduced to fraction concepts, and that understanding fractions in a part of a set context is not part of the standards for third grade. One mathematics content specialist then asked the teacher to provide a rationale for pursuing the part of a set context. No participant in either of the other groups commented on this aspect of the lesson. Although I cannot say for certain without a follow up conversation with the participants, it is likely that the observers in the other groups were either unaware of this aspect of the content or did not feel it important enough to comment on.

Regardless of whether participants from other groups recognized the “part of a set” connection, they did not comment on it. This omission may be due to not understanding the importance of that concept to other mathematics content or to how students make, or fail to make, appropriate connections that lead to a deep understanding free of misconceptions. Mathematics content specialists not only recognized the meaning of “part of a set,” they understood the implications of that mathematical idea in connection with other mathematical ideas as well as possible implications for students as they encounter that topic. Having an understanding of these connections allowed the mathematics content specialists to identify and address this aspect of the lesson, provide feedback on it to the teacher, and then if needed, follow up with suggestions to the teacher on how to monitor their instructional planning and delivery, as well as the students’ thinking, in order to ensure that students understand and apply the content knowledge they are learning.

Content knowledge is foundational to providing effective feedback to teachers of mathematics. This knowledge allows an observer to: 1) recognize the existence and significance of mathematical ideas addressed in a lesson, 2) look for evidence of student understanding and misconceptions related to a particular mathematical idea, 3) provide appropriate feedback to teachers of mathematics in order to enhance future learning opportunities, and 4) provide correction when needed. Content knowledge of mathematics then has a direct impact on the ability of an observer to provide feedback that is specific, detailed, and corrective in nature and targets the implementation of instructional practices that have been shown to increase student achievement in mathematics.

While additional research is still needed to better understand how to best address the noticing needs of principles and administrators (Bennett et al., 2015), it is clear that observers' content knowledge has an impact on what they attend to during an observation, their ability to conduct an in-depth interpretation of the evidence attended to, and to provide supportive suggestions in improving instructional planning and delivery of mathematics. It would follow that continued professional development in the content of mathematics should be a foundational component for the professional development of building administrators who observe and evaluate mathematics instruction.

However, an important challenge remains. Principals and administrators at both the elementary and secondary levels are not only responsible for mathematics as a content area, they are also responsible for all content areas, including but not limited to reading, writing, science, music, art, and physical education. Depending on school size, configuration, and delegation of responsibilities, principals may also be required to oversee varying numbers of personnel and other areas outside of the regular classroom. This begs the question, if all of the content areas require specialized knowledge to make the most of instructional improvement opportunities, and if principals are only able to observe and provide feedback several times per year due to the extensive nature of their other responsibilities, can one or two formal observations and follow up conferences per year effect the level of change needed to develop the deep student understanding called for in the twenty-first century?

Implications of Mathematical Content Knowledge for Supervision of Instruction

The awareness of the need for deep content knowledge when providing instructional leadership is fairly recent. In reviewing the case for mathematics supervision, Nelson and Sassi (2000) described the change in supervision models that has taken place since the 1980s.

During the early part of that time-frame principals were taught to use a technical-didactic model of supervision that had its foundations in behavioral psychology (Garmston et al., 1998). This model led to supervision focused on increasing and replicating desired teacher behaviors, and reducing or eliminating less effective behaviors. These effective or ineffective behaviors were generic in nature and applied across all content areas. Under this supervision model, a single administrator could operate effectively as a supervisor of instruction within the given guidelines.

Another branch of thought emerged in the late 1980s relative to supervision of instruction (Nelson & Sassi, 2000) and focused primarily on the thoughts of teachers rather than their behaviors. This shift to a cognitive focus from a behavioral focus led to the use of models of instructional improvement such as cognitive coaching (Costa & Garmston, 1994) and a reflective version of clinical supervision (Garman, 1986), generating the view that as professionals, teachers should be responsible for their own ongoing learning (Darling-Hammond & Sclan, 1992). However, as Nelson and Sassi (2000) pointed out, “by and large that thinking was still about behaviors rather than about subject-matter ideas” (p. 557).

Since that time, ideas regarding the nature and focus of instruction have continued to shift to more student centered supervision (Tracy, 1998) which includes a focus on student thinking. This shift to a focus on student thinking has necessitated the development of content specific instructional practices in mathematics (Nolan & Francis, 1992). Since the work completed by Nelson and Sassi (2000), other changes have been suggested for the focus of supervision of instruction such as Stein and Nelson’s (2003) work focusing on Leadership Content Knowledge which the authors argue allows administrators to observe how teachers make connections between specific content and how students learn that content, and a shift in mathematics from teacher-centered observation to a focus on student thinking (Bennett et al., 2015; Jacobs et al., 2010; van Es, 2011). This latest shift has increased the need for instructional leaders to have a stronger foundation in content knowledge of mathematics.

It may not be feasible or even possible for one person to shoulder the role of instructional supervision for all content areas. While professional development for administrators or supervisors centered on content knowledge is one way to support improvement in supervision of instruction of mathematics, it may not be reasonable to expect that the school principal take the time for such additional learning with all of the other responsibilities a principal must shoulder. Developing a strong level of LCK in just the content area of mathematics is a significant undertaking in and of itself. Researchers such as Nelson and Sassi (2000), as well as the National Council of Teachers of Mathematics (2014) support the proposition of having multiple individuals assigned to supervise instructional improvement and have identified the need for principals, coaches, specialists, and other school leaders to “make the mathematical success of every student a nonnegotiable priority” (p. 112).

Nelson and Sassi (2000) suggested it might be more beneficial for an administrator to have a deeper knowledge of learning and teaching within one content area than a shallow content knowledge in many areas. In this case, building administrators would need to rely on others who have a deeper understanding of content in areas in which the building administrator is not as strong to provide the instructional leadership in those areas. The authors described two models that could be incorporated to meet this purpose: 1) distributed leadership, in which principals, teachers, mathematics specialists, and other staff members may work together to collaboratively enact leadership roles, and 2) supervision as a responsibility of a school-based community of learners such as principals and teachers who work together to study what learning and instruction look like in a variety of subject areas. Regardless of the model used, it is clear that educational entities will need to consider the importance of content knowledge as they structure responsibilities for instructional leadership and supervision of instruction.

Recommendations

Prior to addressing specific recommendations, an important limitation must be revisited. Although qualitative differences and statistically significant quantitative differences were identified in the findings of this study, the number of participants is small: $N = 15$. Due to the small *N*, care must be taken not to generalize these findings to a larger population. This limitation increases the importance of continued research on this topic as recommended in the next steps for research section of this chapter.

Approaches to mathematics instruction have changed. Instructional supervision and support of mathematics education must evolve as well. Feedback given to teachers of mathematics needs to be specific, detailed, and focused on the procedural and conceptual knowledge of mathematics of both the teacher and the students. An understanding of the interrelationship between these types of knowledge supports an observer’s ability to recognize and make use of the connections found within mathematics, and also supports them in identifying how those connections can be used to support student learning. Once an observer has learned to recognize the existence or absence of these connections in a mathematics lesson, they may then develop feedback designed to help the teacher increase their instructional effectiveness in teaching mathematics.

A beginning step in helping observers to identify these mathematical conceptual connections would be the creation, selection and use of observation tools specific for mathematics lessons. Such tools would help highlight the elements of conceptual thinking, the connection of those elements to procedural understanding, and maintain a focus on student thinking as a driver of instructional practices. One such tool is the Developing Mathematical Thinking for Instruction observation protocol (Brendefur et al., 2010a, 2010b). It is conceivable that the use of this tool will help instructional supervisors notice important mathematical connections, teaching moments, and identify student understanding and misconceptions. Such a tool may also aid in the identification of weak teacher content knowledge, although additional research will be needed to verify this conclusion. Once specific weaknesses have been identified, professional development could then be delivered to address those needs.

Second, it is imperative that supervisors of mathematics instruction participate in professional development aimed at deepening their own understanding of the content and connections found in the subject of mathematics. While feedback regarding general instructional practices is still an important component of instructional supervision and improvement, in order to truly notice, interpret, and provide effective feedback to teachers of mathematics, the person giving the feedback needs to have a strong content knowledge of the subject of mathematics.

Third, administrators and others who observe teachers of mathematics should engage in additional training in identifying, interpreting, and communicating the most important elements of mathematics instruction, including but not limited to conceptual understanding, procedural understanding, attending to student thinking and work, and communication during a mathematics lesson. The van Es (2011) framework has been used as a tool to effectively support the professional noticing of student thinking during instruction, and has been used successfully to improve the noticing skills of administrators as they observe mathematics instruction (Bennett et al., 2015) and would be a useful tool to use in working with a broader group of observers.

Finally, it will be important for educational entities to review their own system for supervision of instruction to determine 1) if there are individuals who have the requisite content knowledge to effectively provide ongoing instructional observation and support, and 2) if those individuals are in a position to be able to lend that support. Regardless of the system used for supervision of instruction, it is vital that the system ensure that those who observe and provide feedback about mathematics instruction possess the requisite mathematics knowledge for teaching and are available and trained to support teachers of mathematics in a formative manner, and if possible in a summative manner as well.

Next Steps for Research

Now that both form and focus themes specific to mathematics in observation comments have been identified, research should be conducted both to verify the results of this study and to determine the power of each focus theme to affect instructional change. Such research may also determine the impact of feedback based on each of these themes on increasing instructional effectiveness. The resulting effective instruction could also be examined for related increases in mathematics student achievement in both short-term and longitudinal studies.

Within the field of instructional supervision it remains to be seen which model or models of supervision have the greatest impact on instructional improvement of mathematics.

In recent years mathematics instructional coaches and specialists have become more common in K-12 schools. However their skill and expertise are not available to all teachers. As content knowledge plays an increasing role in specific observations, research will be needed to determine how best to support the needed changes in instructional leadership and support approaches. Doubtless more individuals will need to be involved in the process of instructional improvement. However, is improvement best facilitated through formative processes involving instructional coaches through an increased evaluative focus from administrators, or with some other model? What are effective practices for sharing this responsibility? The education community as a whole would benefit from research to answer questions such as these.

Finally, the parameters of this study have been purposefully constrained to the content area of mathematics. While the results of this study are not generalizable to other content areas, it would be reasonable to assume that content knowledge specific to reading, writing, social studies, science, art, music, physical education and so on would inform specific feedback themes as well. Research leading to the identification of these content specific themes and whether these themes make a difference in instruction and/or student achievement seem to be reasonable next steps.

Powerful feedback can effect powerful instructional change. In mathematics teaching, the observer or evaluator’s content knowledge for teaching plays a key role in the identification and interpretation of important instructional episodes and related student thinking, which then plays a role in the subsequent forming of feedback that is given to the teacher for the purposes of instructional support and improvement.

It is time to break the cycle described by Ball et al. (2005) in which today's students are learning from teachers who were the students in a system that did not support the development of a strong conceptual understanding of mathematics. One important key to accomplishing this mission is to provide meaningful content specific feedback and support to teachers on a regular basis. Doing so will help increase the instructional effectiveness of our mathematics classrooms in order to see students leave our schools with the mathematical knowledge and skills needed to adapt to and make a positive difference in our ever-changing world.

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APPENDIX A: CONSENT FORM AND INSTRUCTIONS

Thank you for taking the time to review this consent form. The University of Idaho Institutional Review Board has certified this project as Exempt.

Purpose of the Study

The purpose of this study is to examine the types of written feedback given to elementary teachers after observing a mathematics lesson the teacher has taught.

Procedures

If you agree to participate you will be asked to view two 30-45 minute instructional video clips from elementary classrooms filmed during a mathematics lesson. You will be provided an observation form to use for the observation. You will then be asked to provide written feedback to the teacher based on your observation. You will be asked to use the observation form to the best of your ability without training on the instrument itself. This is an intentional element of the study and will be taken into account during the analysis phase of the project. The study should take approximately 2 ½ to 3 hours.

Costs to Participant

There are no costs to you associated with this study other than the approximately 3 hours of time required to observe the videos, provide the feedback, and take a short survey.

Participant Payments

There will be no payments related to participation in this study.

Possible Discomforts

There are minimal possible discomforts during this study. It is possible that you may feel uncomfortable using the observation form without training or in providing feedback to the teacher. This study focuses only on the feedback given by an observer and the teacher will not see the feedback that you provide.

Benefits

It is hoped that the results and conclusions of this study will 1) add to the current body of observation and feedback literature, and 2) provide a critical lens from which to begin a discussion on how observers' backgrounds and experience impact the content of the feedback provided to educators.

Confidentiality and Privacy

We will make every effort to maintain the privacy of any data you provide. There will be an initial survey to collect information about your educational expertise and content focus, as well as experience in conducting observations and providing feedback. This information will be used to help create a frame of reference from which to make comparisons. During the initial data gathering phase your name will be linked to an identification number that will be used to identify each piece of feedback in order to ensure that the data submitted is catalogued appropriately. This file will be maintained separately from the feedback data on a password-protected laptop. Subsequent responses will be kept separate from your name. All responses will stored on a password protected computer, and any hard copies of the feedback will be kept in a locked office.

Other than the researcher and his committee members, only regulatory agencies such as the University of Idaho Institutional Review Board may see your individual data as part of routine audits.

Withdrawal from the Study
Please know that you may discontinue your participation in this study at any time without penalty. Simply discontinue your survey and close your web browser.

Contact Information
Should you have any questions or concerns now or during the study, please contact me or Dr. Adams using one of the following methods:

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Consent to Participate
I have reviewed this consent form and understand and agree to its contents. I understand that by continuing with the survey and observations I am giving my consent to participate in the study, and that should I wish to discontinue, I may do so at any time without penalty. I understand that completion of the survey will constitute consent to participate in this study.

Instructions to Participants

Thank you for taking the time to participate in this study. As previously mentioned your partition in this study will contribute to the body of knowledge on teacher observation and feedback within an educational setting, as well as assist me in collecting data leading to discourse on the impact of an observer's background on the feedback they provide following an observation.

In order to participate, please e-mail me at duanepeck@boisestate.edu. I will then send you the following: 1) the observation form, 2) a link to the study videos, 3) a link to the survey site that will collect the feedback you would provide, and 4) a random identification number that you will enter as the first response in the survey.

Step 1: Please view video one. Please use the observation form to the best of your ability. Following your viewing of the first video, please script in a word processor the feedback that you would provide based on your observation. You may provide any feedback that you would like, including any comments that may not be reflected on the observation instrument.

Step 2: Please view video two: As with video one, use the observation form to the best of your ability and script your feedback in a word processor. **Please do not alter any feedback from the first video based on your second observation.**

Step 3: Follow the link to the Qualtrics survey, and using the username and password provided, complete the survey form. There will be a number of demographic questions followed by a text box for each video observation that will allow you to cut and paste your scripts.

Again thank you for taking the time to participate in this study!

APPENDIX B: FEEDBACK SURVEY 1

Q1.1 Thank you for participating in this study.

Consent to participate: I understand that by continuing with this survey I am giving my consent to participate in the study, and that should I wish to discontinue, I may do so at any time without penalty. I understand that completion of the survey will constitute consent to participate in this study.

Prior to completing this survey, you should have reviewed 2 videos (Teacher 1 and Teacher 2) using the Mathematics Instruction Observation Protocol, and prepared written feedback for each of these teachers. The survey instrument below is divided into three blocks. The first block will collect demographic information regarding your educational and observation experience. Block 2 will be used to collect the feedback that you have prepared for Teacher 1, and Block 2 will collect the feedback that you have prepared for Teacher 2. The survey reflects the same categories as the observation instrument. You may either type your responses in the appropriate fields, or cut and paste responses that you already have in an electronic format. Please remember that your participation is voluntary, and you may choose to end your participation at any time by closing your web browser.

You may choose not to answer any question below, except for the first question that asks about your current position. This question must have a valid response before the survey will allow you to continue. Thank you again for your time and effort in making this study possible!

Q1.2 Which of the following best describes your current position?

- ❍ Working at an institute of higher learning, and have mathematics as a content specialty. (1)
- ❍ Working at an institute of higher learning, and have a content specialty other than mathematics. (2)
- ❍ A building administrator in a K-12 public school. (3)

Q1.3 How many years have you taught in a K-12 classroom?

- ❍ 0 to 5 years (1)
- ❍ 6 to 10 years (2)
- ❍ 11 to 15 years (3)
- ❍ 16 to 20 years (4)
- ❍ 21 to 25 years (5)
- ❍ 26 and up (6)

Q1.4 How many years have you served as a K-12 administrator?
- ❍ 0 to 5 years (1)
- ❍ 6 to 10 years (2)
- ❍ 11 to 15 years (3)
- ❍ 16 to 20 years (4)
- ❍ 21 to 25 years (5)
- ❍ 26 years and up (6)

Q1.5 How many years experience do you have working in higher education?
- ❍ 0 to 5 years (1)
- ❍ 6 to 10 years (2)
- ❍ 11 to 15 years (3)
- ❍ 16 to 20 years (4)
- ❍ 21 to 25 years (5)
- ❍ 26 years and up (6)

Q1.6 How many total years of experience do you have in education as a profession?
- ❍ 0 to 5 years (1)
- ❍ 6 to 10 years (2)
- ❍ 11 to 15 years (3)
- ❍ 16 to 20 years (4)
- ❍ 21 to 25 years (5)
- ❍ 26 years and up (6)

Q1.7 What is your content area specialty?
- ❍ Language Arts (1)
- ❍ Mathematics (2)
- ❍ Science (3)
- ❍ Social Studies (4)
- ❍ Art (5)
- ❍ Physical Education (6)
- ❍ Music (7)
- ❍ Special Education (8)
- ❍ Other (9)

Q1.8 If you answered "other" to the previous question, please specify your content area specialty.

Q1.9 To the best of your recollection, how many mathematics courses (content or professional development) did you take as an undergraduate student?
- 0 to 3 (1)
- 4 to 6 (2)
- 7 to 10 (3)
- 11 and up (4)

Q1.10 To the best of your recollection, how many mathematics courses (content or professional development) have you taken since receiving your Bachelor's degree?
- 0 to 3 (1)
- 4 to 6 (2)
- 7 to 10 (3)
- 11 and up (4)

Q1.11 How many years of experience do you have observing preservice and/or inservice teachers?
- 0 to 5 years (1)
- 6 to 10 years (2)
- 11 to 15 years (3)
- 16 to 20 years (4)
- 21 to 25 years (5)
- 26 years and up (6)

Q1.12 Please briefly describe any mathematics professional development you have been a part of since completing your undergraduate degree.

Q2.1 Please use the following fields to capture the feedback you would provide to Teacher 1. The fields coincide with the observation form you were provided. If you have an electronic copy of your feedback, you may copy and paste your responses into the appropriate fields.

Q2.2 Section 1: Taking Students' Ideas Seriously

Q2.3 Section 2: Press Students Conceptually

Q2.4 Section 3: Encouraging the Use of Multiple Models and Strategies

Q2.5 Section 4: Address Misconceptions

Q2.6 Section 5: Focus on the Structure of Mathematics

Q2.7 Section 6: Other

Q3.1 Please use the following fields to capture the feedback you would provide to Teacher 2. The fields coincide with the observation form you were provided. If you have an electronic copy of your feedback, you may copy and paste your responses into the appropriate fields.

Q3.2 Section 1: Taking Students' Ideas Seriously

Q3.3 Section 2: Press Students Conceptually

Q3.4 Section 3: Encouraging the Use of Multiple Models and Strategies

Q3.5 Section 4: Address Misconceptions

Q3.6 Section 5: Focus on the Structure of Mathematics

Q3.7 Section 6: Other

Q3.8 Thank you for participating in this study!

APPENDIX C: DMT OBSERVATION PROTOCOL FULL VERSION

Observer		**Date**		**District**	
Teacher		**Time**		**Topic**	
School		**Grade Level**		**Lesson Type**	

Notes

Time	**Tasks/Dialogue/Events /Student Work** *Use T for 'teacher', S1 for 'student 1', P for 'paraprofessional', etc.*	**Notes**

OBSERVATION SCALES

Attribute	Description	Scoring L1 L2 L3 L4 L5	
Taking Students' Ideas Seriously	A. Classroom **activities** are focused on, and adapted to, the responses and experiences of the students	1…………2…………3………...4………….5	
	B. Students are placed within the context of their own lives, experiences, and cultures	1…………2…………3………...4………….5	
	C. Students choose and share their methods	1…………2…………3………...4………….5	
	D. Ideas and methods are valued	1…………2…………3………...4………….5	
Pressing Students Conceptually	A. Students are asked to justify their strategy	1…………2…………3………...4………….5	
	B. Students explore the benefits and limitations of the strategy they are using	1…………2…………3………...4………….5	
	C. Students explore the benefits, limitations, and make connections between other strategies used in the classroom	1…………2…………3………...4………….5	
	D. There is evidence of progressive formalization	1…………2…………3………...4………….5	
Encourage Multiple Strategies	A. Multiple strategies and representations are used by students for recording, communicating, and thinking	1…………2…………3………...4………….5	
	B. The model used, or introduced by the teacher, reflects the strategy used by the student(s)	1…………2…………3………...4………….5	
	C. Models reflect the context of the problem and/or are appropriate for the given task	1…………2…………3………...4………….5	
	D. There is utilization of the progression of representational modes from enactive, to iconic, to symbolic	1…………2…………3………...4………….5	
Addressing Misconceptions	A. Incorrect answers or inappropriate strategies are used as learning opportunities for individual students	1…………2…………3………...4………….5	
	B. Incorrect answers or inappropriate strategies are used as learning opportunities for many students	1…………2…………3………...4………….5	
	C. Misconceptions are addressed in a manner that focuses on fundamental mathematics, provides the student an opportunity to understand the needed correction, and is mindful of students' feelings of self-efficacy	1…………2…………3………...4………….5	
	D. Instruction does not contain misleading or incorrect information that hinders future learning	1…………2…………3………...4………….5	
Focus on the Structure of Mathematics	A. Correctness resides in mathematical argument	1…………2…………3………...4………….5	
	B. Recognition of relationships to other mathematical topics are part of the instructional process	1…………2…………3………...4………….5	
	C. Classroom activities leave behind something of mathematical value (e.g. residue)	1…………2…………3………...4………….5	
	D. Opportunities for mathematical generalization are evident through the development of both procedural and conceptual knowledge	1…………2…………3………...4………….5	

APPENDIX D: DMT OBSERVATION PROTOCOL SIMPLIFIED VERSION

Teacher:____________________________ Date:__________________________________

Grade or Class:______________________ Topic:_________________________________

Attribute	Description	Observation Notes
Taking Students' Ideas Seriously	A. Classroom **<u>activities</u>** are focused on, and adapted to, the responses and experiences of the students	
	B. Students are placed within the context of their own lives, experiences, and cultures	
	C. Students choose and share their methods	
	D. Ideas and methods are valued	
Pressing Students Conceptually	A. Students are asked to justify their strategy	
	B. Students explore the benefits and limitations of the strategy they are using	
	C. Students explore the benefits, limitations, and make connections between other strategies used in the classroom	
	D. There is evidence of progressive formalization	
Encourage Multiple Strategies	A. Multiple strategies and representations are used by students for recording, communicating, and thinking	
	B. The model used, or introduced by the teacher, reflects the strategy used by the student(s)	
	C. Models reflect the context of the problem and/or are appropriate for the given task	
	D. There is utilization of the progression of representational modes from enactive, to iconic, to symbolic	
Addressing Misconceptions	A. Incorrect answers or inappropriate strategies are used as learning opportunities for individual students	
	B. Incorrect answers or inappropriate strategies are used as learning opportunities for many students	
	C. Misconceptions are addressed in a manner that focuses on fundamental mathematics, provides the student an opportunity to understand the needed correction, and is mindful of students' feelings of self-efficacy	

	D. Instruction does not contain misleading or incorrect information that hinders future learning	
Focus on the Structure of Mathematics	A. Correctness resides in mathematical argument	
	B. Recognition of relationships to other mathematical topics are part of the instructional process	
	C. Classroom activities leave behind something of mathematical value (e.g. residue)	
	D. Opportunities for mathematical generalization are evident through the development of both procedural and conceptual knowledge	
Other Notes, Observations, or Comments		

APPENDIX E: IRB EXEMPT CERTIFICATION

University of Idaho
Office of Research Assurances (ORA)
Institutional Review Board (IRB)
875 Perimeter
Drive, MS 3010
Moscow ID 83844-3010
Phone: 208-885-6162
Fax: 208-885-5752
irb@uidaho.edu

November 1, 2013

To: Anne Adams
Cc: Duane Peck

From: IRB, University of Idaho Institutional Review Board

Subject: Exempt Certification for IRB project number 13-270

Determination: November 1, 2013
Certified as Exempt under category 1 at 45 CFR 46.101(b)(1)
IRB project number 13-270: A Comparison of Written Feedback Given to Instructors of Mathematics

This study may be conducted according to the protocol described in the Application without further review by the IRB. As specific instruments are developed, each should be forwarded to the ORA, in order to allow the IRB to maintain current records. Every effort should be made to ensure that the project is conducted in a manner consistent with the three fundamental principles identified in the Belmont Report: respect for persons; beneficence; and justice.

It is important to note that certification of exemption is NOT approval by the IRB. Do not include the statement that the UI IRB has reviewed and approved the study for human subject participation. Remove all statements of IRB Approval and IRB contact information from study materials that will be disseminated to participants. Instead please indicate, "The University of Idaho Institutional Review Board has Certified this project as Exempt."

Certification of exemption is not to be construed as authorization to recruit participants or conduct research in schools or other institutions, including on Native Reserved lands or within Native Institutions, which have their own policies that require approvals before Human Subjects Research Projects can begin. This authorization must be obtained from the appropriate Tribal Government (or equivalent) and/or Institutional Administration. This may include independent review by a tribal or institutional IRB or equivalent. It is the investigator's responsibility to obtain all such necessary approvals and provide copies of these approvals to ORA, in order to allow the IRB to maintain current records.

This certification is valid only for the study protocol as it was submitted to the ORA. Studies certified as Exempt are not subject to continuing review (this Certification does not expire). If any changes are made to the study protocol, you must submit the changes to the ORA for determination that the study remains Exempt before implementing the changes. The IRB Modification Request Form is available online at:
http://www.uidaho.edu/ora/committees/irb/irbforms