Tidal Evolution of Extrasolar Moons and Its Applications

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Abstract

In this thesis, we consider tidal evolution of extrasolar moons of star-planet-moon systems. We formulated tidal decay lifetimes for hypothetical moons with both lunar and stellar tides. We found four types of trajectories depending on the astronomical parameters and the initial conditions. For each type, we derived a formula that can calculate the lifetimes of moons.

We apply our results to rocky planets at habitable distances. In order to find the conditions that the lifetimes of moons are more than 5 Gyrs. We found that the Moon in the Sun-Earth system must have had an initial orbital period of at most 20 hr/rev to exceed a 5 Gyr lifetime. Because not all extrasolar rocky planets are Earth-like (33% iron, 67% rock) planet, we examined four typical planetary compositions: iron (100%), Earth-like, rock (100%), and rock-ice (50%-50%). We found that there is the minimum stellar mass below which moons cannot survive more than 5 Gyrs. This minimum stellar mass becomes lower if the planetary density is higher.

When we consider the tidal evolution of star-planet-moon systems with both stellar and lunar tides, moons may hit planets or escape from them. We studied two consequences of moon-planet collisions, and found that the moons of terrestrial planets in habitable zones cannot survive 1 Gyrs if stars are 0.3 $M_{\odot}$, however they may survive more than 5 Gyrs if stars are 0.8 $M_{\odot}$ or more. Extrasolar rocky planets are usually hard to detect because they are small and dim. A planet-moon collision would be a good opportunity for detecting such planets from the Earth because the brightness of the planet increases dramatically after the collision. Our results suggest that longer wavelengths (We prefer 5μm) are better for such a detection.

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I would like to thank to my PhD supervisor, Dr. Jason Barnes, for starting me on a very interesting research topic. I feel I am very lucky because I have really enjoyed this research opportunity.

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Chapter 1

Introduction

1.1 Extrasolar Planets

The discovery of the first extrasolar planet orbiting main-sequence star by Mayor & Queloz (1995) [45] was big news in planetary science. Just before this discovery, Gordon Walker concluded that there were no extrasolar planets [63]. He had measured the radial velocities of 21 solar-type stars for 12 years in an effort to find Jupiter-sized extrasolar planets. We now know that many Jupiter-sized extrasolar planets have much shorter periods. But around that time, Jupiter-sized extrasolar planets were expected to have long periods because our solar system was only the planetary system we knew. The detection of the first Jovian-like planet, 51 Pegasus b, in a 4-day orbit was completely unexpected. The closest planet in our solar system, Mercury, has an 88 day period - 51 Pegasus b is more than 7 times closer to its host star than Mercury is to the Sun! 51 Pegasus b is not the only planet whose period is very short. There are more than 600 planets known whose periods are shorter than 10 days and masses are roughly equivalent to Jupiter [1]. For such close-in planets, the stellar insolation that they receive could lead to surface temperature as high as 1500K. They are called "hot Jupiters".

Another interesting type of extrasolar Jovian planet are "eccentric planets". In our solar system, all four Jovian planets have small eccentricities. The eccentricity of Saturn is the largest and its value is 0.056. Fig 1.1 shows the semimajor axes and eccentricities of all detected extrasolar planets. Eccentricities of extrasolar planets are widely distributed up to near 1. HD 20782 b has the most eccentric orbit and its eccentricity is 0.97. Because the semimajor axis of HD 20782 b is 1.381, the apsides are 0.04 and 2.72 AU.

The existence of many "hot Jupiters" and "eccentric planets" challenges the con-

[1] http://exoplanet.eu/

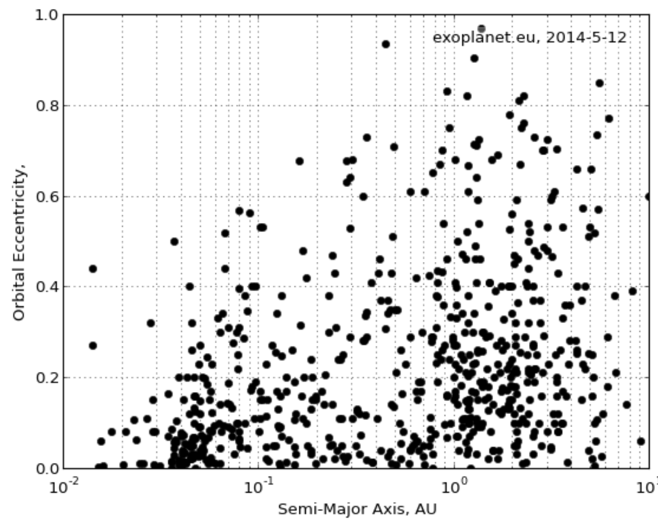

Figure 1.1: This graph, taken from http://exoplanet.eu, shows the distribution of extrasolar planetary eccentricities and semimajor axes.

ventional planet formation theory. Because our solar system was before 1995 the only planetary system we knew, conventional planet formation theory explains our solar system: terrestrial planets are inside of the ice line and gas giants are beyond; all planets have enough space not to perturb each other; and planets have mostly circular orbits. Actual extrasolar planets are much more varied than we expected. From Fig 1.1, for semimajor axis greater than 0.2 AU, extrasolar planets can have values of eccentricities up to 0.97. For semimajor axis less than 0.2 AU, the values of eccentricities are at most 0.7. Even though there seems the maximum values of eccentricity for close extrasolar planets otherwise their periapsis would be inside their star, the distribution of eccentricities is widely spread compared to our solar system. Fig 1.2 shows masses and distances from the stars of extrasolar planets. As mentioned, gas giants are thought to form beyond the ice line according to the conventional planet formation theory. For extrasolar planetary systems, however, gas giants clearly exist inside the ice line.

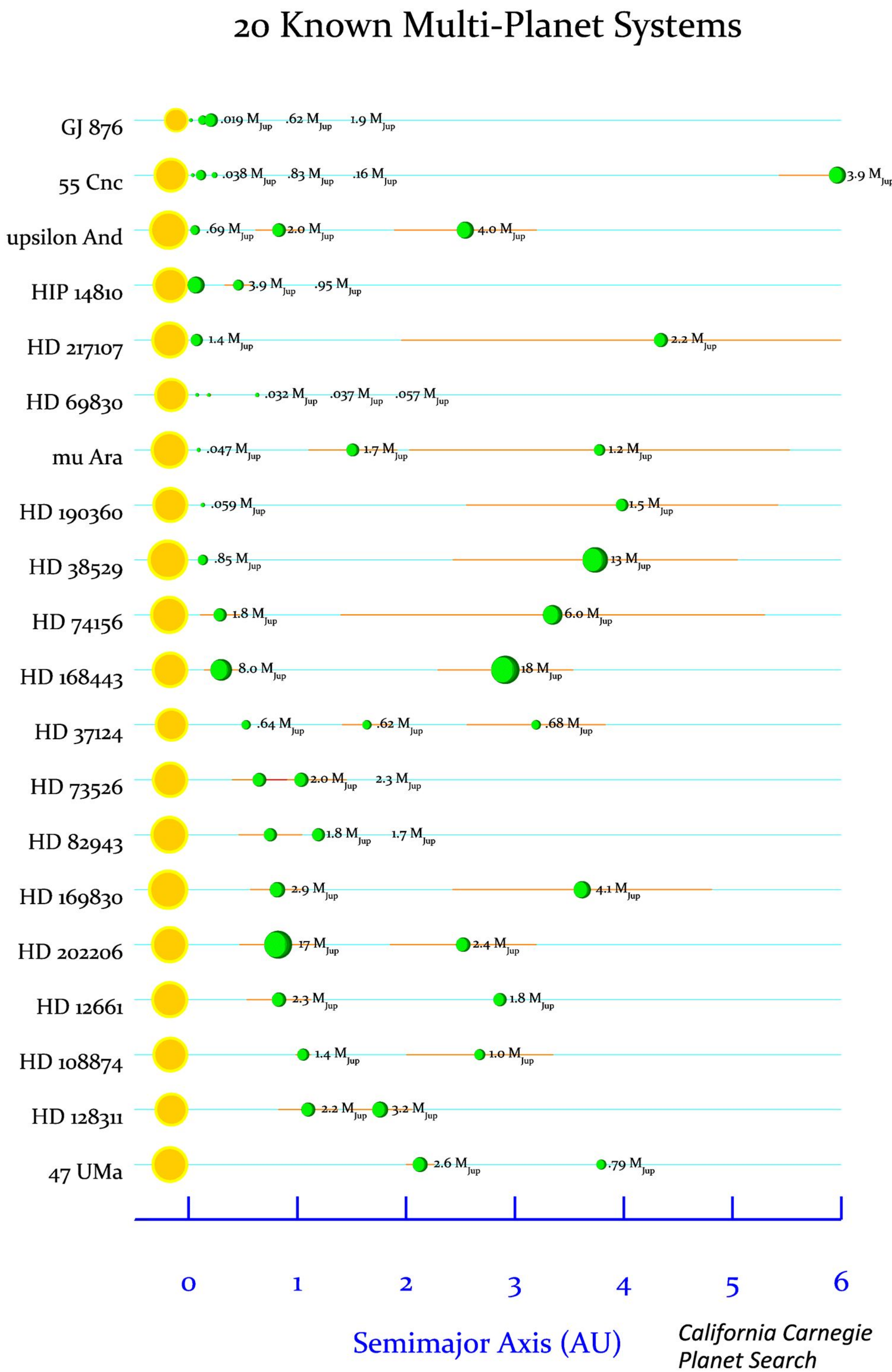

Figure 1.2: This graph shows masses and distances from the stars of extrasolar planets. In extrasolar systems, Jupiter-sized planets can exist inside of the ice line. (Image credit: http://exoplanets.org/exoplanets_pub.html)

1.2 Extrasolar Moons

There are dozens of moons in our solar system. Almost all of them belong to ice and gas giant planets. Jupiter and Saturn, gas giant planets, have 67 and 62 moons, respectively. Uranus and Neptune, ice giant planets, have 27 and 14 moons, respectively. These moons have a wide range of diversity. The size of these moons ranges from less than 1 km diameter to larger than planet Mercury. Titan, the largest moon of Saturn, has an atmosphere. So far more than 1700 extrasolar planets [2] have been discovered, but extrasolar moons have yet to be detected.

MOA-2011-BLG-262 the first microlensing candidate for a free-rotating exoplanet-exomoon system with a sub-Earth mass moon orbiting around a ~ 4 Jupiter mass planet [3]. Unfortunately, though, it is not possible to confirm this result because MOA-2011-BLG-262 got accidentally between the observer on the Earth and a distant star. This result is not conclusive evidence of the existence of extrasolar moons, but it does show us the possibility of the existence of extrasolar moons.

Because there are many unique moons in our solar system, I expect that extrasolar moons will also reflect this diversity, such that even Earth-sized, habitable moons are possible. On the other hand, there are only three moons around rocky planets in our solar system. Mercury and Venus do not have moons. Satellite orbital tidal decay may be a reason of the absence of the inner planets' moons [65],[7]. Earth has the Moon. Mars has two moons, but they are very small compared to Mars in mass ratio.

Our Moon may have played a surprisingly important role in the evolution of life on Earth [64]. For life, liquid water is a very important material. In addition to liquid water, stable climate on giga-year timescales may be one of the essential conditions for planets to be habitable. Planetary climate is greatly influenced by obliquity. For Earth, planetary obliquity is stabilized by the Moon [38]. For extrasolar planets, the situation may be the same: moons stabilize planetary obliquity.

[2]http://exoplanet.eu/

1.3 Tidal Theory

The effects of tidal interaction in the solar system were discussed in Goldreich & Soter (1966) [22]. The tidal dissipation quality factor Q is defined by the following equation

$$Q^{-1} = \frac{1}{2\pi E_0} \oint \left(-\frac{dE}{dt}\right) dt, \tag{1.1}$$

where E_0 is the maximum energy stored in the tides, and dE/dt is the rate of dissipation. The integral is evaluated over one tidal period. Hence, Q is inversely proportional to the tidal dissipation efficiency. This means that if Q is larger, then less energy is lost due to tidal friction. On the other hand, if Q is smaller, then the planet dissipates energy relatively easily.

There are two groups of planetary Q. The terrestrial planets and satellites of the Jovian planets have values of Q from 10 to 500. Jovian planets, however, have Q values that are greater than 10^4. For example, the estimated value of Q for Saturn is greater than 1.8×10^4 and for the Earth is about 12 [48]. Because Saturn has thick hydrogen/helium atmosphere, energy loss due to tidal friction is very small. Hence, Q for Saturn is very large. Because the Earth loses large amounts of energy in shallow seas, the Q value of the Earth is low.

The relationship between Q and the angle between the high tide and the orbiting body, θ, is:

$$Q^{-1} = \tan(2\theta), \tag{1.2}$$

and because $Q \gg 1$, $Q^{-1} \sim 2\theta$ [43].

The effects of tides are important to the dynamics of star-planet-moon systems. The tides on a planet change the orbits of both the planet and moon. There are three kinds of orbital evolution of the moon when we consider planet-moon systems: the orbits of moons (i) migrate inward, (ii) migrate outward, and (iii) reach a stable synchronous state [13]. Our Earth-Moon system is now in (i) state because the Earth-Moon distance increases about 3.8 cm/year [37], [48]. The orbits of close extrasolar giant planets will circularize due to the tides on the planets from their parent stars [62]. The circularization of the orbits of extrasolar giant planets may explain why most close-in planets have low eccentricities [30].

1.4 Habitability and Tidal Theory

Detecting extrasolar planets in the habitable zone is exciting, particulary Earth-sized, rocky planets, because life may exist on these planets. Kepler-62e and f, and Kepler-186f are newly-discovered, Earth-sized, rocky extrasolar planets in the habitable zone [52],[5]. Liquid water may exist on their surface; hence, life may also exist. Off course, more detailed investigations are needed to make any detailed conclusions, but our work puts some constrains on the possibilities.

In order for life to reach a complex form, it is not enough just for planets to be in the habitable zone. Long-term stable planetary climate may also be needed. A planet's climate depends heavily on obliquity, or axis tilt [16], [68]. If planetary obliquity changes rapidly, then the planet cannot have a long-term stable climate. Because the Moon stabilizes the Earth's obliquity [38], the Earth has a long-term stable climate. In Chapter 2, we study orbital evolution of the moon in star-planet-moon systems. We found that when we consider tides on the planets due to both the stars and moons, moons will either hit or escape from their planets. There is not stable planet-moon synchronous state. If moons orbit around planets for a long time, then planets have long-term stable planetary climate, making their host planets more amenable to have evolving life forms.

Rocky planets in the habitable zone are not the only places possible for life to exist. Moons of extrasolar giant planets orbiting at habitable distance would also be good candidates. Even if a moon were not in habitable zone, it could still be habitable. If,

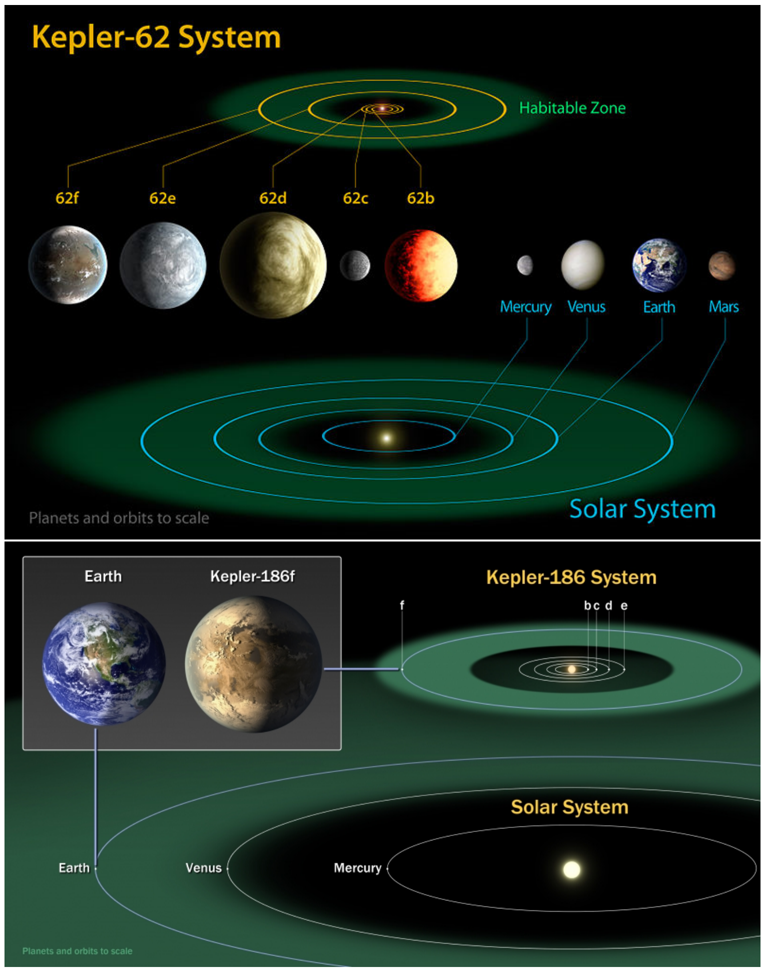

Figure 1.3: Kepler-62e and f, and Kepler-186f are newly discovered Earth-sized rocky extrasolar planets in habitable zone. The green regions around stars are the habitable zone. (Image credit: http://www. nasa.gov)

for example, the moon were close enough to the extrasolar giant planets, tides on the moon could generate large amounts of energy (like on Jupiter's moon Io)[26].

1.5 Outline of This Thesis

In this thesis, I studied the tidal evolution of a star-planet-moon system. Especially, I was interested in the tidal decay lifetimes of moons. If moons are habitable, then their tidal decay lifetimes give us an idea of how long the moons may orbit around their planets. If planets are habitable, then tidal decay lifetimes of moons tell us how long the planetary obliquity may be stable. Hence, the timescale of stable planetary climate can be estimated.

In Chapter 2, we formulated tidal decay lifetimes for hypothetical moons with both lunar and stellar tides. We found four types of trajectories depending on some astronomical parameters such as the masses of the star, planet, and moon; radius of planet; and the initial conditions such as the initial positions of the planet and moon and initial rotation of the planet. For each type of trajectory, I derived a mathematical expression to estimate the lifetimes of moons. In this method, I can estimate lifetimes of moons without using simulations.

I applied my methods and results to rocky planets at habitable distances to find the conditions that the moons may have more than 5 Gyrs lifetimes in Chapter 3.

I studied two consequences of moon-planet collisions in Chapter 4. Chapter 5 is my conclusion.

Chapter 2

Outcomes and Duration of Tidal Evolution in a Star-Planet-Moon System

(This chapter was published: Sasaki, T., Barnes, J. W., & O'Brien, D. P 2012, ApJ, 745, 51)

ABSTRACT

We formulated tidal decay lifetimes for hypothetical moons orbiting extrasolar planets with both lunar and stellar tides. Previous work neglected the effect of lunar tides on planet rotation, and are therefore applicable only to systems in which the moon's mass is much less than that of the planet. This work, in contrast, can be applied to the relatively large moons that might be detected around newly-discovered Neptune-mass and super-Earth planets. We conclude that moons are more stable when the planet/moon systems are further from the parent star, the planets are heavier, or the parent stars are lighter. Inclusion of lunar tides allows for significantly longer lifetimes for a massive moon relative to prior formulations. We expect that the semi-major axis of the planet hosting the first detected exomoon around a G-type star is 0.4-0.6 AU and is 0.2-0.4 AU for an M-type star.

2.1 Introduction

The first discovery of an extrasolar planet in orbit around a main-sequence star was made by [45]. Since then, more than 700 extrasolar planets[1] have been discovered. Although extrasolar moons have not yet been detected, they almost certainly exist. Most of the planets in our solar system have satellites. Even Pluto, though no longer officially a planet, has three moons [66]. It is likely that the mechanisms for moon formation in our solar system (impact, capture, and coaccretion) prevail beyond it

[1] http://exoplanet.eu/

[44].

The Earth's obliquity, or axial tilt, is stabilized by the Moon [38]. Mars, on the other hand, has relatively small satellites, and its obliquity changes chaotically, fluctuating on a 100,000-year timescale [39]. Stable obliquity in its star's habitable zone may be necessary for a planet to support life. An Earth-size planet with no moon, or a relatively small one, may be subject to large fluctuations in obliquity. In such a case, favorable conditions may not last long enough for life to become established. In the same way, orbital longevity is required for any life form to have time to become established. Hence, the prospects for habitable planets may hinge on moons [64]; but see also [41].

In 2005, Rivera et al. [54] discovered Gliese 876 d, the first super-Earth around a main sequence star. To date more than thirty super-Earths have been discovered[2]. The discovery of Kepler 22-b in the habitable zone gives rise to the possibility of life beyond our Solar System [6]. It is important to know the lifetime of moons orbiting super-Earths in the habitable zone: while the planet might be unsuited to the evolution of life, its moons might be. Moons with masses of at least one third $M_\oplus$, and orbiting around gas giant planets in the habitable zone may have habitable environments [69]. The moon's orbital stability plays a role in habitability as well. Clearly, if the moon leaves orbit, it will probably leave the habitable zone.

Although extrasolar moons have not yet been found, several methods to detect them have been investigated. After Kaltenegger (2010) [32], the following methods can detect extrasolar moons:

1. Transit timing variations (Satoretti & Schneider 1999; Agol et al. 2005; Holman & Murray 2005) [56, 1, 27].

2. Transit duration variations (Kipping 2009) [34].

[2] http://exoplanet.eu/

3. Light curve distortions (Szabó et al. 2006) [61].

4. Planet-moon eclipses (Cabrera & Schneider 2007) [8].

5. Microlensing (Han 2008) [23].

6. Pulsar timing (Lewis et al. 2008) [40].

7. Distortion of the Rossiter-McLaughlin effect of a transiting planet (Simon et al. 2009) [59].

Considering the speed at which observational instrumentation has developed, it is only a matter of time before extrasolar moons are discovered.

Tidal torque is important to the long-term orbital stability of extrasolar moons. A binary system can be in tidal equilibrium only if coplanarity (the equatorial planes of the planet and moon coincide with the orbital plane), circularity (of the orbit), and corotation (the rotation periods of the planet and moon are equal to the revolution period) have been fulfilled. Further, stability occurs only if the orbital angular momentum exceeds the sum of the spin angular momenta of the planet and moon by more than a factor of three [29].

Counselman (1973) [13] studied the stability of these equilibria only with respect to coplanarity and circularity. He pointed out that in a planet-moon system with lunar[3] tides, there are three possible evolutionary states.

Counselman state (i):the semi-major axis of the moon's orbit tidally evolves inward until the moon hits the planet. Example:Phobos around Mars.

Counselman state (ii):the semi-major axis of the moon's orbit tidally evolves outward until the moon escapes from the planet. No solar system examples are available. But this result would happen to the Earth-Moon if Earth's present rotation rate were doubled.

[3]In this paper, we use "lunar" as the adjective of any moons, not just the Moon

Counselman state (iii):lunar orbital and planetary spin angular velocities enter mutual resonance and are kept commensurate by tidal forces. Example:Pluto and Charon. This Counselman state is static, while state (i) and (ii) are evolutionary.

Here, we consider a star-planet-moon system with stellar tides. Although they did not consider the effects of lunar tides or maximum distance from the planet, Ward & Reid (1973) [65] examined the impact of solar tides on planetary rotation in a limited star-planet-moon system.

Barnes & O'Brien (2002) [2] considered a similar case, considering the maximum distance of the moon but neglecting the lunar tide's effect on planetary rotation. They found just two possible final states:the moon may either hit the planet or escape from it.

In this paper, we consider a star-planet-moon system with both stellar and lunar tides, and lunar maximum distance from the planet. Stellar and lunar tides both affect planetary spin, whereas stellar and lunar tides affect planet and moon orbits, respectively. We do not consider the effect of stellar tides on the moon's rotation. Stellar tides should sap angular momentum from the system but this effect is less important if the mass of the planet is at least ten times greater than the mass of the moon. We apply tidal theory and set up a system of differential equations that govern the planetary rotational rate and orbital mean motion as well as the orbital mean motion of the moon. The system of differential equations is solved numerically. Finally, a formula for the length of time the moon will stably orbit is found. We then apply this result to hypothetical extrasolar planet moon systems.

2.2 Theory

In this paper, we use standard tidal evolution theory with the constant Q approach [22], along with the following assumptions:

1. The spin angular momentum of the planet is parallel to the orbital angular

momenta of both the moon about the planet and the planet about the star; i.e. the planet has 0° obliquity, the moon orbits in the planet's equatorial plane, and the planet and moon motions are prograde.

2. The total angular momentum, that is the sum of the moon's orbital angular momentum and the planet's rotational and orbital angular momenta, is constant. We neglect the orbital angular momentum of the moon about the star and the moon's rotational angular momentum.

3. The moon's orbit about the planet and the planet's orbit about the star are circular.

4. The moon is less (at most $\sim 1/10$) massive than the planet and the planet is also less (at most $\sim 1/10$) massive than the star.

5. The star's spin angular momentum is not considered nor are the planet's tides on the star or the star's tides on the moon.

6. The specific dissipation function of the planet, Q_p, is independent of the tidal forcing frequency and does not change as a function of time.

Planetary Q_p falls into two groups. The first group has values of Q_p that range from 10 to 500. The terrestrial planets and satellites of the Jovian planets are in this group. The other group has Q_p values greater than 10^4. The Jovian planets are in this group. In the case of the Earth, tidal dissipation is due to friction between the tidally generated currents and the ocean floor and occurs mostly in shallow seas. For Mercury and Venus, tidal dissipation is driven by viscous dissipation within the bulk planetary interior. The mechanism for tidal dissipation within giant planets remains unknown.

In that case, the torque on the planet due to the moon τ_{p-m} is given by Barnes & O'Brien (2002) [2], Goldreich & Soter (1966) [22], and Murray & Dermott (2000)

[48] in Chapter 4 as

$$\tau_{p-m} = -\frac{3}{2}\frac{k_{2p}GM_m^2R_p^5}{Q_p a_m^6}\mathrm{sgn}(\Omega_p - n_m), \tag{2.1}$$

where Ω_p is the rotational rate of the planet, k_{2p} is the tidal Love number of the planet, G is the gravitational constant, R_p is the radius of the planet, M_m is the mass of the moon, a_m is the semimajor axis of the moon's orbit, and n_m is the orbital mean motion of the moon. The function $\mathrm{sgn}(\Omega_p - n_m)$ is 1 if $(\Omega_p - n_m)$ is positive, -1 if $(\Omega_p - n_m)$ is negative, and undefined if $(\Omega_p - n_m) = 0$.

Similarly, the torque on the planet due to the star τ_{p-s} is

$$\tau_{p-s} = -\frac{3}{2}\frac{k_{2p}GM_s^2R_p^5}{Q_p a_p^6}\mathrm{sgn}(\Omega_p - n_p), \tag{2.2}$$

where M_s is the mass of the star, a_p is the semimajor axis of the planet's orbit, n_p is the orbital mean motion of the planet.

For the spin angular momentum of the planet,

$$I_p\frac{d\Omega_p}{dt} = \frac{dL_{pspin}}{dt} = \tau_{p-m} + \tau_{p-s}, \tag{2.3}$$

where the planet's rotational moment of inertia $I_p = \alpha M_p R_p^2$. α is the moment of inertia constant. For Earth and Jupiter, α's are 0.3308 and 0.254, respectively [15].

The change in orbital angular momenta of the moon about the planet and the planet about the star, by Newton's Third Law, are equal and opposite the moon's and star's torques on the planet, respectively:

$$\frac{dL_m}{dt} = \tau_{m-p} = -\tau_{p-m} \tag{2.4}$$

and

$$\frac{dL_p}{dt} = \tau_{s-p} = -\tau_{p-s}, \tag{2.5}$$

where $L_m = M_m a_m^2 n_m$ and $L_s = M_p a_p^2 n_p$.

Using Kepler's Third Law, $n_m^2 a_m^3 \approx GM_p$ and $n_p^2 a_p^3 \approx GM_s$ because we assume that $M_m \ll M_p$ and $M_p \ll M_s$. This allows us to derive these expressions for L_m and L_p

$$L_m = \frac{M_m (GM_p)^{2/3}}{n_m^{1/3}} \tag{2.6}$$

and

$$L_p = \frac{M_p (GM_s)^{2/3}}{n_p^{1/3}}. \tag{2.7}$$

To determine how n_m and n_p vary with time, we take the derivative of L_m and L_p with respect to t, set the results equal to equation (2.4) and equation (2.5), and solve for $\frac{dn_m}{dt}$ and $\frac{dn_p}{dt}$.

Then, we have

$$\frac{dn_m}{dt} = \frac{3\tau_{p-m}}{M_m (GM_p)^{2/3}} n_m^{4/3} \tag{2.8}$$

and

$$\frac{dn_p}{dt} = \frac{3\tau_{p-s}}{M_p (GM_s)^{2/3}} n_p^{4/3}. \tag{2.9}$$

When we combine equation (5.39), equation (5.40), equation (2.3), equation (2.8), and equation (2.9), we obtain the differential equations that govern the time-evolution of the star-planet-moon system:

$$\begin{cases} \frac{dn_m}{dt} = -\frac{9}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}} n_m^{16/3}\mathrm{sgn}(\Omega_p - n_m) \\ \frac{dn_p}{dt} = -\frac{9}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}} n_p^{16/3}\mathrm{sgn}(\Omega_p - n_p) \\ \frac{d\Omega_p}{dt} = -\frac{3}{2}\frac{k_{2p}R_p^3}{Q_p}\frac{(GM_m)^2}{\alpha(GM_p)^3} n_m^4\mathrm{sgn}(\Omega_p - n_m) \\ \qquad\qquad -\frac{3}{2}\frac{k_{2p}R_p^3}{Q_p}\frac{1}{\alpha(GM_p)} n_p^4\mathrm{sgn}(\Omega_p - n_p). \end{cases} \tag{2.10}$$

The solutions to these differential equations are

$$n_m(t) = \Bigg(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\,t\,\mathrm{sgn}(\Omega_p - n_m) \\ + n_m^{-13/3}(t=0)\Bigg)^{-3/13} \tag{2.11a}$$

$$n_p(t) = \Bigg(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\,t\,\mathrm{sgn}(\Omega_p - n_p) \\ + n_p^{-13/3}(0)\Bigg)^{-3/13} \tag{2.11b}$$

$$\Omega_p(t) = -\frac{1}{\alpha R_p^2}\Bigg\{\frac{GM_m}{(GM_p)^{1/3}}\left(n_m^{-1/3}(t) - n_m^{-1/3}(0)\right) \\ +(GM_s)^{2/3}\left(n_p^{-1/3}(t) - n_p^{-1/3}(0)\right)\Bigg\} \\ +\Omega_p(0). \tag{2.11c}$$

These solutions are only valid if each of $\mathrm{sgn}(\Omega_p - n_m)$ and $\mathrm{sgn}(\Omega_p - n_p)$ are constant in time. Also, these solutions are only valid when the planet's rotation is not tidally synchronous with either the star or the moon, i.e. $\Omega_p - n_m \neq 0$ and $\Omega_p - n_p \neq 0$. When the planet's rotation is synchronized, we must use an alternative approach.

When synchronization has occurred, i.e. when $\Omega_p = n_m$ or $\Omega_p = n_p$, we follow the evolution of the system using conservation of angular momentum:

$$\frac{M_m(GM_p)^{2/3}}{n_m^{1/3}(t)} + \alpha R_p^2 M_p \Omega_p(t) + \frac{M_p(GM_s)^{2/3}}{n_p^{1/3}(t)} = L_0, \tag{2.12}$$

where $L_0 = \frac{M_m(GM_p)^{2/3}}{n_m^{1/3}(0)} + \alpha R_p^2 M_p \Omega_p(0) + \frac{M_p(GM_s)^{2/3}}{n_p^{1/3}(0)}$ is the initial angular momentum.

By our assumption 2, the total angular momentum is the sum of the moon's orbital angular momentum, which is the first term, the planet's rotational angular momentum, which is the second term, and orbital angular momentum, which is the third term.

When the planet is not tidally locked with either the star or the moon, these three equations are valid:

$$n_m(t) = \Bigg(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\, t\, \mathrm{sgn}(\Omega_p - n_m) \\ + n_m^{-13/3}(0)\Bigg)^{-3/13} \tag{2.13a}$$

$$n_p(t) = \Bigg(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\, t\, \mathrm{sgn}(\Omega_p - n_p) \\ + n_p^{-13/3}(0)\Bigg)^{-3/13} \tag{2.13b}$$

$$\frac{(GM_m)(GM_p)^{2/3}}{n_m^{1/3}(t)} + \alpha R_p^2 (GM_p)\Omega_p(t) \\ + \frac{(GM_p)(GM_s)^{2/3}}{n_p^{1/3}(t)} = GL_0. \tag{2.13c}$$

Even though equation (2.11c) and equation (2.13c) are equivalent, equation (2.13c) is valid when the planet's rotation is tidally locked to the moon because equation (2.13c) is derived from the conservation of angular momentum.

When the planet is tidally locked with the moon, i.e. $n_m = \Omega_p$, equation (2.13a) is not valid. Hence, in that case,

$$n_p(t) = \Bigg(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\, t\, \mathrm{sgn}(n_m - n_p) \\ + n_p^{-13/3}(0)\Bigg)^{-3/13} \tag{2.14a}$$

$$\frac{(GM_m)(GM_p)^{2/3}}{n_m^{1/3}(t)} + \alpha R_p^2 (GM_p) n_m(t) \\ + \frac{(GM_p)(GM_s)^{2/3}}{n_p^{1/3}(t)} = GL_0. \tag{2.14b}$$

When the planet is tidally locked with the star, i.e. $n_p = \Omega_p$, equation (2.13b) is not valid. Hence, in that case,

$$n_m(t) = \Big(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\, t\, \mathrm{sgn}(n_p - n_m) + n_m^{-13/3}(0)\Big)^{-3/13} \tag{2.15a}$$

$$\frac{(GM_m)(GM_p)^{2/3}}{n_m^{1/3}(t)} + \alpha R_p^2 (GM_p) n_p(t) + \frac{(GM_p)(GM_s)^{2/3}}{n_p^{1/3}(t)} = GL_0. \tag{2.15b}$$

2.3 Numerical Solutions

We first explore the implications of equations (2.13), (2.14), and (2.15) numerically. In these simulations, we start with $\Omega_p(0) > n_m(0) > n_p(0)$. Physically, this condition implies that one planet year is longer than one planet day and that the orbital period of the moon is between them. A typical example is our Sun-Earth-Moon system. The shapes of the resulting graphs of Ω_p, n_m, and n_p as a function of time depend on the torques due to the planet and moon, but also on the orbital angular velocities of the planet and moon. If both the orbital angular velocities of the moon and planet are slower than the spin angular velocity of the planet, then both the torques due to the moon and those due to the star brake the rotation of the planet. If the orbital angular velocity of the moon is faster than the spin angular velocity of the planet, then the spin angular velocity of the planet may increase or decrease, depending on the relative magnitude of the torque due to the moon and star.

One important lunar escape condition for the calculation is the critical semimajor axis; this is the outermost stable orbit for the moon. Barnes & O'Brien (2002) [2] stated that the critical semimajor axis, a_{crit}, is

$$a_{crit} = f R_H, \tag{2.16}$$

where f is a constant and R_H is the radius of the Hill's sphere [15]

$$R_H = a_p \left(\frac{M_p}{3M_s} \right)^{1/3}, \tag{2.17}$$

where a_p is the semimajor axis of the planet. Orbits outside a_{crit} are not stable. In this paper, we follow the orbits using angular velocity instead of semimajor axis. Considering n_{crit} such that

$$n_{crit}^2 a_{crit}^3 = GM_p, \tag{2.18}$$

then, using (2.16) and (2.17), and Kepler's Third Law for the planet, $n_p^2 a_p^3 = GM_s$, we calculate n_{crit} as a function of f:

$$n_{crit}(t) = \left(\frac{3}{f^3} \right)^{1/2} n_p(t). \tag{2.19}$$

The value of f is not well-determined. Barnes & O'Brien (2002)[2] used $f = 0.36$. Domingos et al. (2006) [18] suggested $f = 0.49$. In this paper, we use $f = 0.36$ for numerical calculations because it is the most conservative estimate for the moon to remain bound.

This is the critical mean motion that is the lowest stable angular velocity for the moon. From section 2.4.1 to section 2.4.3, we enforce that

$$n_{crit}(t) < n_m(t). \tag{2.20}$$

This means that the moon has a stable orbit.

In the resulting numerical integrations, we found three classes of stable outcomes. We call the three stable outcomes :

- Type I(Fig.2.1 and 2.2)
  - planet-moon become synchronous

- Type II(Fig.2.3)
  - planet-star become synchronous first, then planet-moon become synchronous later
- Type III(Fig.2.4)
  - planet-moon never synchronous.

For Type I, there are two subcases.

In each of the three stable outcomes, the first part is common. Initially, $\Omega_p(0) > n_m(0) > n_p(0)$. Since the orbital angular velocities of the moon and the planet are slower than the spin angular velocity of the planet, the torques due to the moon and the star brake the rotation of the planet. This continues until the spin angular velocity of the planet is equal to the angular velocity of the moon. We call this time $T1$. From the beginning to $T1$, the planet loses rotational angular momentum to the orbital motions of the moon and the planet. By gaining angular momentum, the orbital motions of the moon and the planet slow down and their semimajor axes increase.

After $T1$, each type has its own characteristics. Another feature that Type I, II, and III have in common is that the planet's angular velocity, $n_p(t)$, always decreases due to solar tides. This indicates that the orbital angular momentum of the planet always increases.

In Type I, if the tidal locking starts at $T1$, then the system is Type I Case 1 (Fig.2.1). The system is Type I Case 2 if the tidal locking starts after $T1$ (Fig.2.2).

- Type I
  - Case1 (Fig.2.1)
    * In our Type I Case 1 star-planet-moon system, the torque on the planet due to the moon is greater than that due to the star at $t = T1$.

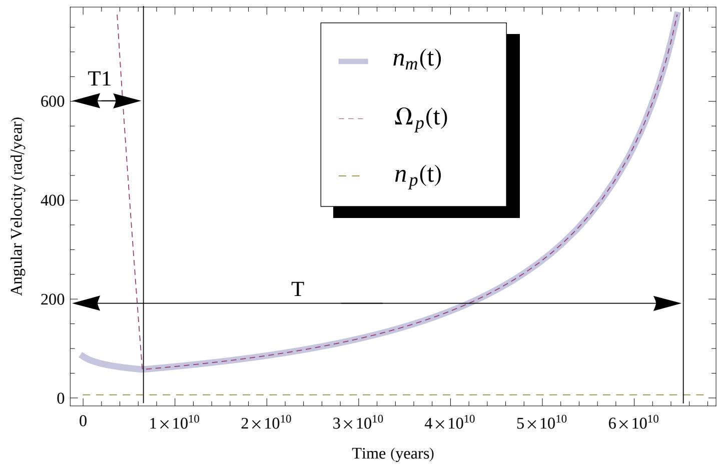

Figure 2.1: Here we graph the time evolution of Ω_p, n_p, and n_m for Type I Case 1. We use the present data of our Sun-Earth-Moon system for the initial condition, i.e. $n_m(0)$=84 rad/year, $\Omega_p(0) = 730\pi$ rad/year, and $n_p(0) = 2\pi$ rad/year. We take k_{2p} and Q_p for Earth to be 0.299 and 12 respectively (Murray & Dermott [48], pg166). The black vertical line on the right corresponds to the lunar orbital frequency at which the moon is orbiting at the planetary radius, i.e.when it crashes into the Earth and is destroyed.

At $T1$, the planet and the moon then assume a synchronized state with $\Omega_p = n_m$. Once they reach this synchronized state, they will stay in this state until the end for Type I Case 1. Since the tidal torque on the planet due to the moon is greater than that due to the star, the moon's orbital velocity, n_m, and the planet's spin angular velocity, Ω_p, are kept equal. In this synchronized state, only the orbital motion of the moon loses angular momentum; the planet's orbital and spin motion gain angular momentum.

- Case2(Fig.2.2)

  * In Type I Case 2, the moon's tidal torque on the planet is slightly smaller than the stellar torque at $T1$, but the planet's rotation never becomes tidally locked to the star. There is a brief period when Ω_p is between n_m and n_p, tidally locked to neither the star nor the planet.

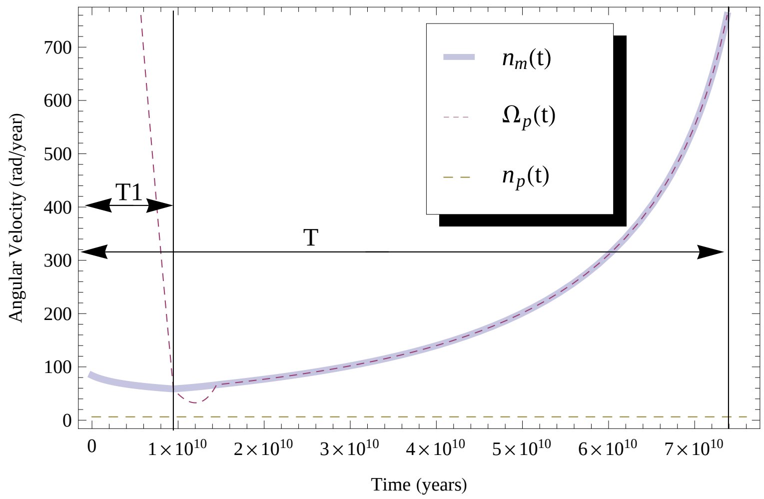

Figure 2.2: This graph is for Type I Case 2. We use the same conditions as in Fig.2.1 except the planet's mass is 1.2 $M_{\oplus}$. Note the "notch" in Ω_p just after $T1$; it is what differentiate Case2 from Case1.

At $t = T1$, the planet and the moon cannot reach the synchronized state because the torque due to the moon is smaller than that due to the star. The planet's spin keeps decreasing for a while. In this period, the moon's orbital motion and the planet's spin motions lose angular momentum, and the planet's orbital angular momentum increases because of the decreasing semimajor axis of the moon. As the moon's orbital angular velocity, $n_m(t)$, increases, so does the tidal torque due to the moon. Shortly thereafter, the torque due to the moon overcomes the torque due to the star. The planet's spin angular velocity, $\Omega_p(t)$, starts to increase. Then, the planet and the moon reach the synchronized state. Once synchronous, the moon's orbital motion loses angular momentum, and the planet's orbital and spin motions increase angular momentum. This case is distinct from Case 1 in that there is a period when the moon is migrating inward, but is not synchronized with the planet's spin.

- Type II(Fig.2.3)

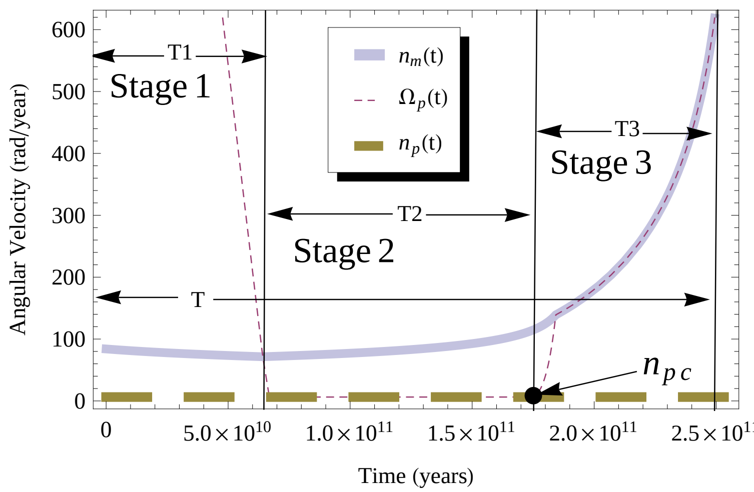

Figure 2.3: This graph is for Type II. We use the same conditions as in Fig.2.1 except the planet's mass is 4 $M_{\oplus}$. Here the planetary spin becomes synchronous with its orbit after $T1$ for a time $T2$. But spins up to become synchronous with the moon thereafter.

– For Type II, at $t = T1$, the tidal torque due to the star is greater than that due to the moon, which forces the planet's rotation to continue to slow down until it becomes synchronized to the star ($\Omega_p = n_p$).

In Stage 2, then, the planet and star remain in a synchronized state because the torque due to the moon does not overcome the torque due to the star. Until the planet and star reach a synchronized state, the moon's orbital motion and the planet's spin motion both lose angular momentum. At the star-planet synchronized state, only the moon's orbital motion loses angular momentum.

We can see the difference between Type II and Type III in Stage 3. Roughly speaking, if we can see Stage 3, then the system is Type II. If Stage 3 is so short that we cannot see it, the system is Type III. For Type II, the torque due to the star becomes smaller than the torque due to the moon as the moon spirals inward. The planet's rotation becomes tidally locked to the moon, after which only the moon's orbital motion loses angular momentum.

- Type III(Fig.2.4)

    – In this case, the tidal torque due to the star is always greater than that due to the moon. The amount of loss or gain in angular momentum for the moon's orbital motion is so small that we can treat the sum of the orbital angular momentum and the spin angular momentum of the planet as a constant. In essence, the planet's spin evolves as if the moon does not exist - this corresponds to the Barnes & O'Brien [2] condition.

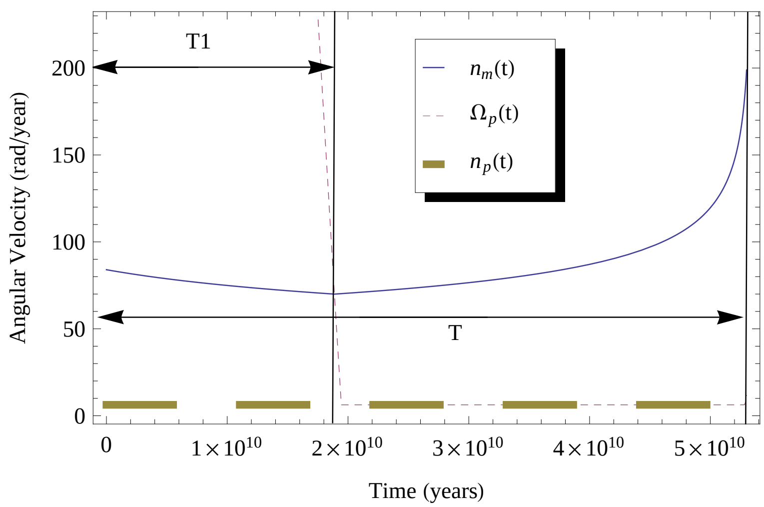

Figure 2.4: This graph represents Type III. We use the same conditions as in Fig.2.1 except that the moon's mass is set to 0.1 mass of the Moon. The planetary rotation becomes synchronous with the star, and never with its moon.

All n_ps in Fig.2.1-Fig.2.4 decrease only a very small amount. These changes are almost unnoticeable. However, the differences between the outcomes of tidal evolution in a two body system and a three body system come from these small changes. Because n_p decreases in our model, which does not include the star's tidal response to the planet, our results have a systematic error with respect to the lifetime of moons for close-in planets which experience orbital decay.

2.4 Analytical Lifetimes

In this section, we derive analytical formulae for the total moon lifetime for each of Type I, II, and III. The total moon lifetime means the time it takes for the moon to either hit the planet or escape. We will discuss the results here; The full derivations of the formulae are described in the Appendix.

2.4.1 Type I Solution

As discussed in section 3, Type I has two different cases. Because we can use the same formula to calculate the lifetime of the moon in each case though, we call both cases Type I. By creating a new function, $\widetilde{n}_m(t)$, which coincided with $n_m(t)$ after $T1$, we found the formula for the lifetime of the moons.

The formula for the lifetime of the moons for Type I, T, is:

$$T = \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}(GM_p)(GM_s)^{2/3} \left[\left(\frac{3^{3/4}GL_0-4\{(GM_m)^3(GM_p)^3\alpha R_p^2\}^{1/4}}{3^{3/4}(GM_p)(GM_s)^{2/3}}\right)^{13} - \left(\frac{1}{n_p(0)}\right)^{13/3}\right]. \tag{2.21}$$

After the synchronized state is broken at the very end of the moon's life, the moon has a spiral inward orbit (Fig 2.5). We do not explicitly include this period because it is very small compared to T. For Type II, we have the same situation.

Interestingly, a Type I star-planet-moon system experiences all three Counselman states. From $t = 0$ to $T1$, the orbital velocity of the moon, n_m, decreases. This indicates that the orbital semimajor axis of the moon increases. This is Counselman state (ii), except that the moon does not escape from the planet. At the synchronized state, the orbital velocity of the moon is equal to the spin angular velocity of the planet, i.e. $n_m = \Omega_p$. This corresponds to Counselman state (iii). After the synchronized state, the moon has a brief spiral inward orbit (Fig 2.5); this is Counselman state (i).

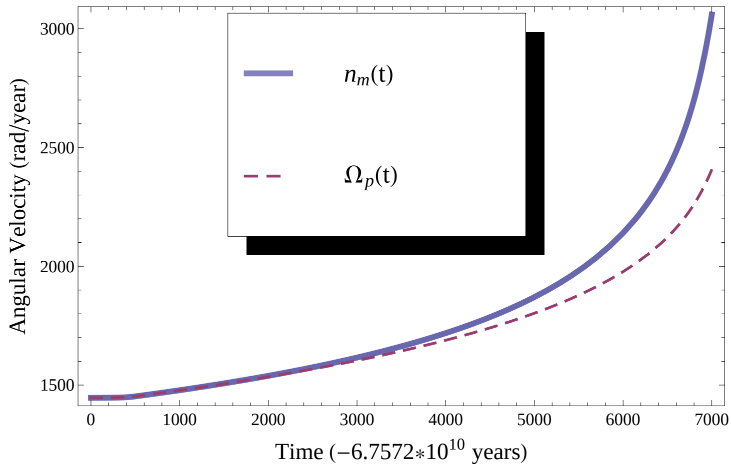

Figure 2.5: This graph is a magnification of the last part of Fig 2.1. At the very end, the synchronized state is over. The orbit of the moon decays inward much faster than before. Because the duration of this final death spiral is so short - 7000 years in a 67.5 Gyr evolution - we neglect it in our analytical formulations.

2.4.2 Type II Solution

For Type II, there are three stages (Fig.2.3). We calculated the time intervals for $T1$, $T2$, and $T3$, respectively. By adding them up, we found the lifetime of the moons for Type II.

The formula for the lifetime of the moons for Type II, T, is:

$$\begin{aligned} T &= T1 + T2 + T3 \\ &= 2T1 + \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\left[\frac{(GM_p)^{8/3}}{(GM_m)} n_m^{-13/3}(0) \right. \\ &\quad + \frac{\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13}}{3^{39/4}(GM_p)^{12}(GM_s)^8} \\ &\quad \left. - \frac{(GL_0)^{13}}{\left\{(GM_p)^{1/2}(GM_m)^{7/6} + (GM_p)(GM_s)^{2/3}\right\}^{12}}\right]. \end{aligned} \tag{2.22}$$

We could not find the analytical expression for $T1$. However, we can calculate $T1$

by solving the following systems of equations numerically for t

$$\begin{cases} n_m(t) = \left(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\,t + n_m^{-13/3}(0)\right)^{-3/13} \\ n_p(t) = \left(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\,t + n_p^{-13/3}(0)\right)^{-3/13} \\ \frac{M_m(GM_p)^{2/3}}{n_m^{1/3}(t)} + \alpha R_p^2 M_p n_m(t) \\ \qquad\qquad + \frac{M_p(GM_s)^{2/3}}{n_p^{1/3}(t)} = L_0. \end{cases} \tag{2.23}$$

After Stage 3, there is a brief Stage 4 wherein the moon makes its final death spiral into the planet's cloud tops. At Stage 4, $n_m(t) \neq \Omega_p(t)$ - actually, $n_m(t) > \Omega_p(t)$. Since $T4$ is very small compared to $T1$, $T2$, and $T3$, we do not explicitly include $T4$ in our calculation.

Type II has all three Counselman states, like Type I, plus one extra state. Stage 1 corresponds to Counselman state (ii) except that the moon does not escape from the planet. Stage 2 is the extra state. At this stage, the planet and the star are tidally locked. Because Counselman (1973)[13] considered a planet-satellite system, the planet could not be tidally locked with the star, hence Stage 2 has no corresponding Counselman state. Stage 3 corresponds to Counselman state (iii). The planet and the moon reach a synchronized state. At Stage 4, the moon has a spiral inward orbit. This is Counselman state (i).

2.4.3 Type III Solution

For Type III, we can calculate the lifetime of the moon using symmetry (Fig 2.6) as for Barens & O'Brien (2002) [2].

The formula for the lifetime of the moons for Type III, T, is:

$$T = 2T1 + \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\frac{(GM_p)^{8/3}}{GM_m}n_m^{-13/3}(0). \tag{2.24}$$

In general, the system is Type III if the moon is very small compared to the planet.

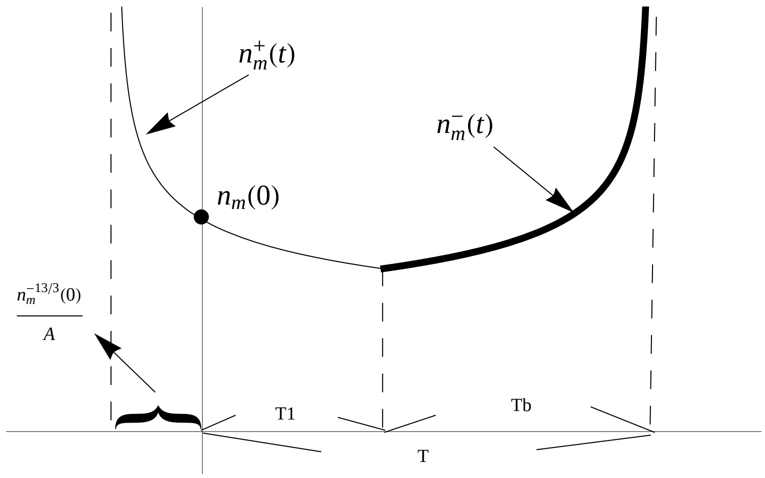

Figure 2.6: This is the graph of $n_m(t)$ in Type III. In this graph, $A = \frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}$.

If we set $(GM_m)^3(GM_p)^3\alpha R_p^2$ and $(GM_p)^{1/2}(GM_m)^{7/6}$ in equation (2.22) equal to zero, then we get equation (2.24).

Type III has one Counselman state and one extra state. The first part, from $t = 0$ to $T1$, is Counselman state (ii) except the moon does not escape from the planet. The second part, $t > T1$, is the extra state which is the same as Stage 2 in Type II.

2.4.4 Type IV Solution

So far, we assume that the orbit of the moon is always stable. In this section, we calculate when the orbit of the moon becomes unstable. This means

$$n_{crit}(t) > n_m(t) \tag{2.25}$$

at some t (Fig.2.7). In this case, we can use equation (2.13) with sgn($\Omega_p - n_m$)=1 and sgn($\Omega_p - n_p$)=1 because the planet is not tidally locked with either the star or the moon. To find the time when the orbit of the moon becomes unstable, we set

$n_{crit}(t) = n_m(t)$. Then we solve for t. The lifetime of the moon in this case is

$$T = \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\left(\frac{(GM_p)^{8/3}(GM_s)^{2/3}}{(GM_m)(GM_s)^{2/3}-(f^3/3)^{13/6}(GM_p)^{5/3}}\right) \times \left((f^3/3)^{13/6} n_p^{-13/3}(0) - n_m^{-13/3}(0)\right). \tag{2.26}$$

Type IV has just one Counselman state: Counselman's state (ii).

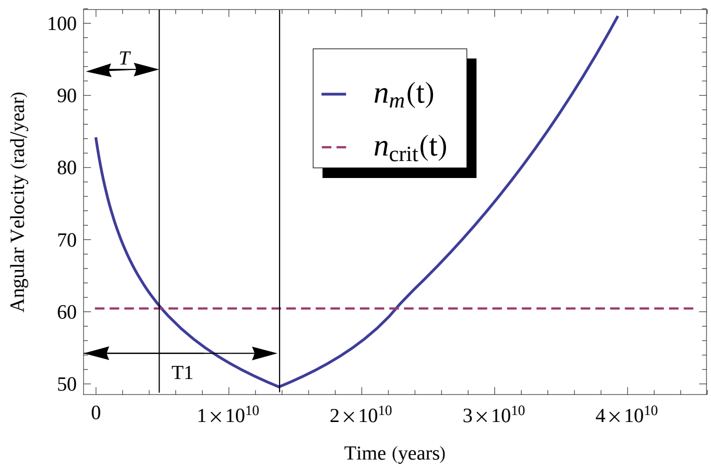

Figure 2.7: This is the graph of $n_m(t)$ and $n_{crit}(t)$ for Type IV. The dashed line is $n_{crit}(t)$ and the solid line is $n_m(t)$. At $t = T$, the moon is on the outermost stable orbit. After that, the orbit becomes unstable. Then, the planet loses the moon to interplanetary space. In this case, the lifetime of the moon is T. We use the data of the present Sun-Earth-Moon system except with an initial Earth spin angular velocity, $\Omega_p(0)$, of 1200π.

2.5 Determine the Type of the System

The expressions in section 4 can be used to calculate the ultimate lifetime of any star-planet-moon system, provided you know which type of system it is. In this section, we show how to determine a system's type.

2.5.1 Condition for Type I Case 1

The condition for Type I Case 1 is that the magnitude of the torque due to the moon is greater than the magnitude of the torque due to the star at $t = T1$ (Fig.2.1),

$$|\tau_{p-m}(T1)| \geq |\tau_{p-s}(T1)|. \tag{2.27}$$

This condition implies

$$\begin{aligned} T1 \leq \frac{2}{39} \frac{Q_p}{k_{2p} R_p^5} \frac{(GM_p)(GM_m)^{7/6}(GM_s)^{2/3}}{(GM_p)^{1/2}(GM_s)^{2/3} - (GM_m)^{7/6}} \\ \times \left\{ n_p^{-13/3}(0) - \left(\frac{GM_p}{GM_m} \right)^{13/6} n_m^{-13/3}(0) \right\}. \end{aligned} \tag{2.28}$$

This is the condition for the Type I Case 1. We can get $T1$ by solving equation (2.23) numerically.

If the system satisfies equation (2.28), then we can conclude that it is of Type I Case 1. If not, the system may be Type I Case 2, Type II, or Type III.

The sign on the right side of equation (2.28) depends on

$$\left\{ n_p^{-13/3}(0) - \left(\frac{GM_p}{GM_m} \right)^{13/6} n_m^{-13/3}(0) \right\} \tag{2.29}$$

because $(GM_p)^{1/2}(GM_s)^{2/3} - (GM_m)^{7/6} > (10^{11/6} - 1)(GM_m)^{7/6} > 0$ by our assumption 4. If equation (2.29) is negative, then the system cannot satisfy equation (2.28). Hence, the system is Type I Case 2, Type II, or Type III. The inequality $\left\{ n_p^{-13/3}(0) - \left(\frac{GM_p}{GM_m} \right)^{13/6} n_m^{-13/3}(0) \right\} \leq 0$ implies that $|\tau_{p-m}(0)| \leq |\tau_{p-s}(0)|$. This means that if the initial torque due to the star is greater than the initial torque due to the moon, the system cannot be Type I Case 1.

2.5.2 Condition for Type I Case 2

Assume $n_p(T1)$ and $n_m(T1)$ are known. Let t^+ be the time from $T1$, when the magnitudes of the two torques are equal (Fig.2.8).

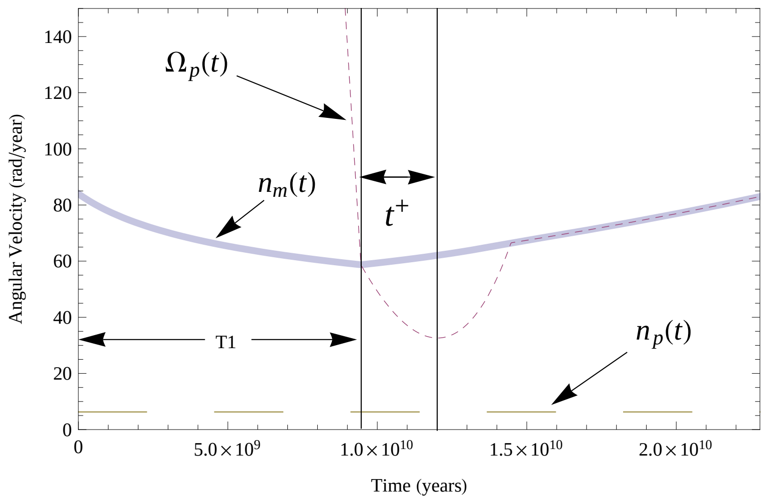

Figure 2.8: This graph is a magnification of Fig 2.2. The rotational rate of the planet, $\Omega_p(t)$, decreases until $t = T1 + t^+$ because the torque due to the star is greater than the torque due to the moon. At $t = T1 + t^+$, these two torques are equal. After that, the torque due to the moon exceeds the torque due to the star. The rotational rate of the planet, $\Omega_p(t)$, starts to increase. Then, the planet and the moon reach a synchronized state.

The condition for Type I Case 2 is that

$$\Omega_p(t^+) \geq n_p(t^+) \tag{2.30}$$

where t_* satisfies

$$|\tau_{p-m}(t^+)| = |\tau_{p-s}(t^+)|. \tag{2.31}$$

This condition implies that

$$a_1 b^{12} X^4 - G L_0 X^3 + \frac{a_2}{c} b^3 \leq 0, \tag{2.32}$$

where

$$
\begin{aligned}
c &= \tfrac{1}{\alpha R_p^2 (GM_p)} \\
a_1 &= (GM_p)^{1/2}(GM_m)^{7/78} \\
a_2 &= \tfrac{1}{(GM_m)^{7/26}} \\
b &= \left\{(GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right\}^{1/13} \\
X &= \left\{\left(\tfrac{GM_p}{GM_m}\right)^{13/6} n_m^{-13/3}(T1) + \tfrac{(GM_p)^{1/2}(GM_s)^{2/3}}{(GM_m)^{7/6}} n_p^{-13/3}(T1)\right\}^{1/13}.
\end{aligned}
$$

If the system is not Type I and satisfies equation (2.32), then it is Type I Case 2.

2.5.3 Conditions for Type II and III

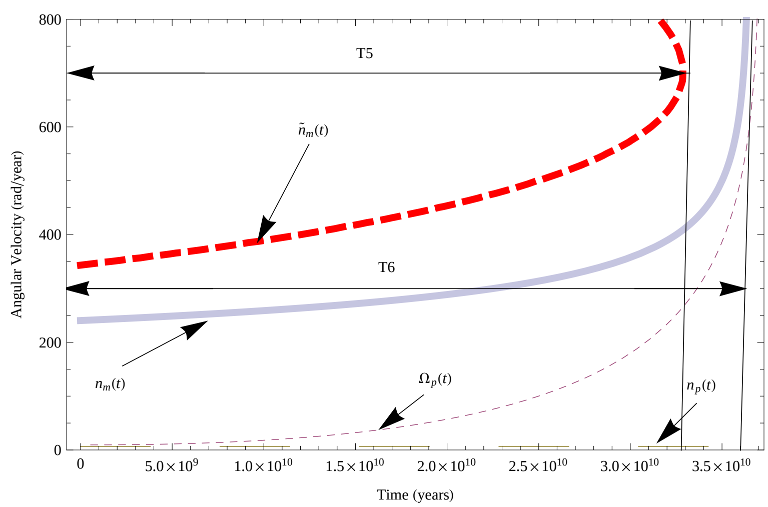

Figure 2.9a: This is a graph of Type III after the synchronized state of the star and planet end. In this graph, $t = 0$ means the time when the synchronized state of the star and planet ends. We used the present data of our Sun-Earth-Moon system except $M_p = 18M_\oplus$, $n_m(0) = 240$, $n_p(0) = \Omega_p(0) = 6.28$ for $n_m(t)$. For $\widetilde{n}_m(t)$, we used $n_p(0) = 6.28$, and $\widetilde{n}_m(0) = \Omega_p(0) = 343$.

As you can see in Fig.2.3, the planet and the moon reach a synchronized state at Stage 3 for Type II. In Fig.2.4, the synchronized state of the star and planet seems to continue indefinitely. The planet and star seem to be tidally locked until the end. Actually, it ends when the moon spirals into the planet, which occurs when $n_m(t)$ is

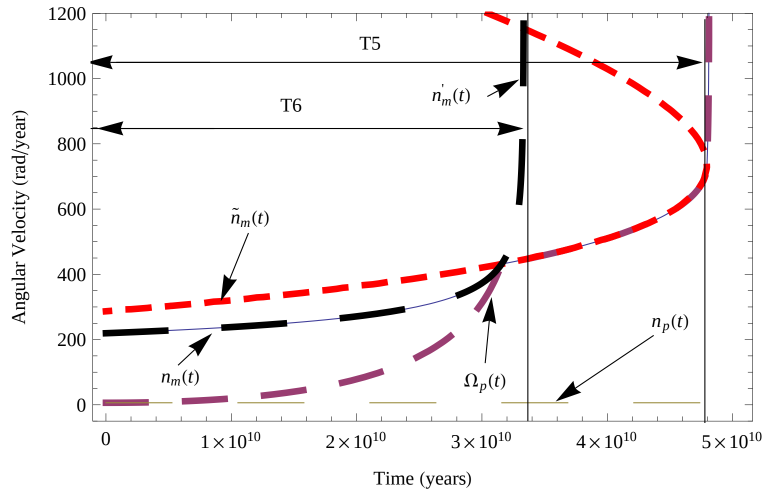

Figure 2.9b: This is a graph of Type II at Stage 3. In this graph, $t = 0$ means $t = T1 + T2$ in the original Type II graph. We used the present data of our Sun-Earth-Moon system except $M_p = 15M_\oplus$, $n_m(0) = 219$, $n_p(0) = \Omega_p(0) = 6.28$. For $\widetilde{n}_m(t)$, we used $n_p(0) = 6.28$, and $\widetilde{n}_m(0) = \Omega_p(0) = 286$.

large enough. In Fig.2.4, the total lifetime of the moon, T, is about 53.3 billion years. The synchronized state of the star and planet ends at about 52.7 billion years. In Type III, there is a stage that corresponds to Stage 3 for Type II. However, because that stage is short compared to the total lifetime, it is hard for us to see it.

Fig.2.9a is a graph of Type III after the synchronized state of the star and planet ends. The planet is so big that the spin angular velocity of the planet cannot increase fast enough to catch up to the orbital angular velocity of the moon. $\widetilde{n}_m(t)$ is the hypothetical situation in which the planet and the moon are tidally locked from the beginning. We introduced $\widetilde{n}_m(t)$ in 2.4.1.

Fig.2.9b is a graph of Type II at Stage 3. The planet is not big enough so that the spin angular velocity of the planet increases fast enough to catch up the orbital angular velocity of the planet. $n'_m(t)$ is the graph of

$$\left(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\,t + n_m^{-13/3}(T1+T2)\right)^{-3/13} \tag{2.33}$$

that is the same equation of $n_m(t)$ in Type III.

$T5$ is the maximum range of $\widetilde{n}_m(t)$ and $T6$ is the maximum range of $n_m(t)$ in Type II and $n'_m(t)$ in Type III.

From Fig.2.9a and Fig.2.9b, the condition for Type II is $T5 > T6$ and the condition for Type III is $T5 \leq T6$.

The condition for Type II, $T5 > T6$, implies that

$$\begin{aligned}
&\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13} \times \\
&\left((GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right)^{12} \\
&> 3^{39/4}(GM_p)^6(GM_s)^8(GL_0)^{13}.
\end{aligned} \tag{2.34}$$

Similarly, the condition for Type III, $T5 \leq T6$, implies that

$$\begin{aligned}
&\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13} \times \\
&\left((GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right)^{12} \\
&\leq 3^{39/4}(GM_p)^6(GM_s)^8(GL_0)^{13}.
\end{aligned} \tag{2.35}$$

2.5.4 Condition for Type IV

Looking at the graphs of Type I (Fig.2.1, Fig.2.2), Type II (Fig.2.3), and Type III (Fig.2.4), we know $n_m(t)$ has a minimum at $t = T1$. Hence, the condition for Type IV is

$$\begin{aligned}
T1 > T = {} & \tfrac{2}{39}\tfrac{Q_p}{k_{2p}R_p^5}\left(\tfrac{(GM_p)^{8/3}(GM_s)^{2/3}}{(GM_m)(GM_s)^{2/3}-(f^3/3)^{13/6}(GM_p)^{5/3}}\right) \\
& \times\left((f^3/3)^{13/6}n_p^{-13/3}(0) - n_m^{-13/3}(0)\right).
\end{aligned} \tag{2.36}$$

We can find $T1$ by solving the system of equations (2.23) numerically.

2.6 Applications

In this section, we check the formulae for the lifetime of the moon and the condition for the type of system first in two examples, and then showing an application of our results.

In the first two examples, we use the present data of our Sun-Earth-Moon system. We take k_{2p} and Q_p for Earth to be 0.299 and 12, respectively (Murray & Dermott [48], pg166). In the first example, we survey a range for the mass of the planet from 0.1 to 25 $M_\oplus$. In the second example, we explore moon masses from 0.01 to 2 mass of the Moon. For Type I, we can plot the graph of the lifetime of the moon easily because every parameter is constant (Eq.2.21). For Type II and III, we need to know the expression for $T1$ to plot the graph (Eq.2.22,2.24). In order to have $T1$, we must calculate $T1$ numerically. Then, by using an approximation method, such as best-fitting or interpolation, we can obtain the expression of $T1$.

The fractional error of our analytical formulae is always smaller than 10^{-3} when compared to numerical integration and usually much smaller. The fractional error is defined as the absolute value of the lifetime from simulation minus lifetime from formula divided by lifetime from simulation.

In the first example, Q_p=12 is not realistic for a high mass planet. However, we want to show just how the mass of the planet affects the lifetime of the moon. To get a more realistic result, we should change both k_{2p} and Q_p for Jovian planets. If we did that, we would expect that the lifetime of the moons would be much longer.

In both examples, the lifetime in most of these figures is longer than the typical main sequence lifetimes of solar-type stars. Over such a timescale additional processes, such as inflation of the stellar radius, and the resulting changes to tidal evolution become important.

2.6.1 First Example - Changing The Mass of The Planet

Fig.2.10 shows how the lifetime of a 1 mass of the Moon varies as the mass of the planet changes from 0.1 to 2 $M_{\oplus}$. Fig.2.11 is similar, but with the mass of the planet varying from 2 to 25 $M_{\oplus}$. The difference between the data from numerical simulations and the graph generated by the formula is very small. From the graph Fig.2.10, and Fig.2.11, every data point is on the curve. The transition from one type to another is smooth.

As you can see in Fig.2.10 and Fig.2.11, the lifetime of the moon increases as the planet mass increases. The increased longevity occurs because the effects of the lunar and stellar tides on the planetary spin evolution are reduced as we increase M_p, i.e. the planet is not braked as easily by each of the these effects as we go to more massive planets. The system therefore continues to evolve with $\Omega_p > n_m$ for longer, in which the tidal torque on the moon is positive, lengthening the overall evolution.

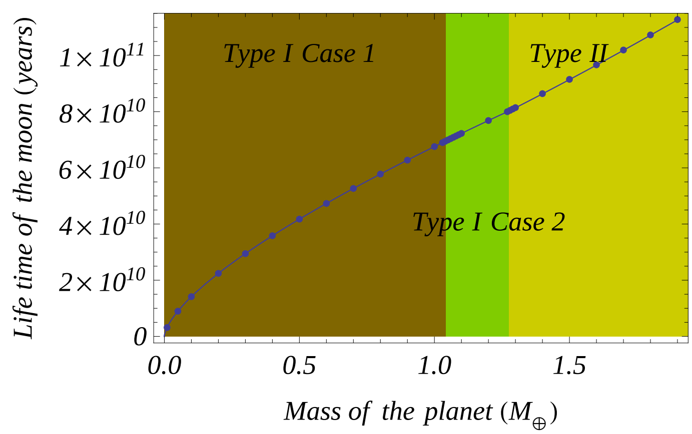

Figure 2.10: This graph shows the lifetime of a hypothetical system with Sun-Earth-Moon parameters with varying planetary mass from 0.1 to 2 $M_{\oplus}$. Each dot represents the result from numerical solutions, and the curve is generated by equation (2.21) or equation (2.22), depending on the type of system. The lifetime of the moon increases linearly as the mass of the planet increases in this region. The borders between Case 1 and Case 2, and Case 2 and Type II are $M_p = 1.04M_{\oplus}$ and $M_p = 1.27M_{\oplus}$, respectively. The border between different types depends on the initial conditions and Q_p.

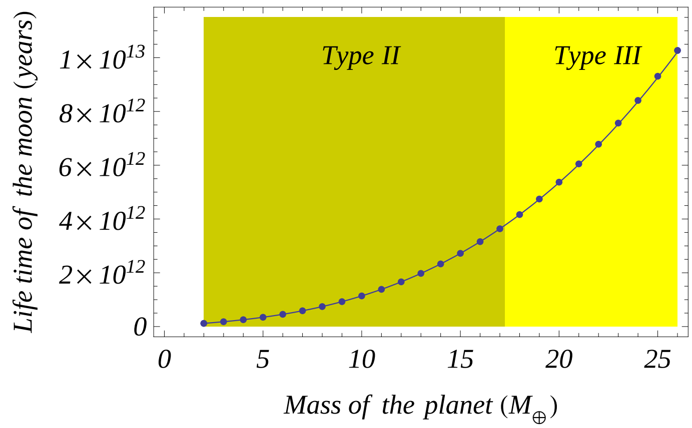

Figure 2.11: Here we show the lifetime of a hypothetical system with Sun-Earth-Moon parameters with varying planetary mass from 2 to 25 $M_{\oplus}$. Each dot represents the result from numerical solutions. The curve is generated by equation (2.22), and equation (2.24). In this region, the lifetime of the moon increases exponentially as the mass of the planet increases. The border between Type II and Type III is $M_p = 17.2M_{\oplus}$. The border between different types depends on the initial conditions and Q_p.

2.6.2 Second Example - Changing The Mass of the Moon

Fig.2.12 shows how a moon's lifetime would change as the moon's mass varies from 0.01 to 2 mass of the Moon. For low-mass moons, the greater the mass of the moon, the more quickly the moon evolves from equation (2.10). This is why the heavier moon has the shorter lifetime in Type III (Barnes & O'Brien (2002) [2]). Additionally, at Type I Stage 1, a heavy moon evolves faster than a light moon. Indeed, $T1$ decreases as the mass of the moon increases.

However, once the moon and the planet reach their synchronized state, the planet keeps the heavier moon for a longer time. When the planet and the moon become tidally locked, this planet-moon system behaves like one object. As the mass of the moon increases, the moment of inertia of the planet-moon system increases. Because the star saps angular momentum at a constant rate, the planet-moon system evolves more slowly when the system has the heavier moon.

Between Types III and I, the lifetime of the moon has a minimum value. When the mass of the moon is 0.307 mass of the Moon, it has a minimum lifetime of 4.24×10^{10} years.

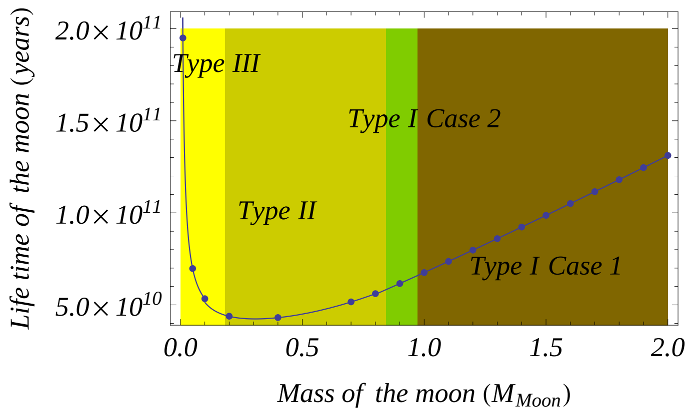

Figure 2.12: This graph shows the lifetime of a hypothetical system with Sun-Earth-Moon parameters with varying moon mass from 0.01 to 2 mass of the Moon. When the mass of the moon is small, the system is Type III. As the mass of the moon increases, the system becomes Type II, Type I Case 2, and then Type I Case 1. There is a minimum lifetime of 4.24×10^{10} years at 0.307 mass of the Moon. This minimum arises because the mass of the moon has different effects in Type I and III. In Type III, the heavier moon has a shorter lifetime due to faster tidal evolution. While in Type I, the heavier moon has the longer lifetime because the moon has grater orbital angular momentum. When these effects cancel each other out, there is a minimum value. The borders between the Type III and Type II, Type II and Case 2, and Case 2 and Case 1 are $M_m = 0.181$ mass of the Moon, $M_m = 0.842$ mass of the Moon, and $M_m = 0.972$ mass of the Moon, respectively. The border between different types depends on the initial conditions and Q_p.

We know that the tidal effect of the real Moon is important on the Earth. To show the utility of our approach, we calculate the lifetime of hypothetical moons with and without lunar tidal effect (Fig.2.13). For low masses of the moon there is no difference between these results and previous results of [2] because the effect of the lunar tides is small. For the high mass moons, the lifetime of the moon with the lunar tidal effect is significantly longer than that without lunar tidal effect.

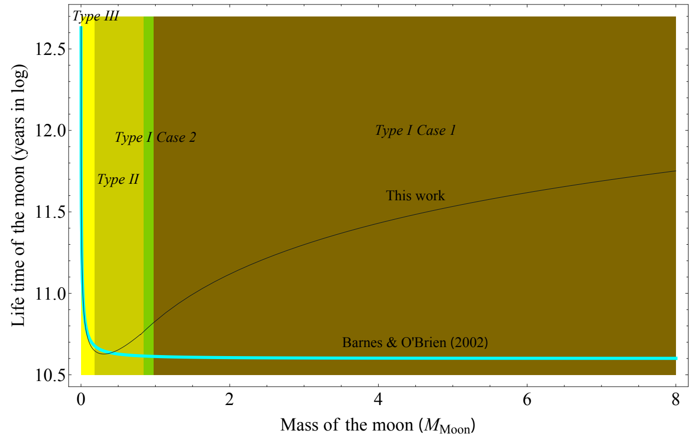

Figure 2.13: This graph shows that the lifetime of a hypothetical system using Sun-Earth-Moon present parameter with (this work) and without (Barnes & O'Brien (2002) [2]) lunar tides in log scale. The black thin line represents the lifetime of the moon including lunar tides. The light blue thick line is the lifetime of the moon not including lunar tides. In the Type III region, both results agree very well. However, in the Type I region, the necessity of incorporating lunar tides' effect on the planet's rotation, as we have introduced in this paper, becomes clear.

2.6.3 Third Example - Other Systems

Finally, we apply our results to extrasolar star-planet-moon systems where we expect that the first exomoons will be discovered. To see the big picture, we chose some typical combinations of the stars, planets, and moons (Fig.2.14). We choose 1.0 $M_{\bigodot}$ and 0.3 $M_{\bigodot}$ stars as the parent stars. For each parent star, we investigate 7 planet/moon systems. For rocky planets, we chose Earth/Moon, Earth/Mars, and 8-$M_{\bigoplus}$-super-earth-planet/Venus systems. For ice giant planets, we choose hypothetical Neptune/Earth system. For gas giant plants, we choose Saturn/Earth, Jupiter/Earth, and 10-M_{Jup}-planet/Earth systems. To see the trend of each type of the planets, we also separate the systems by Q_p (Fig.2.15). We use $Q_p = 100$, 10^4, and 10^5 for rocky, ice giant, and gas giant planets, respectively.

Fig.2.14 shows the moon stability lines for 1 to 10 Gyr applied to types of planet/moon systems. It is worth noting that a star of mass 0.3 $M_{\odot}$ has a main sequence lifetime much longer than that of the Sun. The lifetimes of the 0.3 $M_{\odot}$ and the Sun are the order of 100 Gyr and 10 Gyr, respectively. Like the 'ice line' with respect to planet formation, we define the 'moon stability line' as the location beyond which a moon is stable for the life of the stellar system. Therefore no such primordial moons can presently exist inside the moon stability line, though moons are possible outside it. Each point represents a moon stability line. In each case, we assumed the planet-moon synchronized state, i.e. $n_m(0) = \Omega_p(0)$ as the initial condition, and the moon almost reached the outer most stable radius, i.e. $n_m(T1) \approx n_{crit}(T1)$ and $n_m(T1) > n_{crit}(T1)$(Fig.2.16). Because the moon did not reach the outer most stable radius, after that the system continued to evolve back inwards towards the planet.

It is worth noting that the moon stability lines depend on Q. In Fig.2.14, we adopt Q=12 for rocky planets, Q=$10^4 \sim 10^5$ for ice and gas giants. Increasing Q increases the tidal evolutionary timescales.

Overall, the moon stability line moves inward for massive planets, for less massive parent stars, and for younger systems. In other words, moons are more stable when the planet/moon systems are further from the parent star, the planets are heavier, or the parent stars are lighter. This result can be explained by the size of Hill radius. The planet has a larger Hill radius for larger planet mass, smaller stellar mass, or larger planetary semi-major axis. In general, the moon has longer lifetime for the larger Hill radius of the planet.

Gas giant systems For gas giants, the moon stability line moves inward as the mass of the planet increases. In other words, moons are more stable when gas giants are heavier. Barnes & O'Brien (2002) [2] studied this type of system. They concluded that smaller moons are more stable around gas giant planets. With their high mass ratios, from these results we agree that smaller moons are more stable around heavier gas giants.

Ice giant systems In Fig.2.15 top right, we use $k_{2p} = 0.4$, $\alpha = 0.23$, $Q_p = 10^4$ which are the parameters of Neptune, and use the Earth as a moon. In this case, the moon stability line does not move much even the mass of the planet increases.

Rocky planet systems Compared to ice and gas giant planets, rocky planets are small and light. Therefore, the mass of the moon is one of the factors that moves the moon stability line when compared to the giants. Look at the Earth/Moon system and the Earth/Mars system in Fig.2.14. The only difference between these systems is the mass of the moon. As you can see, the moon stability line moves inward as the mass of the moon increases. Look at the Earth/Mars system and an 8-Earth-mass-planet/Venus system in Fig.2.14. The differences between these systems are the masses of the planet and moon. But the mass ratio between planet and moon is about 10 to 1, which is the maximum ratio of planet and moon for which our formulation is valid, in both cases. As you can see, the moon stability line moves inward as the mass of the planet and moon increase to a ratio of 10 to 1. Fig.2.15 top left shows the moon stability lines for rocky planets with our Moon mass moon. We use $k_{2p} = 0.299$, $\alpha = 1/3$, and $Q_p = 100$. The moon stability line moves inward as the mass of the planet increases except below the 1 $M_{\oplus}$.

The planet/moon system is preferred to be closer to the parent star to detect an extrasolar moon because we can make many observations of the planet and moon transiting . However, our result shows that the moon stability line moves inward for a younger system. If the planet/moon system to the parent star is close, we may find the planet but it's moon has already gone. If the planet is far from the parent star, it's moon may exist but the planet is hard to detect. We expect that the semi-major axis of the planet around which the first extramoon of a G-type star is 0.4-0.6 AU because the lifetime of the moon is more than 10 Gyr in most cases and we can observe the transiting planet two to four times in a year. For M-type star, we expect that the planet/moon system locate 0.2-0.4 AU because the lifetime of the moon is more than 10 Gyr in most cases and we can observe the transiting planet three to six times in a year.

2.7 Conclusion

We derive analytical expressions for determining the lifetime of hypothetical moons in star-planet-moon systems. Our solutions allow us to find the type of system and the lifetime of the moon without the need to numerically solve a system of differential equations. The flow chart in Fig.2.17 summarizes how to calculate the lifetime for any star-planet-moon systems. We first determine whether the moon remains within the planet's outermost stable orbit. If not, the moon is lost and the system is Type IV. If the moon remains in orbit, there are three possible outcomes: Types I, II, and III. In Type I, the planet is tidally locked with the moon. In Type II, the planet is tidally locked first with the star, and later with the moon. In Type III, the planet is not tidally locked with the moon. The type of system depends on characteristics of the star, planet, and moon (mass, radius, Love number Q_p, etc.) as well as the initial conditions of the planet and the moon.

Once we determine the system type, we can calculate the lifetime of the moon. To find the type of system and the lifetime of the moon, we need $T1$, which is the time when the spin angular velocity of the planet is equal to the angular velocity of the moon; See Fig.2.1, Fig.2.3 and Fig.2.4. We should use a numerical method to find $T1$.

Our results are extension of Ward & Reid (1973) [65] and Barnes & O'Brien (2002) [2]. At the range that they considered, our results agree to their results. Ward & Reid (1973) [65] considered Type III without critical mean motion. In this case, the planet will lose its moon only if the moon collides with the planet. Barnes & O'Brien (2002) [2] considered Type III with critical mean motion. In this case, the moon may either hit the planet or escape from it. In both cases, the planet and moon are asynchronous.

Barnes & O'Brien (2002) [2] concluded that the heavier the moon, the shorter the lifetime of the moon. Because they considered only systems of Type III, this result agree to our result (Fig.2.12). On the other hand, the heavier the moon, the longer the lifetime of the moon for Type I and II.

Our Moon stabilizes Earth obliquity - a key reason for the development of life on Earth (Ward & Browmlee 2000 [64]). Stable obliquity in its star's habitable zone may be necessary for a planet to support life. An extrasolar moon of sufficient mass could stabilize the obliquity of an Earth-size extrasolar planet. However even if a planet has a relatively large moon like the Earth does, the planetary obliquity may not be stable in some cases such as the moon is far from the planet, the planet is close to the star, there is a Jupiter-size-planet close enough to the planet, etc (Lissauer et al. 2012 [41]). On the other hand, Mars has relatively small satellites, and its obliquity changes chaotically, fluctuating on a 100,000-year timescale (Laskar & Robutel 1993 [39]). Having a relatively large moon is not enough in and of itself to provide a sufficient condition for an extrasolar planet to stabilize its obliquity meaning support life. Hence, our results give a condition needed to support life on a planet in the habitable zone.

Suppose we find a Jupiter-size planet in the habitable zone. This planet may have an Earth-sized moon. If the lifetime of that extrasolar moon is equal to or greater than the age of Earth, then the moon may support life. Hence, our results gives a condition needed for potentially habitable moons.

In the third example, we show the moon stability lines for 1 to 10 Gyr applied to types of planet/moon systems. We define the 'moon stability line' to be the location beyond which a moon is stable for the life of the stellar system. In general, the moon stability line moves inward for more massive planet, for a less massive parent star, and for younger systems. In other words, moons are more stable when the planet/moon systems are further from the parent star, the planets are heavier, or the parent stars are lighter. We expect that the semi-major axis of the planet for the first extramoon of a G-type star will be 0.4-0.6 AU and for an M-type star 0.2-0.4 AU.

This lays the ground work for the tidal evolution of a star-planet-moon system and makes it possible to classify star-planet-moon systems and providing useful estimates of the lifetime of a moon.

In some cases, we may not necessarily be able to accurately predict the long-term survival of the moon. The value of Q_p, the specific dissipation function of the planet, is assumed to be constant in time. However, Q_p is not known theoretically, and may depend on the planetary internal structure. For the sake of simplicity, we considered a star-planet-moon system with a single planet and a single moon. But we do not consider any interactions between the star and the moon. This deficiency may be addressed in future work. Gravitational perturbations caused by other planets or moons may be significant. For close-in planets, the stellar gravitational perturbations of the moon's orbit are important (Cassidy et al. 2009 [12]). In these situations, our method may not predict the lifetime of the moon accurately. Despite its shortcomings, our approach provides an important step toward understanding the tidal evolution and longevity of extrasolar moons, and will form both a basis for future theoretical investigations and direction for future searches to detect extrasolar moons.

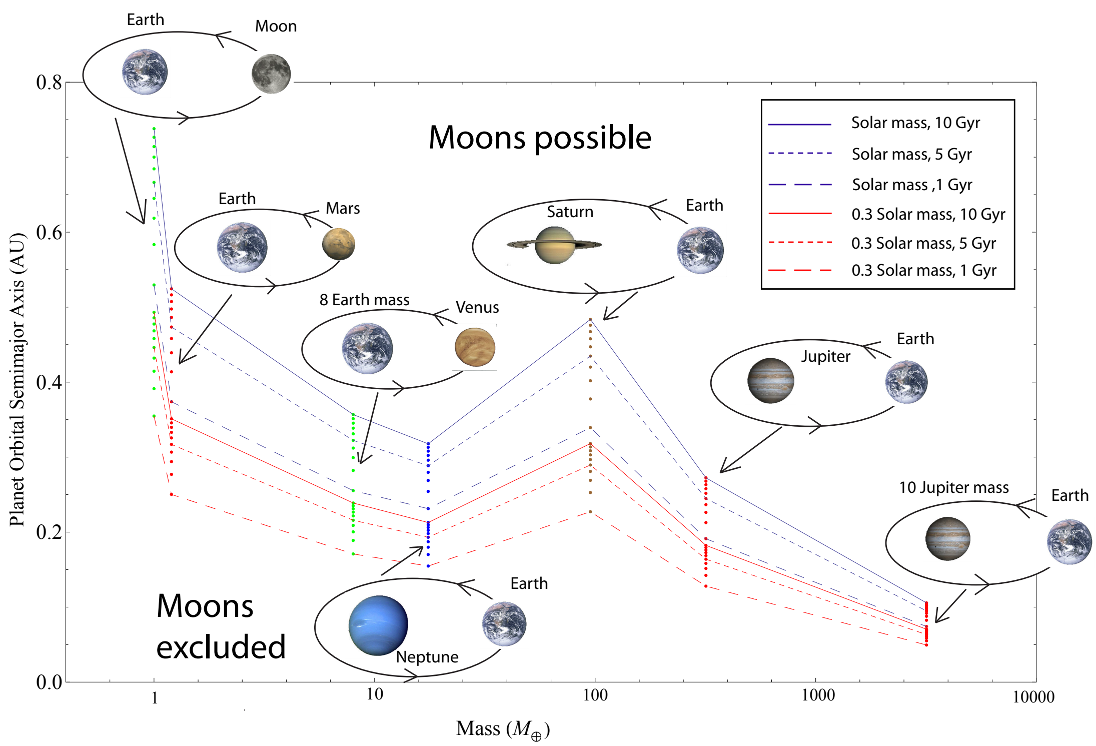

Figure 2.14: This graph shows the location required for the planet/moon system to have 1 Gyr, 2 Gyr, ..., up to 10 Gyr lunar lifetimes. The blue lines are for 1.0 $M_\odot$ stars. The red lines are for 0.3 $M_\odot$ stars. The pictures show which planet-moon pair the system has. The size of the pictures do not accurately depict the size of the planet and the moon. The top solid lines, both red and blue, are for 10 Gyr lunar lifetimes. The second and third dashed-lines are for 5 Gyr and 1 Gyr in both red and blue lines. We used the planet-moon synchronized state as the initial condition for each case, i.e $n_m(0) = \Omega_p(0)$. Each star-planet-moon system has ten dots. From the bottom to the top, the dots represent 1 Gyr, 2 Gyr, ..., up to 10 Gyr lunar lifetimes. For the Earth, we used a k_{2p} of 0.299, a Q of 12, and the moment inertia constant, α of 0.33. For an 8-Earth-mass-planet, k_{2p} is 0.299, Q is 12, α is 0.33, and the radius of the planet is 1.8 times the Earth radius. For Neptune, k_{2p} is 0.4, Q is 10^4, and α is 0.23. For Saturn, k_{2p} is 0.35, Q is 1.8×10^4, and α is 0.21. For Jupiter and the 10 Jupiter mass planet, k_{2p} is 0.5, Q is 10^5, and α is 0.254. We assume that the 10-Jupiter-mass-planet has the same radius as Jupiter. Note that the moon stability lines depend on Q. We adopt Q=12 for rocky planets, Q=$10^4 \sim 10^5$ for ice and gas giants. Increasing Q increases the tidal evolutionary timescales.

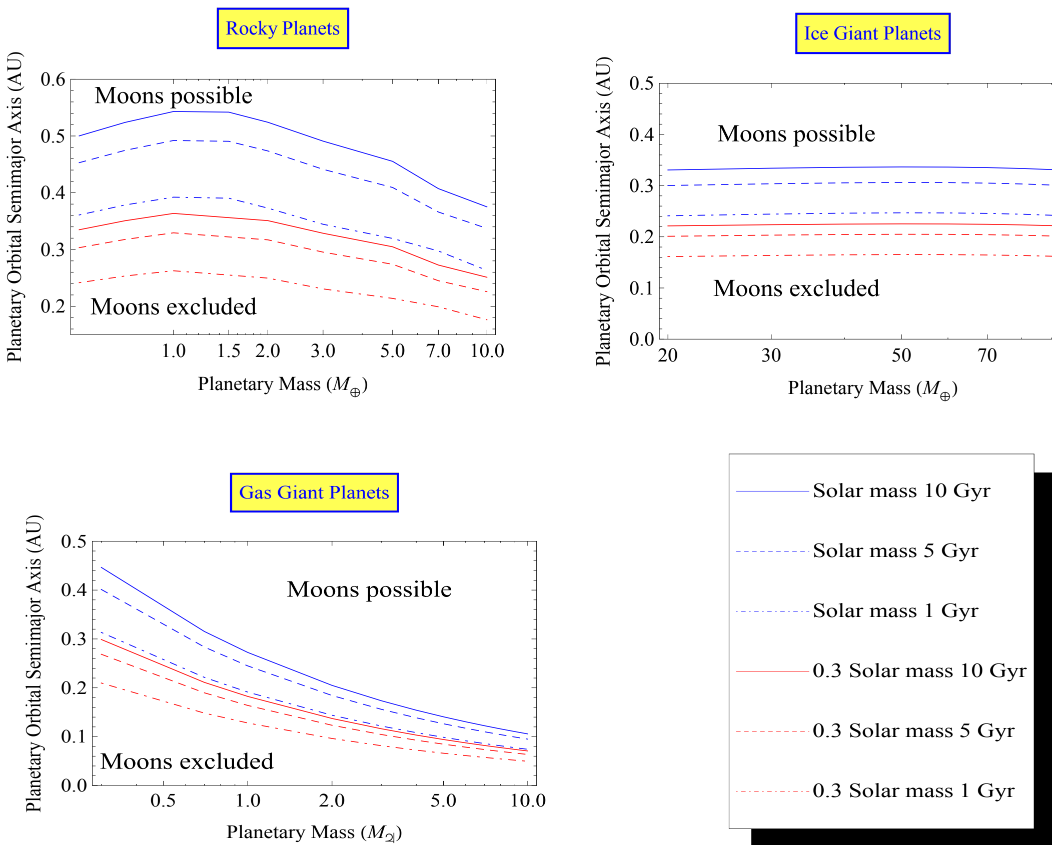

Figure 2.15: The top left graph shows the stability line for rocky planets with our Moon mass moon. We use k_{2p} of 0.299, α of 1/3 and Q_p of 100. We find the planetary radius from Fortney et al. (2007) [20] with the same composition rate of the Earth. The top right graph shows the stability line for ice giant planets with the Earth mass moon. We use k_{2p} of 0.4, α of 0.23 and Q_p of 10^4. We find the planetary radius from Fortney et al. (2007) [20] with the same composition rate of Neptune. The bottom left graph shows the stability line for gas giant planets with the Earth mass moon. We use k_{2p} of 0.5, α of 0.254 and Q_p of 10^5. We assume that the planetary radius is one Jupiter radius.

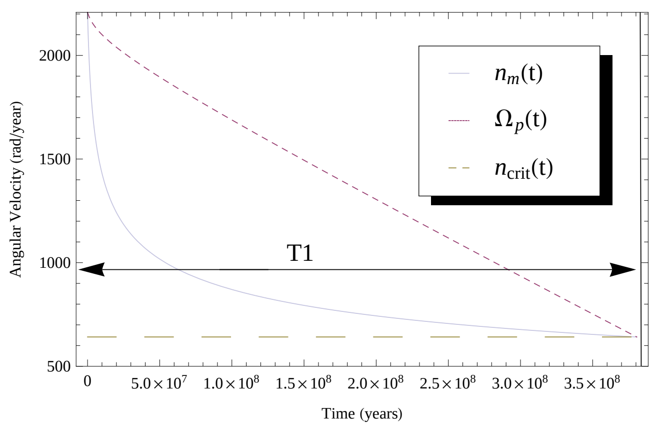

Figure 2.16: This is the first part of a typical graph for the third example. The initial condition is the planet-moon synchronized state, i.e $n_m(0) = \Omega_p(0)$. The moon almost reached the outer most stable radius, i.e.$n_m(T1) \approx n_{crit}(T1)$ and $n_m(T1) > n_{crit}(T1)$. The system continues to evolve back inwards towards the planet.

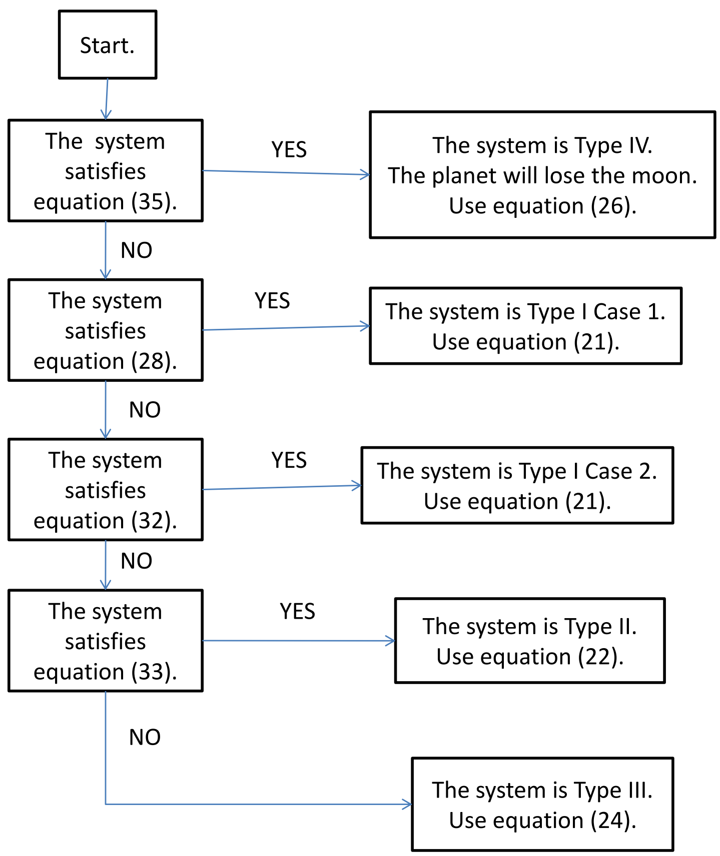

Figure 2.17: Flow-chart for calculating moon lifetimes in a star-planet-moon system. First, check the type of system. Then, calculate the lifetime of the moon.

Chapter 3

Longevity of Moons around Habitable Planets

(This chapter was submitted to: Sasaki, T., & Barnes, J. W., International Journal of Astrobiology, 2014)

ABSTRACT

We consider tidal decay lifetimes for moons orbiting habitable extrasolar planets using the constant Q approach for tidal evolution theory. Large moons stabilize planetary obliquity in some cases, and it has been suggested that large moons are necessary for the evolution of complex life. We find that the Moon in the Sun-Earth system must have had an initial orbital period of not slower than 20 hr/rev for the moon's lifetime to exceed a 5 Gyr lifetime. We assume that 5 Gyrs is long enough for life on planets to evolve complex life. We show that moons of habitable planets cannot survive for more than 5 Gyrs if the stellar mass is less than 0.55 and 0.42 $M_{\odot}$ for Q_p=10 and 100, respectively where Q_p is the planetary tidal dissipation quality factor. Kepler-62e and f are of particular interest because they are two actually known rocky planets in the habitable zone. Kepler-62e would need to be made of iron and have Q_p=100 for its hypothetical moon to live for longer than 5 Gyrs. A hypothetical moon of Kepler-62f, by contrast, may have a lifetime greater than 5-Gyrs under several scenarios, and particularly for Q_p=100.

3.1 Introduction

Detecting terrestrial planets in habitable zones is exciting because life may exist on such planets. To support life, a planet must be in the habitable zone and have a moderate climate. It may take a long time for life to reach complex, multicellular forms of life. For example, it took about 4 billion years for life on Earth to evolve from single-celled organisms to multicellular creatures such as plants, animals and fungi. A moderate long-term climate is crucial for life to reach complex life form. In this paper, we assume that 5 billion years is long enough for life on other planets to evolve from simple life to complex life.

Earth's obliquity, or axis tilt, is stabilized by the Moon [38]. Mars, on the other hand, has relatively small satellites and its obliquity changes chaotically, fluctuating on a 100,000-year timescale [39]. Hence, even if an Earth-sized planet has a moon, the planetary obliquity may fluctuate wildly if that moon is too small. Because planetary climate depends heavily on obliquity ([16], [68]), such a planet may not maintain a favorable climate for evolutionarily-relevant timescales. Therefore, orbital longevity of a moon may be important for a planet to have a moderate long-term climate. The prospects for habitable planets may hinge on moons [64]; but see also [41].

Tidal torque is important to the long-term orbital stability of extrasolar moons. Counselman (1973) [13] pointed out that in a planet-moon system with lunar[1] tides, there are three possible evolutionary states.

1. The semi-major axis of the moon's orbit tidally evolves inward until the moon hits the planet. Mars' moon Phobos is one such example.

2. The semi-major axis of the moon's orbit tidally evolves outward until the moon escapes from the planet. While no solar system examples exist for this case, such a result would happen to the Earth-Moon system if Earth's present rotation rate were doubled.

3. Lunar orbital and planetary spin angular velocities enter mutual resonance and are kept commensurate by tidal forces. This is the case for Charon, the dwarf planet Pluto's moon. Unlike the first two states, which are evolutionary, this state is static.

Ward & Reid (1973) [65] considered a star-planet-moon system with stellar tides

[1] In this paper, we use "lunar" as the adjective of any moons, not just the Moon.

and examined the impact of solar tides on planetary rotation in a limited star-planet-moon without the considering of the effects of lunar tides or maximum distance from the planet. Barnes & O'Brien (2002) [2] considered a similar tidal evolution senario, incorporating the maximum distance of the moon but not the lunar tide's effect on planetary rotation. According to their work, the moon may either hit the planet or escape from it. Sasaki et al. (2012) [57] studied the general tidal evolution of star-planet-moon systems, extending Barnes & O'Brien (2002) [2] to include the lunar effect on the planetary rotation. Sasaki et al. (2012) [57] also found the same two possible final states. Their result is applicable to a star-planet-moon system whose rocky planet orbiting at habitable distance. We are using lunar and stellar tides to refer to the tides raised on a planet by a moon and star, respectively.

In this paper, we investigate the conditions of star-planet-moon systems for moons to have lifetimes grater than 5 billion years. We are especially interested in rocky planets within habitable distances. In Section 2.1, we give a brief introduction of some important parameters such as the planetary tidal dissipation values and Love numbers. In Section 2.2, we introduce tidal evolution trajectories. We consider the Earth in Section 3. We then calculate the lifetime of moons with hypothetical moon/planet mass ratio and initial planetary rotational periods. We consider rocky planets with the same planet composition of the Earth Section 4. Because not all extrasolar rocky planets are Earth-like, we examine four typical planet compositions, which are 50% ice-50% rock, 100% rock, Earth-like (67% rock, 33% iron), and 100% iron in Section 4.3. In Section 4.4, we discuss "critical line" of moon-stability. In Section 5, we study the lifetimes of the hypothetical moons of two known rocky planets in their stars' habitable zone: Kepler-62e and f. Section 6 is discussion, and the conclusions are summarized in Section 7.

3.2 Method

We consider a star-planet-moon system and focus on the tidal effects on the planets due to the star and moon. We use standard tidal evolution theory with the constant Q approach (Goldreich & Soter 1966)[22]. In our model, tides on a planet are induced by both star and moon. Sasaki et al. (2012) [57] formulated tidal decay lifetimes for hypothetical moons orbiting extrasolar planets with both lunar and stellar tides. In this research, we apply the Sasaki et al. (2012) [57] method to $\sim 1.0\ M_{\odot}$ star systems with 0.1 - 10 $M_{\oplus}$ terrestrial planets at habitable distances. Because we use [57] method and apply the results, it is important to show the major assumptions of the model and our own assumptions:

1. The planet has 0° obliquity, the moon orbits in the planet's equatorial plane, and the planet and moon motions are prograde.

2. We neglect the orbital angular momentum of the moon about the star and the moon's rotational angular momentum.

3. The moon's orbit about the planet and the planet's orbit about the star are circular.

4. The star's spin angular momentum is not considered, nor are the planet's tides on the star or the star's tides on the moon.

5. The specific dissipation function of the planet, Q_p, is independent of the tidal forcing frequency and does not change as a function of time.

6. The systems start in a planet-moon synchronized state, i.e. the planetary angular spin velocity is equal to the moon's orbital angular velocity. This initial state is unstable, and the moon's orbit evolve rapidly outward thereafter.

Note that Sasaki et al.(2012) [57] does not use the assumption (6). That assumption is added for the applications in this paper. These assumptions simplify the calculation allowing us to apply them generally. They also reflect our goal of constraining the existence of moons because non-zero obliquity and eccentricities would only shorten the moons' lifetimes.

Regarding assumption 6, if a planet-moon system does not start with the synchronized state, we can always find the synchronized state by integrating the equations of the planetary angular spin velocity and the moon's orbital angular velocity backwards in time. A planet-moon system evolves quickly at first if it starts with the synchronized state. Hence, the error by assuming the synchronized state as initial condition is small. With these assumptions, the upper bound of moons' lifetimes can be estimated.

3.2.1 Parameters

Habitable distance is a controversial concept in planetary science. Even for the Sun, there are several estimations. Dole (1964) [17] predicted that the habitable distance is from 0.725 AU to 1.24 AU. Hart (1979) [25] concluded that the habitable distance is from 0.95 AU to 1.01 AU. Kopparapu et al.(2013) [35] estimated the habitable distance is from 0.99 AU to 1.7 AU. Calculating the habitable distance is difficult process even if restricted to the classical circumstellar habitable zone (CHZ), which is based on sustainability of liquid water on the surface. The difference between the Sun and lower mass star is not only the radiant energy but also wavelength. Wavelength is important because it is closely related to planetary albedo. Ice on the surface is very reflective in the visible light from the Sun-type stars, but its albedo is low in infrared region, which is the peak emission from low-mass stars ([31], [58]). In this study, we take the habitable distance to be the distance at which the radiant energy of the center star that the planet receives is the same as that of the Earth. At least at this distance, we know of at least one case in which a planet retains liquid water on its surface.

We use the following equation to find the habitable distance:

$$d(AU) = \sqrt{\frac{L_*}{L_\odot}} \tag{3.1}$$

where L_* is the stellar luminosity. In lieu of a complex suit of stellar models, adopt the rough approximation [24]:

$$\frac{L_*}{L_\odot} = \left(\frac{M_*}{M_\odot}\right)^{3.5} \tag{3.2}$$

to estimate the luminosity as a function of stellar mass. Fig.3.1 shows the habitable distance as a function of stellar mass.

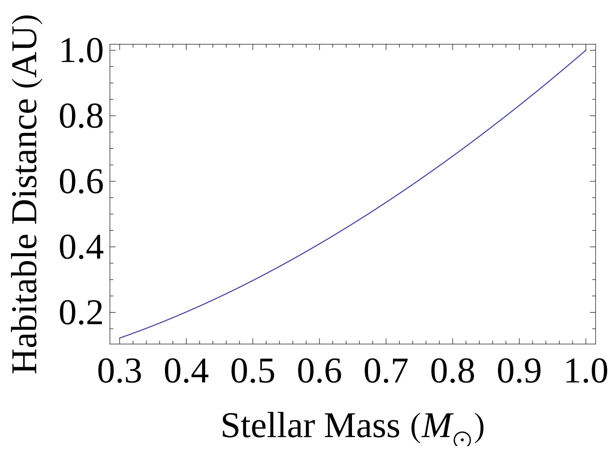

Figure 3.1: This graph shows the habitable distance calculated from Equations (3.1) and (3.2). The habitable distance moves outward almost linearly for higher mass stars.

Planetary radii, the planetary tidal dissipation values, and the Love numbers are important input values for the tidal theory. Planetary radii depend on the planet's composition [20]. The three main ingredients of rocky planets are ice, rock, and iron. We consider four planetary compositions to be 50% ice-50% rock, 100% rock, Earth-like (67% rock, 33% iron), and 100% iron. 100%-iron-planets may not commonly exist in the universe. However, we consider these types of planets as end-member cases because, for a given planetary mass, a 100% iron planet would have the smallest radius.

The planetary tidal dissipation value Q_p was introduced in Goldreich & Soter (1966) [22]. Its definition comes from the analogy with force damping oscillators and evaluates the ratio between the maximum energy stored in during the cycle and the energy dissipated over one cycle by friction. A small value of Q_p means large energy dissipation and vice versa. Estimating Q_p is not an easy task because it depends on planetary structure such as composition, equation of the state, properties of material, and so on. The exact nature of a planet's tidal response is still under investigation([28], [21], [49], and [50]). For the rocky planets, the values of Q_p range between 10 and 500 [22].

The Love number, k_2, characterizes the overall elastic response of the planet to the tides and depends on the mass and composition of the planet. Earth's k_2 value is 0.299, for example. In Appendix, we show the k_2 values in this study. As we mentioned in introduction, a small moon cannot stabilize planetary obliquity. To stabilize planetary obliquity, the moon must be large enough. We estimate the minimum lunar mass to stabilize planetary obliquity, that is

$$\frac{M_m}{M_p} \gtrsim \frac{\beta^3}{3}, \tag{3.3}$$

where β is the distance of a moon in terms of the Hill's radius, and M_m and M_p are the masses of moon and planet, respectively. This equation indicates that if we know the distance of a moon in terms of the Hill's radius, we can estimate the minimum lunar mass required to stabilize a planet's obliquity. The derivation of equation (3.3) is in Appendix.

3.2.2 Tidal Evolution Trajectories

In order to calculate the lifetime of the moon, we first determine the type of the system. There are four types of star-planet-moon systems based on the trajectories of the planets and moons: three "colliding" (Type I, II, and III) and one "escaping" (Type IV) [57]. The colliding type is defined by the radius of moon's orbit being continuously less than 0.36 Hill's radius all the time, with the moon hitting the planet in the end. The escaping type requires the radius of the moon's orbit be above this ratio. We think that a moon can maintain stable circular orbit inside of 0.36 Hill's radius because the perturbation from the Sun is small within this region. The planet also has difficulty holding on to the moon if the moon orbits outside of 0.36 Hill radii. While Barnes & O'Brien (2002) [2] suggests 0.36 for critical ratio, Domingos et al.(2006) [18] suggests 0.49. We use 0.36 for critical ratio because it is the most conservative estimate for the moon to remain bound.

Here is the summary of the types of outcomes defined by Sasaki et al.(2012)[57]. If the tidal torque on the planet from the moon is always greater than that from star, then the star-planet-moon system will be Type I. Because the tidal torque on the planet from the moon is greater than that from the star, the planet and moon do not leave the synchronized state once they reach this state. When the moon is larger, the system tends to be Type I. Our Sun-Earth-Moon system is Type I.

On the other hand, if the tidal torque on the planet from the moon is always smaller than that of the star, then the system will be Type III. Because the tidal torque on the planet from the moon is smaller than that from the star, the planet and star will reach synchronized state. When the moon is smaller, the system tends to be Type III.

Type II is between Type I and Type III. First, the planet and star reaches synchronized state, and then the planet and moon reaches synchronized state. If the moon migrate outward more than 0.36 Hill's radius, then the system will be Type

IV. The planet looses the moon interstellar space.

3.3 Sun/Earth System Q_p=12

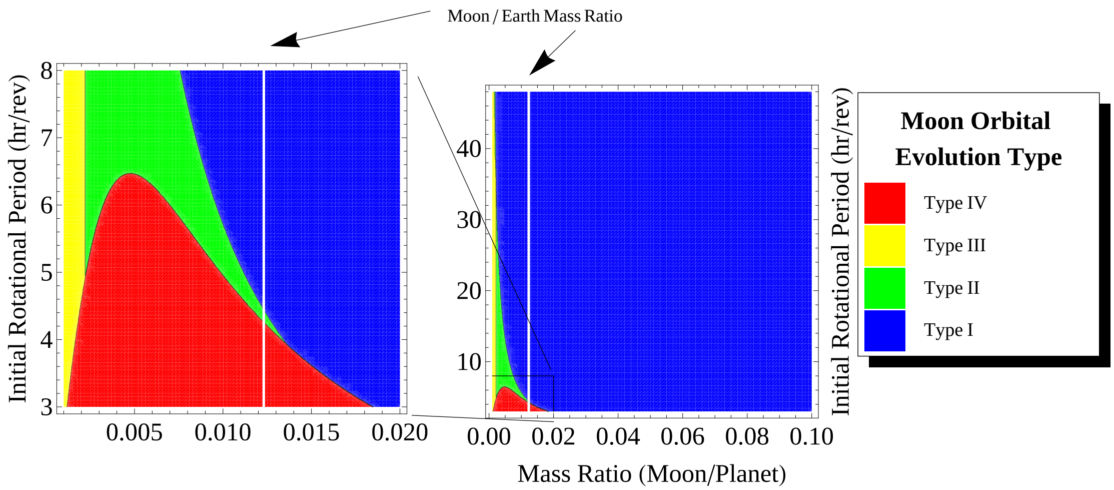

Figure 3.2: This graph shows the moon orbital evolution type of Sun-Earth system according to types defined by [57]. Type IV is an unstable orbit where Earth loses the moon. Type I, II, and III are stable orbits, which mean that Earth keeps the moon. The white vertical line represents the mass ratio of the real Moon and Earth. For the cases, our Sun-Earth system is Type I. Note that the orbital evolution types shown here are not function of Q_p.

In this section, we apply the method that Sasaki et al.(2012) [57] introduced to the Sun/Earth System. Fig.3.2 shows the moon orbital evolution type of Sun-Earth system as a function of moon mass and initial planetary rotation. The white vertical line represents the mass ratio of real Moon and Earth.

The giant impact hypothesis is currently the favoured hypothesis for the origin of the Moon [10]. While this hypothesis explains the current angular momentum of the Earth-Moon system, the Moon's small iron core, and the compositional similarity between the Moon and Earth [60], it does not explain how the oxygen isotropic composition of the Moon could be indistinguishable from that of the Earth [67]. Pahlevan & Stevenson (2007)[51], Canup (2012)[9], and Ćuk & Stewart (2012) [14]

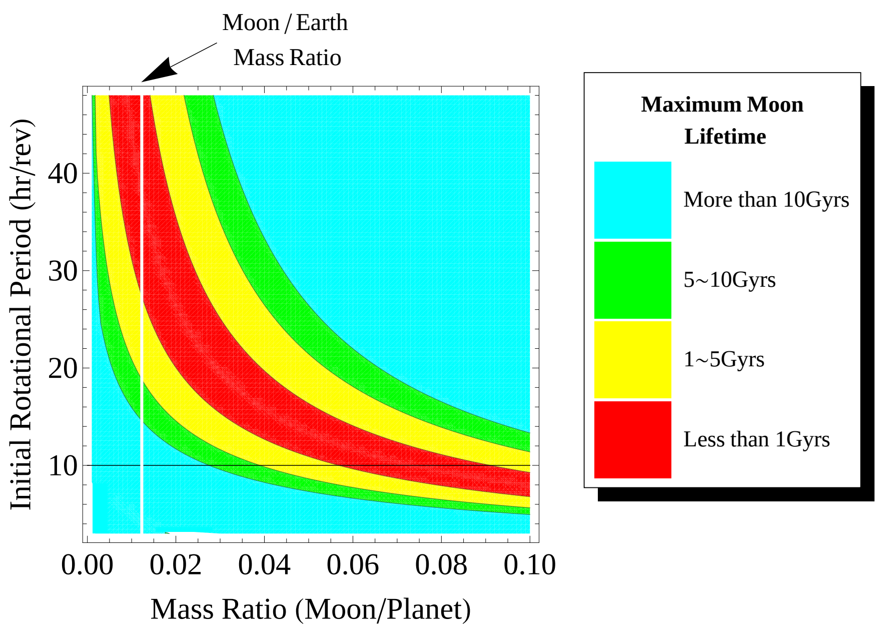

Figure 3.3: This graph shows the lifetime of hypothetical moons in the Sun-Earth system. The white vertical line represents the mass ratio of the actual Moon and Earth. The black horizontal line is 10 hr/rev. The real Earth-Moon situation is on the white line and may be below the black line. Our result suggests that the lifetime of our Moon is more than 10 Gyrs.

suggested different models to solve this problem. It is the beyond the scope of this paper to specifically model the formation of moons. The giant impact scenario might be the most common way for a rocky planet to have a moon. By this hypothesis, Earth's initial angular spin velocity would have been from 5 to 8 hr/rev and the initial Earth-Moon distance is ∼20000 km, 7.8 hr/rev. The Earth-Moon system thus may start near a planet-moon synchronized state, but an unstable one from which it evolves rapidly.

Our calculations suggest that the Sun-Earth-Moon system would be Type I if both Earth's initial spin velocity and Moon's initial orbital angular velocity are from 5 to 8 hr/rev. This means that the torque on the Earth from the Moon is greater than that from the Sun. Earth's spin velocity slows down and the Moon is spiraling outward until the Moon reaches synchronous distance (we are in this stage now). Once the Moon reaches Earth's synchronous radius, the Moon's orbital angular velocity will continue to be equal to Earth's spin angular velocity. However, because solar tides continue to rob angular momentum from the system, Earth's spin angular velocity and Moon's orbital angular velocity will both increase until the Moon hits the Earth. When moons are low-mass and planets have short rotational periods, systems tend to be Type IV (bottom left corner, red). It is hard for the planet to keep a light and fast-moving moon, which would spiral away until it is lost to interplanetary space.

If the Moon were less massive, our Sun-Earth-Moon system would be Type II. The fate of the hypothetical Sun-Earth-Moon system would be different. Earth's spin velocity slows down and the Moon is spiraling outward until the Moon reaches synchronous distance like Type I. Because the Moon is less massive, the tidal torque due to the Moon is not large enough to keep the planet-moon synchronous state. Earth's spin velocity keeps slowing down until it equals Earth's angular orbital velocity. In other words, Earth's one day becomes longer and longer until Earth's one day equals Earth's one year. The system is in the planet-star synchronous state. Meanwhile, the Moon is spiraling inward. Because the Moon is spiraling inward, the tidal torque due to the Moon becomes larger and larger. When the Moon is sufficiently close, the tidal torque due to the Moon overcomes that due to the Sun. At this point, the planet-star synchronous state ends. Earth's spin velocity starts increasing, which means that Earth's one day becomes shorter and shorter. Then, the Earth and the Moon reaches the planet-moon synchronous state, which means that the Moon stays in one position in the sky. When the system reaches the planet-moon synchronous state, Earth's spin velocity increases from solar tides and the Moon spirals inward until the Moon hits Earth.

Fig.3.3 shows the lifetime for moons of Earth with differing initial planetary rotation and moon mass: less than 1 Gyr (red), 1-5 Gyrs (yellow), 5-10 Gyrs (green) and more than 10 Gyrs (blue). Each graph of the lifetime of the moon like Fig.3.3 has its own graph of the type of the system like Fig.3.2. However, we only show Sun-Earth case, Fig.3.2, here because the basic features are the same in all cases.

For a fixed small mass ratio, the longer the initial rotational period is, the shorter the lifetime of the moon. The lifetime of the moon depends on the total initial angular momentum of the planet-moon system. When the system has smaller total angular momentum, the lifetime of the moon is shorter. For a fixed small mass ratio, the system has smaller total angular momentum when the initial rotational period is longer. Consider the Earth-Moon case as a example. The white vertical line represents the mass ratio of Moon and Earth. Our result indicates that for initial rotational periods up to 14 hr/rev, the lifetime of the moon is more than 10 Gyrs, which agrees with the predictions of the giant impact hypothesis (5 to 8 hr/rev). On the other hand, the lifetime of the moon is shorter than the age of the Earth if the initial rotational period is 20 hr/rev or slower. This indicates that if the initial rotational period had been 20 hr/rev or slower, the moon would have already hit the Earth, a thankfully unphysical result.

For a fixed fast initial rotational period, say 8 hr/rev, the relationship between the mass ratio and the lifetime is monotonic. The bigger the mass of the moon, the shorter the lifetime. At 8 hr/rev, the lifetime of the moon is more than 10 Gyrs when the moon-planet mass ratio is up to 0.04. The lifetimes are 5 and 1 Gyrs when the mass ratio is 0.055 and 0.08, respectively. The lifetime is less than 1 Gyrs when the mass ratio is more than 0.08.

For a slower rotational period, say 20 hr/rev, the relationship between the mass ratio and the lifetime is not monotonic. There are two ways for the moon to have more than 1 Gyrs lifetime. In this specific case, when the moon-planet mass ratio is either up to 0.02 or more than 0.04, the moon can survive longer than 1Gyrs.

3.4 Generalized Habitable Planets

We now extend the investigation of the previous section to the lifetimes of moons around extrasolar planets by considering $\sim 1.0\ M_{\odot}$ star and 0.1 - 10 $M_{\oplus}$ planet systems with Q_p=10 and 100. We consider planets made of 50% ice-50% rock, 100% rock, Earth-like (67% rock, 33% iron), and 100% iron. We show the results of 0.4 - 1.0 $M_{\odot}$ stars because we had the same results when stars are less than 0.4 $M_{\odot}$.

3.4.1 Earth-like planets with high dissipation

Earth-like planets at a habitable distance might have a similar environment to the Earth. By the definition of the planetary tidal dissipation value, $Q_p = 10$ indicates that there exists a mechanism that dissipates large amounts of tidal energy each cycle. On the Earth it is well-known that tidal dissipation occurs mainly in the oceans ([47], [19], and [53]). Tidal friction takes place mainly in the hydrosphere, and in particular in shallow seas, which is about less than 100m deep on continental shelf [36]. Tidal dissipation was significantly lower over the past three million years on average [42]. This result may be explained by a reduced in a global tidal friction during periods of glacio-eustatic sea level lowering [42]. For the Earth, tidal sloshing in shallow seas may be the mechanism that dissipates large amount of energy. Because it is hard to estimate Q_p from the planetary structure directly, $Q_p = 10$ dose not necessarily mean that a planet has shallow seas. However, we do not know other mechanism that dissipates large energy besides shallow seas. Hence, $Q_p = 10$ indicates that a planet may have shallow seas. Fig.3.4 shows the lifetimes of the moons whose planets have the same compositions of the Earth and orbit at the habitable distance from 0.4 - 1.0 $M_{\odot}$ stars. For 0.4 and 0.6 $M_{\odot}$ stars, moons cannot orbit around their planets for more than 5 Gyrs in any situation. These planets' Hill spheres are too small.

For 0.8 and 1.0 $M_{\odot}$ stars, moons can survive more than 5 Gyrs if the conditions are appropriate. One condition for moons to survive more than 5 Gyrs is that the moon/planet mass ratio is greater than 0.09 and the initial rotational period is 30hr/rev or slower. If we restrict ourselves to relatively fast initial planetary rotational rates (as might result from giant-impact origins for the moon) (below the black line), the results are dramatic: if the stellar mass is 0.8 $M_{\odot}$, it is almost impossible for 1 and 10 $M_{\oplus}$ planet to have a moon with a lifetime more than 5 Gyrs. If the Earth-Moon system were born at the habitable distance around a 0.8 $M_{\odot}$ star, then the moon would have already hit the Earth or been lost to interplanetary space. For 1.0 $M_{\odot}$ stellar mass, small mass planets easily have moons whose lifetimes are longer than 5 Gyrs. On the other hand, large mass planets have moons whose lifetimes are longer than 5 Gyrs if their initial rotational periods are sufficiently fast and moon/planet mass ratios are sufficiently small.

3.4.2 Earth-like planets with low dissipation

In this section, we examine Earth-like planets again. But, this time, we use Q_p=100 as might be fit a planet with no ocean or deep oceans. As we mentioned earlier, calculating Q_p is not easy. Mars has the Q_p value of 86 ([48], [4]) and tidal dissipation

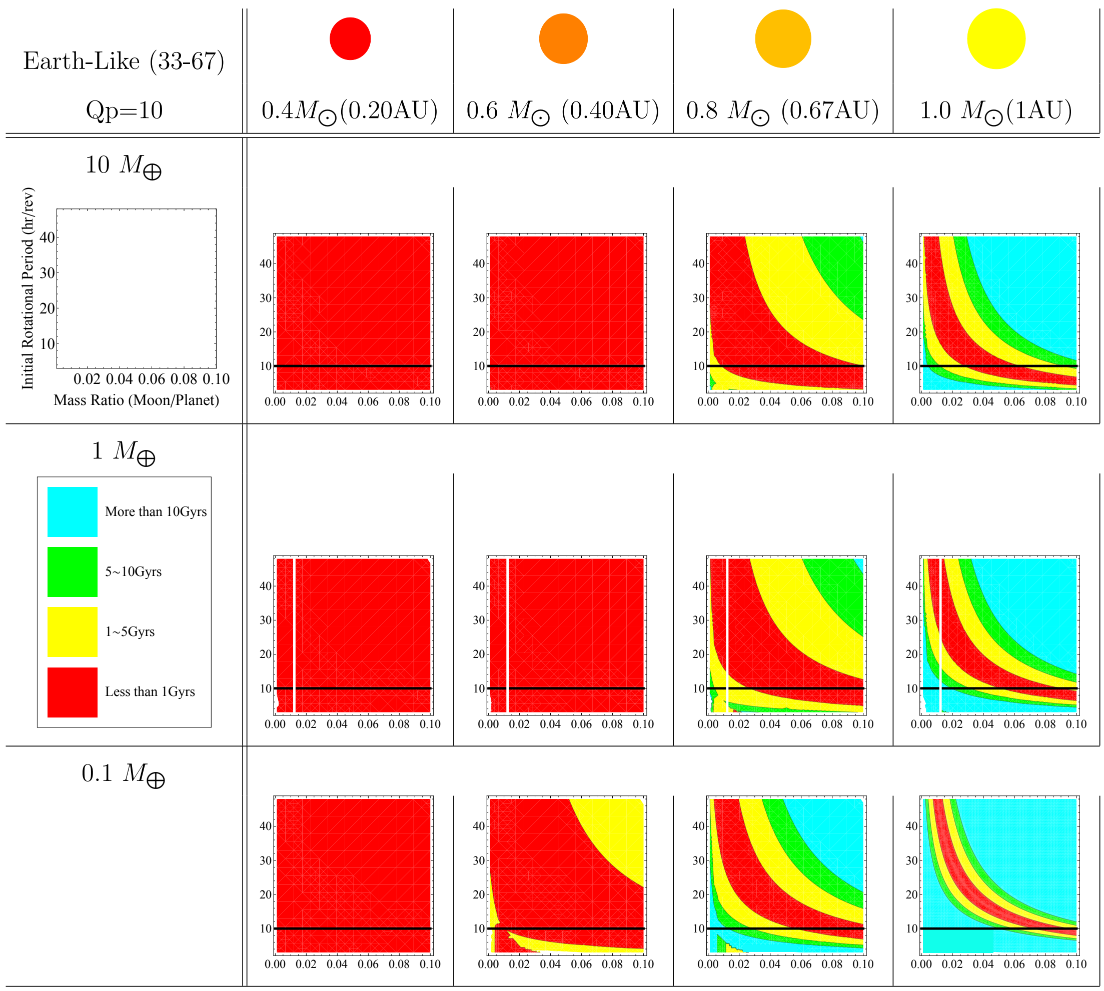

Figure 3.4: This figure shows our calculated lifetimes for moons whose planets have the same compositions of the Earth with high tidal dissipation (Q_p=10) and orbit at the habitable distance for 0.4−1.0 $M_\odot$ stars. The numbers in the parenthesis are the habitable distance for each star. The circles represent the color and relative size of the stars. The white vertical line represents the mass ratio of the actual Moon and Earth. The black horizontal line is 10 hr/rev. For 0.4 and 0.6 $M_\odot$ stars, moons cannot orbit around their planets for more than 5 Gyrs in any situation.

is driven by viscous dissipation within the bulk of the planetary interior [4]. Even though Mars has the Q_p value of 86, we cannot conclude that a planet whose Q_p is about 100 has the same dissipation mechanism of Mars. However, the dissipation mechanism of Mars is one possibility that a planet has the Q_p value of about 100. Also, an ocean planet may have the Q_p value of about 100 [55]. An ocean planet is a type of planet whose surface is completely covered by one to hundreds of kilometers of water.

Fig.3.5 shows the lifetimes of the moons whose planets have the same composition of as Earth and orbit in the habitable distance of 0.4-1.0 $M_{\odot}$ stars. For 0.8 and 1.0 $M_{\odot}$, moons can survive more than 10 Gyrs in most cases. If we restrict relatively fast initial planetary rotaional rate (below the black line), then lifetimes are commonly more than 10 Gyrs.

For 0.4 and 0.6 $M_{\odot}$, it is difficult for moons to have longer lifetime. If the star is 0.4 $M_{\odot}$, then moons cannot survive more than 5 Gyrs. If the star's mass is 0.6 $M_{\odot}$, then the moons' lifetimes are at most 10 Gyrs for 1 and 10 $M_{\oplus}$.

3.4.3 Four Typical Compositions of Planets with Q_p=100

In Section 3.4.1 and 3.4.2, we considered planets with Earth-like bulk composition. However, not all rocky extrasolar planets will be Earth-like. In this section, we examine four typical planet compositions: 50% ice-50% rock, 100% rock, Earth-like (67% rock, 33% iron), and 100% iron. For uniformity, we assume that the mass of the parent star is the same as that of the Sun.

Fig.3.6 shows the lifetimes of moons whose planets are composed of the four typical compositions. We can see that the lifetimes of moons depend on the composition of the planets. Moons can more easily survive for than 10 Gyrs as you go from ice-rock toward iron. This means that a moon has a longer lifetime when its host planet is more dense. For a fixed planetary mass, the planet's radius decreases with increasing

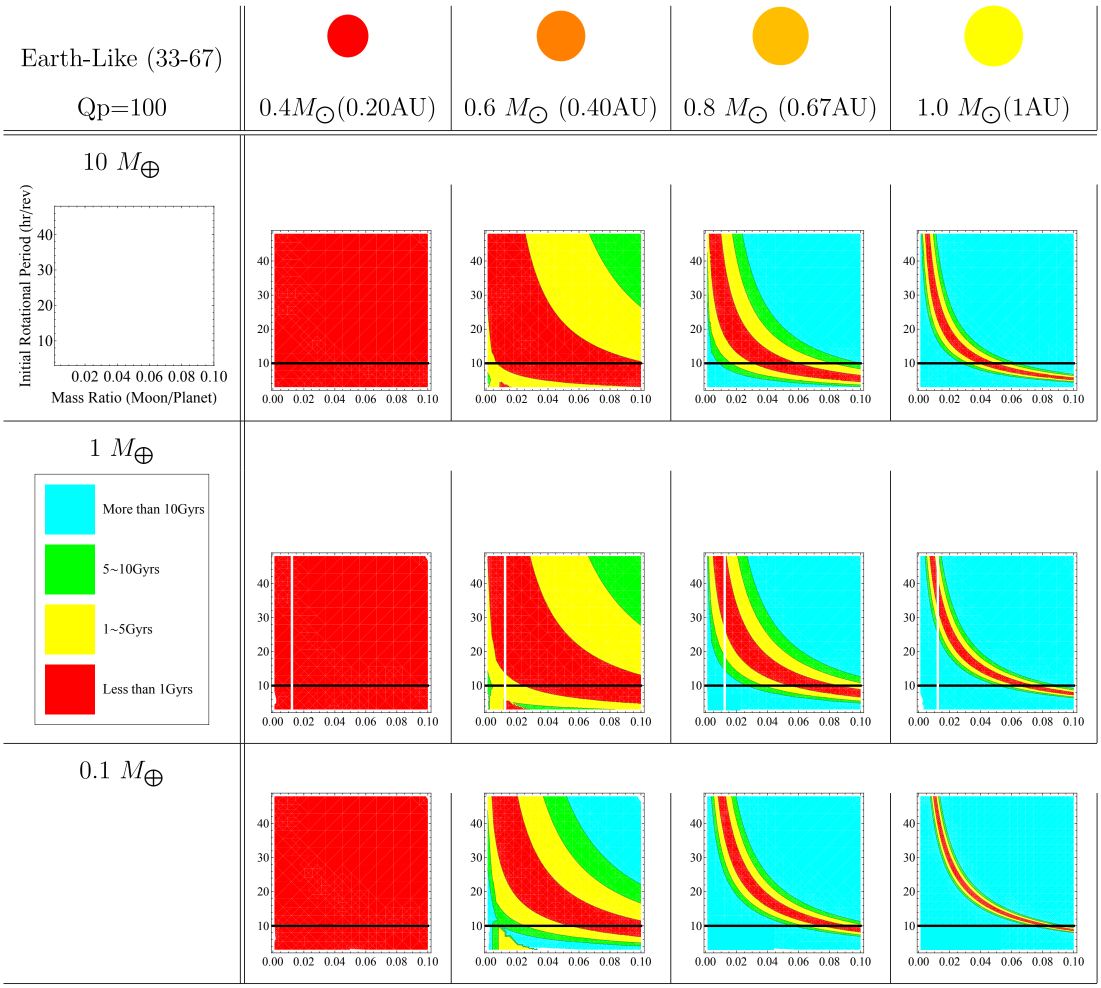

Figure 3.5: This figure shows the lifetime of a hypothetical moon whose planet is made of the same bulk composition as the Earth with low dissipation (Q_p=100) and which orbits at the habitable distance from a 0.4−1.0 $M_\odot$ star. For 0.4 $M_\odot$ stars and later, moons cannot orbit around their planets for more than 5 Gyrs in any situation.

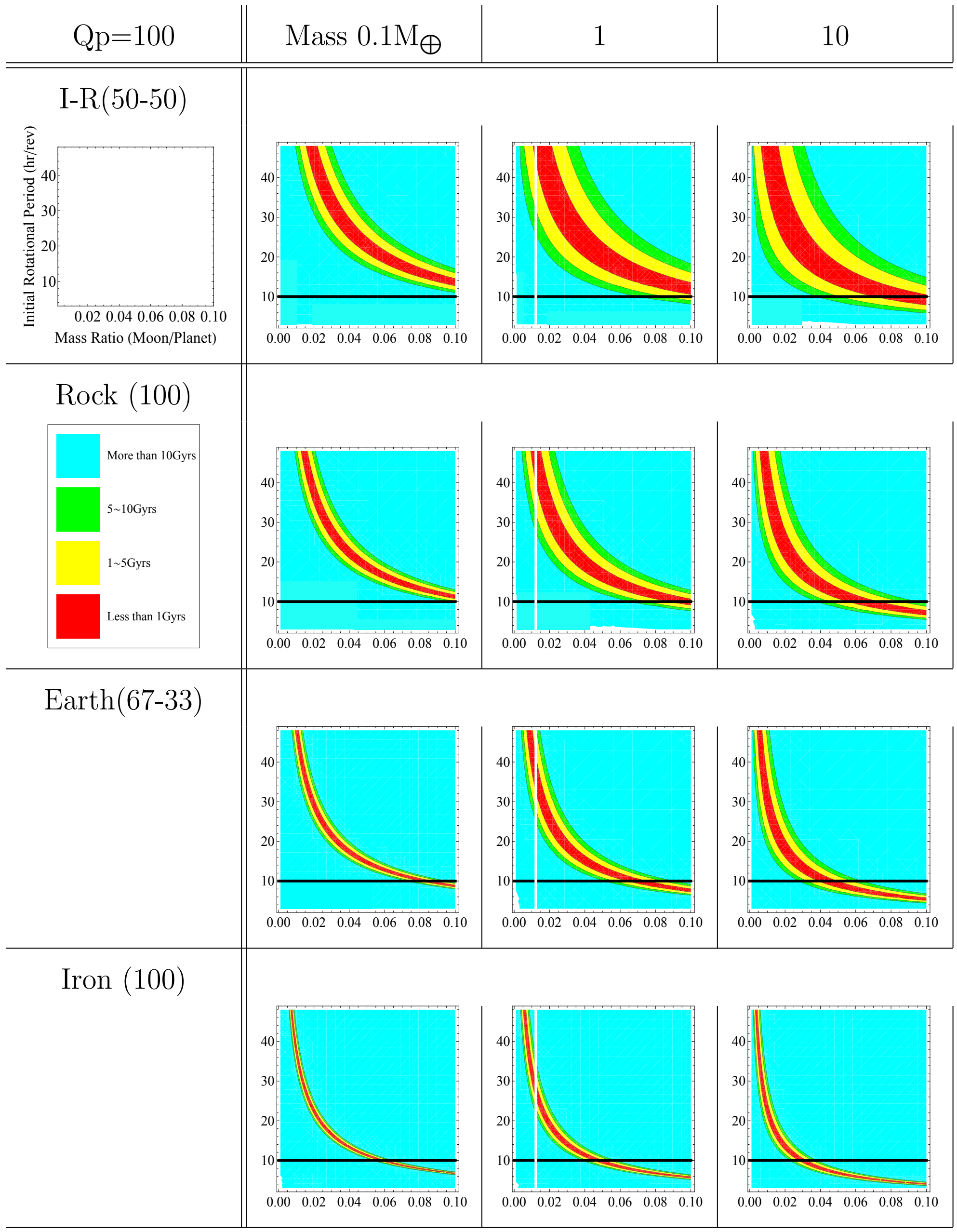

Figure 3.6: This table shows the lifetimes of moons whose planets have different masses and compositions. We use Q_p=100. We show the assumed Love number, k_2, and moment of inertia constant, α, in the Appendix. For the same composition, the heavier the mass of the planet, the shorter the lifetime of the moon. For the same mass, the denser the planet, the longer the lifetime of the moon.

density, as do tidal torques. When the tidal torque is small, the system evolves more slowly. Hence, a moon has longer lifetime when its host planet has higher density. For iron planets, moons can survive more than 10 Gyrs in the majority of cases. On the other hand, for a ice-rock planet, there is relatively small chance for a moon to survive more than 10 Gyrs. Unlike iron planets, moons of ice-rock planets must have very specific initial conditions to have more than 10 Gyrs lifetimes.

3.4.4 The Critical Line

In Section 3.4.1, we see that moons cannot survive more than 5 Gyrs around 0.4 $M_{\odot}$ and 0.6 $M_{\odot}$ stars provided Q_p is 10. In Section 3.4.2, if the stellar mass is 0.4 $M_{\odot}$, the lifetimes of the moons are no more than 5 Gyrs provided Q_p is 100.

We expect that there is the minimum stellar mass below which moons cannot survive more than 5 Gyrs. For Q_p=10, the minimum stellar mass is between 0.6 and 0.8 $M_{\odot}$. For Q_p=100, it should be between 0.4 and 0.6 $M_{\odot}$. In Section 3.4.3, we see that the lifetime of the moon depends on the composition of the planet. The minimum stellar mass for which moons cannot survive more than 5 Gyrs also should depend on the composition of the planet. Thus, our results allow us to draw a "critical line" of moon-stability, inward of which moons are unstable and outside which then can survive for astrobiologically relevant time scales.

In Fig.3.7 we show the critical lines not only for Earth-like planets but also for the other planet compositions such as iron, Earth-like, rock, and ice-rock. For Q_p=10, we do not consider ice-rock planets because these planets are unphysical (a planet cannot be made half of water yet still possess a high energy dissipation mechnism).

We draw two conclusions from the locations of the critical lines in Fig.3.7. First, if Q_p is smaller, then the critical stellar mass is higher. For small Q_p, a star-planet-moon system loses energy easily and the system evolves more quickly. Hence, the critical stellar mass becomes larger.

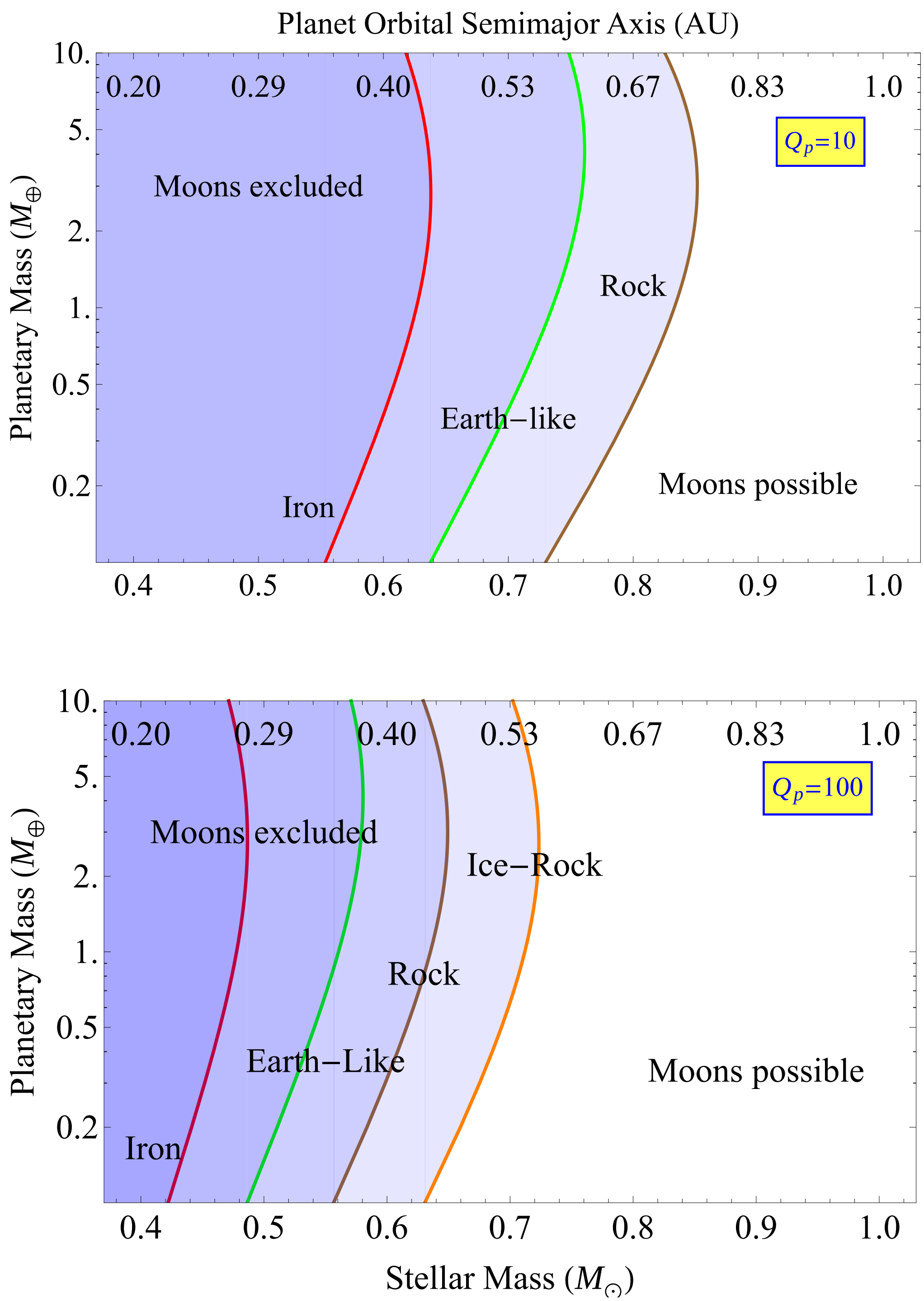

Figure 3.7: This graph shows the 5 Gyrs critical lines for assumed Q_p=10 and Q_p=100. We do not consider ice-rock planets with Q_p=10 because such planets are unphysical. For each planet composition, if a star-planet-moon system is in the left side of the line, then a moon cannot survive more than 5 Gyrs. If a star-planet-moon system is on the right side of the line, then a moon may or may not survive more than 5 Gyrs, depending on the initial rotational period and moon/planet mass ratio.

Planet of Kepler-62	Maximum Mass ($M_\oplus$)	Radius ($R_\oplus$)	Semimajor Axis (AU)
b	< 9	1.31	0.0553
c	< 4	0.54	0.0929
d	< 14	1.95	0.12
e	< 36	1.61	0.427
f	< 35	1.41	0.718

Table 3.1: (Borucki et al. (2013) [5])

Second, if the planetary density is higher, then the critical mass is lower, and more moons are stable. The torque on the planet due to the moon is proportional to the radius of the planet to the fifth power. Therefore, because a higher density indicates a smaller radius, the torque on the planet is lower, all else being equal. The system then evolves slowly. Hence, the critical mass become smaller.

Our results indicate that a rocky planet with Q_p=10 at the habitable distance cannot have a moon whose lifetime is longer than 5 Gyrs if the stellar mass is less than 0.55 $M_\odot$. For Q_p=100, if the stellar mass is less than 0.42 $M_\odot$, the longevity of moon cannot be longer than 5 Gyrs.

3.5 Kepler-62

So far, we have considered hypothetical star-planet-moon systems. In this section, we explore the prospect for moons in a real potentially habitable star-planet system.

Kepler-62, a K-type star with 0.64 $R_\odot$ and 0.69 $M_\odot$, is a five-planet system. Two of these planets have 1.4 and 1.6 Earth radii and orbit in the habitable zone: Kepler-62e and f. Table 3.1 shows maximum masses, radii, and semimajor axes of all five planets in Kepler-62 system. Theoretical models suggest that Kepler-62e and f could be solid, either with a rocky composition or composed of mostly solid water in their bulk [5]. We calculate tidal decay lifetimes for hypothetical moons of Kepler-62e and f using all four possible planetary compositions as well as both Qp=10 and 100.

Fig.3.8 shows lifetimes of the hypothetical moons of Kepler-62e and f. The white

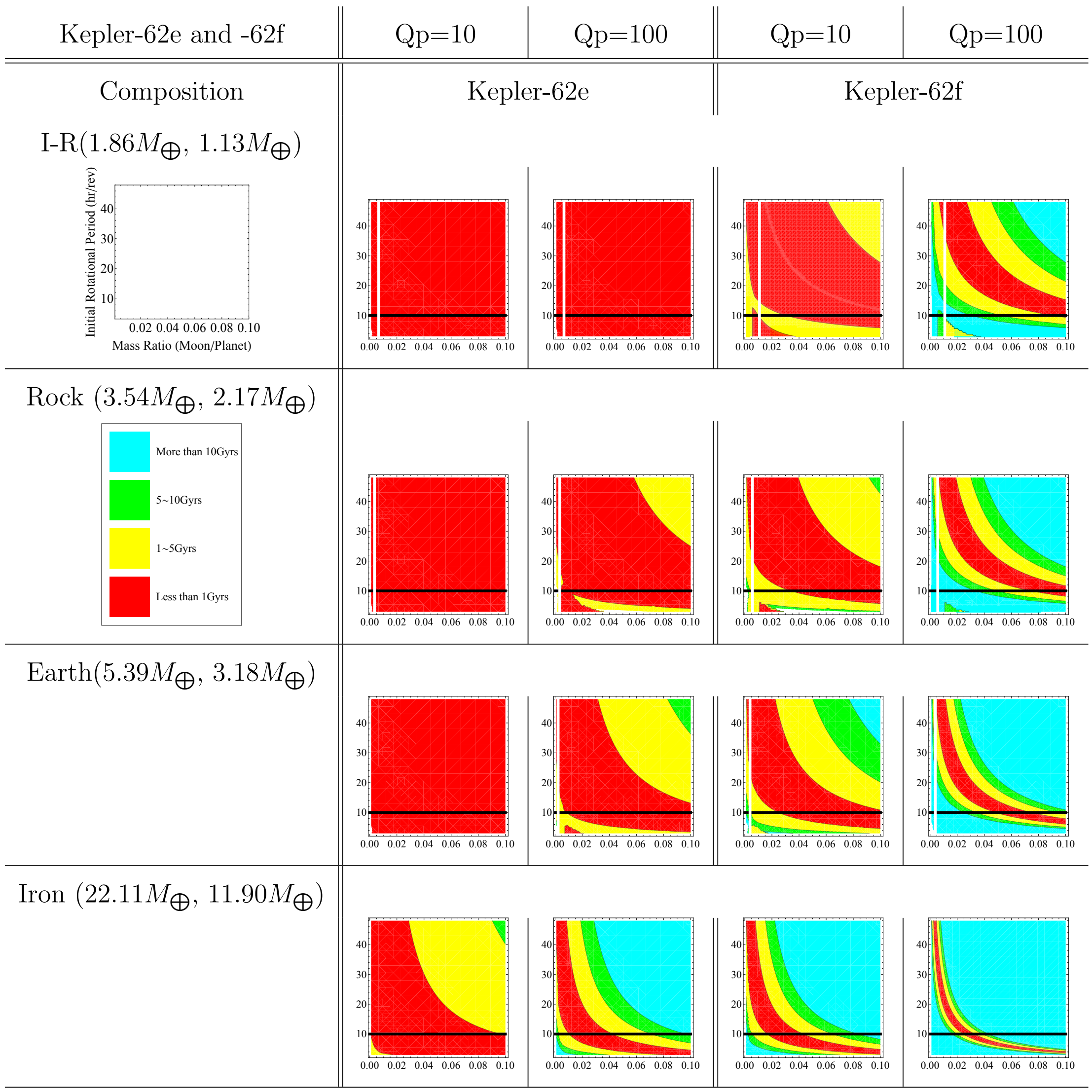

Figure 3.8: This table shows the lifetimes of the hypothetical moons of Kepler-62e and f. The numbers in the parenthesis next to planetary compositions are theoretical mass of Kepler-62e and -62f, respectively. The white vertical lines are the mass ratio of our Moon and Kepler-62e and f. The black horizon line is 10 hr/rev.

vertical lines represent the mass ratio of our Moon and Kepler-62e and f. From the radii of these planets, we can estimate planetary masses depending on the compositions [20]. If the planets are made of low density material, such as ice, their masses are about that of Earth. If the planets are made of high density material, such as iron, Kepler-62e and f are much more massive than the Earth. Our result shows that Kepler-62e could host a moon whose lifetime is longer than 5 Gyrs only if it is made of iron and has Qp=100. On the other hand, many situations exist for a moon of Kepler-62f to have a lifetime longer than 5 Gyrs. Especially for Qp=100, moons of Kepler-62f can have at least 5 Gyr lifetime regardless of planet composition.

3.6 Discussion

Detecting rocky planets in habitable zones is of astrobiological interest because life may be possible on such planets. Kepler-62e and f are two known rocky planets in the habitable zone. However, we show that it is hard for a moon of Kepler-62e to survive more than 5 Gyrs (Fig. 3.8). Without a "long-lived" moon, a planet may not have a long-term moderate climate. Hence, life on Kepler-62e might not have enough time to evolve complex life.

In contrast, it is relatively easy for Kepler-62f to have a surviving moon. However, climate is sensitive. Stable planetary obliquity helps to support but does not guarantee a moderate climate. We would need more detailed calculations of planetary obliquity evolution to test whether Kepler-62f has a long-term moderate obliquity under various conditions. Because Kepler-62 is a newly discovered star-planet system, we do not know if Kepler-62f has a suitable environment for life. We need more information to draw a conclusion. From the stand point of its ability to retain a large moon for potential climatic stability, Kepler-62f could possibility have appropriate conditions for life.

We are interested in searching for rocky planets on which complex life might exist.

Here we define complex life to mean multicellular creatures such as plants, animals, and fungi. Our research shows that the minimum stellar masses below which moons cannot survive more than 5 Gyrs depends on the composition of the planets (Fig.3.7). For $Q_p = 10$, the minimum values of such stellar masses are 0.55 $M_{\odot}$, 0.64 $M_{\odot}$, and 0.73 $M_{\odot}$ for iron, Earth-like, and rock, respectively. For $Q_p = 100$, these masses are 0.42 $M_{\odot}$, 0.49 $M_{\odot}$, 0.56 $M_{\odot}$, and 0.63 $M_{\odot}$ for iron, Earth-like, rock, and ice-rock, respectively (Fig.3.7). If a planet has a long-lived moon, then a planet may have a long-term maderate climate. Hence, the planet has better chance to have complex life on it. Estimating time span for life to evolve from single cell life to complex life form is not easy. To estimate this time span, oxygen is key material. Because complex life needs a large amount of energy to maintain it's body, it needs a system to generate large energy. Complex life on Earth uses oxygen metabolism to create large energy [64]. Because oxygen is a very active material, it is hard for oxygen molecules to exist in planetary atmosphere without a oxygen recycling system. It took about 2 Gyrs that Earth atmosphere began to have oxygen molecules [64]. Since then, Earth has oxygen circulating system and life on Earth has the environment to be able to use oxygen. For complex life on other planets, the situation may be the same. First, there is no oxygen molecules in planetary atmosphere. It may take a few Gyrs for planets to begin to have oxygen molecules in their atmosphere. Once oxygen molecules are in planetary atmosphere, life on the planets can use oxygen to create energy. Hence, life on the planets have possibility to become larger and more complex. It may take another few Gyrs for life to reach complex life from. Therefore, it may takes Gyrs time scale for life on the other planets to evolve from simple life to complex life.

Under the condition that 5 Gyrs is about the time span for life on other planets to evolve from simple life to complex life, star-planet-moon systems whose host stars are less than 0.42 $M_{\odot}$ may not be good choice to look for habitable planets that may have complex life because in any moon/planet ratio and initial planetary rotational rate moons cannot survive more than 5 Gyrs. For $Qp = 10$, the maximum value of the critical line is 0.85 $M_\odot$ when the composition of the planet is rock. For $Qp = 100$, the maximum value of the critical line is 0.72 $M_\odot$ when the composition of the planet is ice-rock. This means that there are the moon/planet mass ratio and the initial planetary rotational rate such that the lifetime of the moon is greater than 5 Gyrs regardless of the composition of the planet if the stellar mass is greater than 0.85 $M_\odot$. Hence, planets whose parent stars are more than 0.85 $M_\odot$ can easily retain large moons. If the evolution of life on other planets is much faster than in our case, then our analysis would need to be modified. If for instance the required time span for life to become mulicellular were 1 or 2 Gyrs, then more worlds around less-massive stars could retain their large obliquity stabilizing moons.

3.7 Conclusion

On our 4.6-billion-year-old Earth, life took about 3.8 billion years to evolve from single-celled organisms to multicellular. A long-term moderate climate is thought to be crucial for life to evolve into complex forms. Stable obliquity of the Earth is key for such a scenario, as Earth's obliquity is stabilized by the Moon [38]. If other habitable planets require moons to maintain obliquity, then the longevity of a planet's moon is also important for life to evolve. We assume that 5 billion years is long enough for life on other planets to become multicellular. In this research, we studied what conditions star-planet-moon systems require in order to have moons with lifetimes longer than 5 billion years.

First, we consider Earth. According to the giant impact hypothesis, the initial rotational period is from 5 to 8 hr/rev. Under this condition, our result suggests that the earth's Moon could survive more than 10 Gyrs. Even if the initial rotational rate were as slow as 20 hr/rev, the Moon would survive more than 5 Gyrs.

Next, we consider hypothetical Earth-like extrasolar planets, with 0.1, 1.0 and

10.0 $M_\oplus$, at the habitable distance from ∼1.0 $M_\odot$ stars. These planets are assumed to have the same composition as the Earth, which is 67% iron and 33% rock, and similar tidal dissipation Q_p=10. For 0.4 and 0.6 $M_\odot$, moons cannot orbit around their planets more than 5 Gyrs in any situation. For 0.8 and 1.0 $M_\odot$, moons can survive more than 5 Gyrs if the initial conditions are appropriate. For the case where Q_p=100 and the star has 0.4 $M_\odot$, it is impossible for moons to have more than 5 Gyrs lifetimes. For 0.6 $M_\odot$ and Q_p=100, moons can survive more than 5 Gyrs if the conditions are appropriate. For 1.0 and 0.8 $M_\odot$, moons can survive more than 10 Gyrs in most cases

Not all extrasolar rocky planets are necessarily Earth-like in composition. We consider five typical planet compositions that are 50% ice-50% rock, 100% rock, Earth-like (67% rock, 33% iron), and 100% iron. Our result indicates that the lifetime of the moon depends on planet compositions and the moon has longer lifetime when its host planet has higher density, for the same planet mass.

The results of Section 3.4.1 and 3.4.2 show that there is a minimum stellar mass below which moons of habitable planets cannot survive for more than 5 Gyrs. We show the minimum stellar mass lines not only for Earth-like planets but also other planet compositions. Our result shows that for Q_p=10, the stellar mass should be larger than 0.55 $M_\odot$ for a rocky planet in the habitable distance to have a moon whose lifetime is longer than 5 Gyrs. For Q_p=100, the stellar mass should be larger than 0.42 $M_\odot$.

Finally, we calculate tidal decay lifetimes for hypothetical moons of Kepler-62e and f, which are in habitable zone. We examine all four possible compositions as well as Qp=10 and 100. Our result shows that Kepler-62e has a moon whose lifetime is longer than 5 Gyrs only if it is made of iron and Q_p=100. On the other hand, there are a lot of situations in which Kepler-62f could have a moon whose lifetime is longer than 5 Gyrs. Especially, for Q_p=100, Kepler-62f could have a 5-Gyr-lifetime-moon for any planetary compositions.

Chapter 4

Prospects for Catastrophic Moon-Planet Collisions in Exoplanetary Systems

4.1 Introduction

Recently, a lot of $\sim 10M_{\oplus}$ extrasolar planets, which may be rocky, have been detected[1]. Unfortunately, most of them are not in habitable zones of their host stars. A habitable zone is the range of distances for a terrestrial planet to maintain liquid water on its surface [33]. In general, organisms we know on the Earth need liquid water at least part of their life cycle. In order for life to spread on a terrestrial planet, planetary orbit should remain inside of habitable zone over the length of time required for biological evolution. In addition of existence of liquid water, stable climate on giga years timescale may be needed for a terrestrial planet to be habitable.

Planetary climate depends heavily on obliquity ([16], [68]). The Earth is a long-term habitat for life because the Moon facilitates a long-term, moderate climate on Earth by stabilizing the plant's obliquity [38]. If an Earth-sized planet has no moon or a relatively small one, the planetary obliquity may fluctuate large angle. Mars is a example of such planet. Mars has relatively small satellites and and its obliquity changes chaotically, fluctuating on a 100,000-year timescale [39]. Hence, in order to have giga years timescale of stable planetary climate, it may be necessary that a moon orbit around a planet in giga years timescale, too.

On the other hand, the moon may become a huge disaster for life on the planet because the moon either hits or escapes from the planet. Counselman (1973)[13] studied a planet-moon system with tides on the planet due to the moon. He found that a planet loses a moon by either increasing moon's semimajor axis until it escapes or migrating inward until it hits the planet's surface. He also found the case that a moon orbited around a planet forever. In this case, the moon does not become

[1] http://exoplanet.eu/

a huge disaster for life on the planet. However, if we consider a star-planet-moon system with tides on the planet due to the moon and the star, the moon either hits or escapes from the planet ([65], [2], [57]).

In general, terrestrial planets are hard to detect because they are small and dim. When a moon hits a terrestrial planet, this event is surely catastrophic for life on the planet but good chance to detect the planet from Earth. Melosh (1990)[46] estimated that the surface temperature of the Earth would be 3200K if a Mars-sized object hit the Earth.

In this paper, we investigate the conditions of star-planet-moon systems for moons with lifetimes 1, 3, and 5 billion years as well as the conditions for planet/star flux ratio of 0.1, 0.5, and 1.0 milimagnitude after the planet-moon collision. In Section 2, we introduce the model and method we use to consider the tidal effects on a planet due to both a star and moon. In Section 3, we show the conditions for a moon to have 1, 3, and 5 Gyrs lifetime by introducing the concept of the "moon stability line". The catastrophic event of planet-moon collision gives us a good chance to detect the planet from the Earth. We consider the observability of such a collision in Section 4. In Section 5, we present the astrobiological consequences of moon-planet collisions, and the conclusions are summarized in Section 6.

4.2 Model and Method

We consider a star-planet-moon system and focus on the tidal effects on the planet due to the star and moon. Both the star and moon are assumed to be point masses, but the planet has physical dimensions. Fig.4.1 shows a model of our study. We assume that a moon and a planet have circular orbits because we are interested in the longest moon's lifetime. Non-zero eccentricities only shorten the lifetime of the moon. There are the tidal bulges on the planet due to both star and moon. Friction between the tidal bulge and the planet slows down the rotational angular velocity of

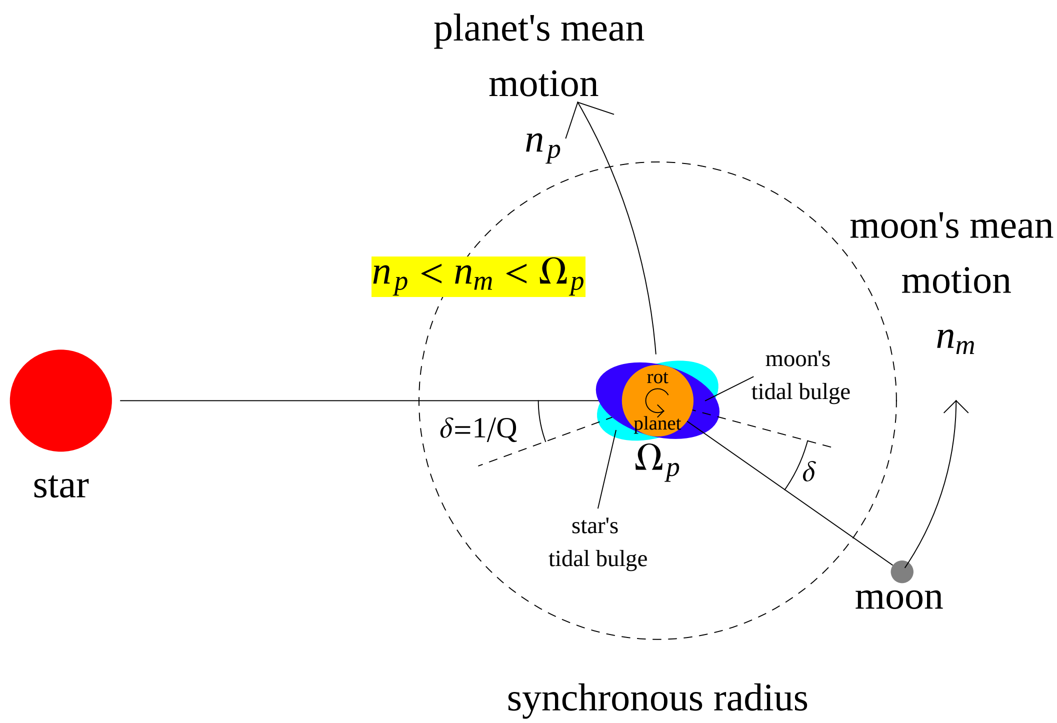

Figure 4.1: This picture shows a typical situation of our study. The planet has tidal bulges induced by both moon and star. Because planetary mean motion is slower than planetary rotation, stellar tidal torque slows planetary rotation. Torque on planet from moon also slows planetary rotation and increases the moon's semimajor axis. If the moon is inside of the synchronous radius, then the planet spins up and the moon spirals inward.

the planet. Meanwhile, the tidal torque due to the tidal bulge transfers the planetary rotational angular momentum to the moon's orbital angular momentum. Hence, the moon's orbital semimajor axis increases. The outward migration of the moon stops when the planet-moon reaches the synchronous state. The evolution of the star-planet-moon system after the planet-moon reaches the synchronous state depends on parameters such as the semimajor axis of the planetary orbit about the star; the initial planetary spin velocity; the initial moon's orbital angular velocity; and masses of the star, planets, and moons. A more detailed discussion can be found in Sasaki et al.(2012)[57].

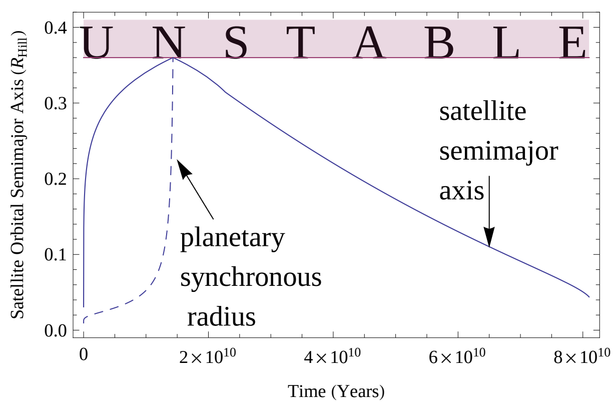

Figure 4.2: This graph shows satellite orbital semimajor axis vs. time with planetary synchronous radius. The solid line represents the satellite semimajor axis and the dashed line indicates the planetary synchronous radius. The moon starts just outside of synchronous radius and almost reaches the outermost stable radius, but later it reverses direction, eventually crashing into the planet.

Sasaki et al.(2012)[57] also formulated tidal decay lifetimes for hypothetical moons orbiting extrasolar planets with both lunar and stellar tides. In this research, we apply their method to 0.3 - 1.0 $M_\odot$ star systems with 0.1 - 10 $M_\oplus$ terrestrial planets. We choose the initial conditions such that the trajectory of the moon with the synchronous radius is similar to that shown in Fig.4.2. In the beginning, the moon is at just outside of the synchronous radius. The moon moves away from the planet very quickly in the first few billion years; then slows it down and almost reaches the outermost stable radius. After that, the moon evolves inward until the moon hits the planet.

4.3 Conditions for Moons' Lifetimes of Billion-Year Timescale

On Earth, it took about 4 billion years for life to evolve from single-celled organisms to multicellular creatures such as plants, animals and fungi. As of yet, we have only one data point from which to estimate the time it takes to develop complex on a planet. Billion-year timescale may be needed to develop complex on a planet.

In this study, we take the habitable zone to be the region where the radiant energy of the center star that the planet receives is $40\% - 140\%$ that of the Earth. For our Sun, the habitable zone is between 0.85 AU to 1.58 AU. The average distances of Venus and Mars from the Sun is 0.72 AU and 1.52 AU, respectively. Venus is outside the habitable zone and Mars is at the border of the habitable zone. When the mass of a star is small, the habitable zone is narrower and closer to the star because the change of the luminosity between a small and large star depends on a reverse square law. the law of the reverse square. For example, the habitable distance of 0.4 $M_{\odot}$ star is between 0.17 AU and 0.32 AU.

Fig. 4.3 shows the moon stability lines for 1-5 Gyrs applied to 0.3-1.0 $M_{\odot}$ stellar mass and 12, 30, 100 planetary dissipation constant, Q_p. Like the "ice line" with respect to planet formation, we define the "moon stability line" as the location beyond which a moon is stable for the life of the stellar system. Therefore, no such primordial moons can presently exist inside the moon stability line, though moons are possible outside.

The basic characteristic of moon stability lines are the same for all cases. At 2.0 $M_{\oplus}$, moon stability lines reach the maximum. When Q_p is smaller, moon stability lines are farther to the star for the same stellar and planetary masses. The planetary tidal dissipation value, Q_p, indicates how easily the planet loses its rotational energy by tidal forces. For small Q_p, a star-planet-moon system loses energy easily. Hence, the system evolves quickly. When stellar mass is smaller, moon stability lines are closer to the star for the same stellar and planetary masses.

For a small mass, say, 0.3 $M_{\odot}$ star, the habitable zone is below the 1 Gyr moon stability line. This means that if a planet-moon system forms in the habitable zone of a 0.3 $M_{\odot}$ or smaller star, then the planet cannot retain a moon for 5 Gyr. If our Earth-Moon system had formed in the habitable zone of such a star, then our Moon would have already hit the Earth. On the other hand, for a large mass star, the habitable zone is above the 5 Gyr moon stability line. Our Sun-Earth-Moon system is an example of this situation.

4.4 Observability

If a moon hits a planet, the event is surely to be disastrous for life on the planet but spectacular event for a observer orbiting around the planet. This event is also good for a observer on the Earth because it is a good chance to detect the planet.

The secondary eclipse method is used to detect extrasolar planets. In this method, planet/star flux ratio is measured. To detect a planet, planet/star flux ratio of 1.0 milimagnitude or more is necessary. Fig. 4.4 shows 0.1, 0.5 and 1.0 milimagnitude lines. We choose M8, M5, and M2 main-sequence stars as parent stars because dimer stars are better. The composition of the hypothetical planets is assumed to be the same of the Earth and choose 1.0, 1.5, and 2.0 $R_{\oplus}$ as planetary radii. At the last stage, moons have circular orbits and migrate inward until moons hit planets. Because moons are point masses in our model, they can migrate inward until moons hit planets. In reality, moons are not point masses and they cannot hit planets because of the Roche limit. We assume that the moons have the same energy when they hit the planets as at the Roche limit of circular orbiting paths.

The planet/star flux ratio depends on the radii and surface temperatures of both planet and star. Because rocky planets are small, detecting these types of planets

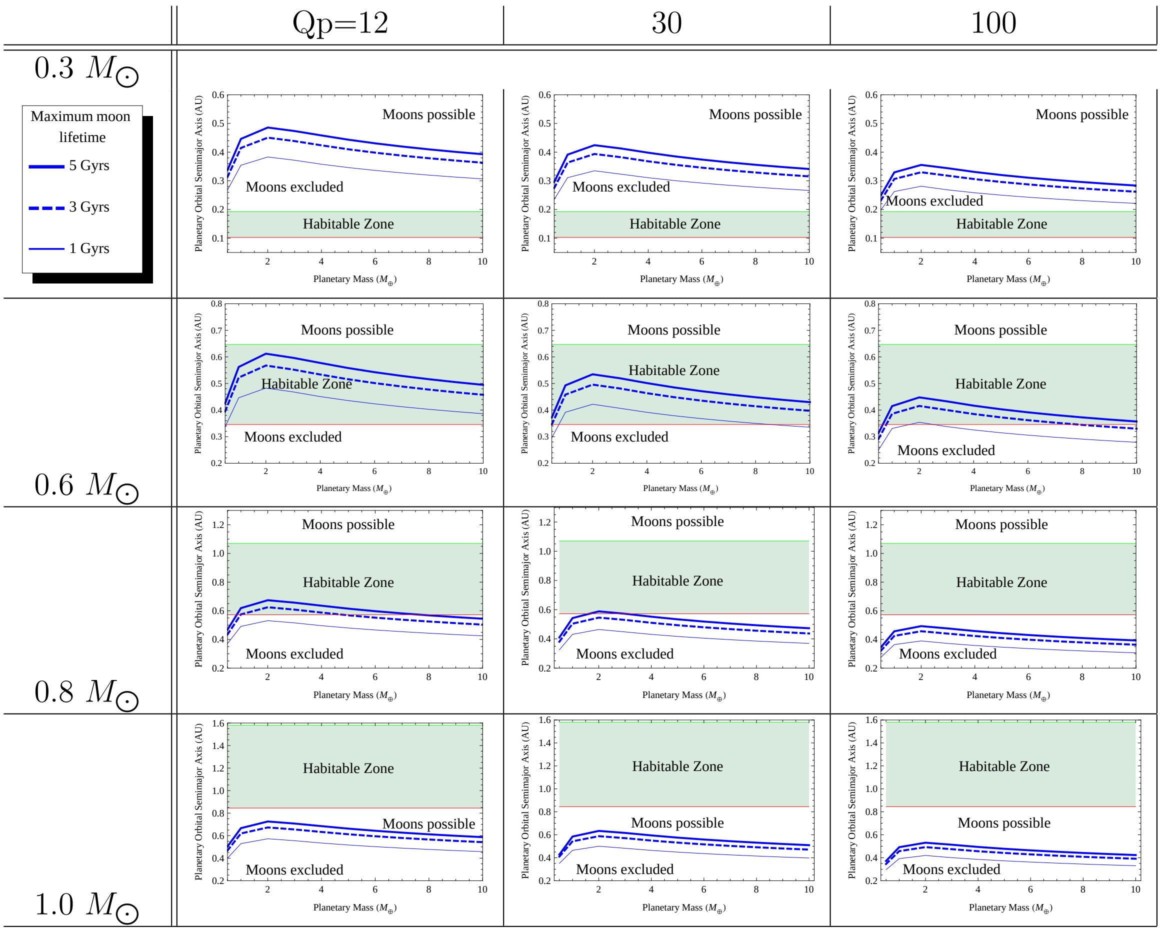

Figure 4.3: This figure shows the moon stability lines for 1, 3, and 5 Gyrs and habitable zones. Below the lines, no moons can survive for 1, 3, and 5 Gyrs, respectively. We assume that the planetary composition is the same as the Earth, which is 33% iron and 67% rock. The mass of the moon is our Moon mass. If the mass of the moon is increased, the moon stability lines shift to right. The shapes of the moon stability lines depends on the mass ratio of moon and planet.

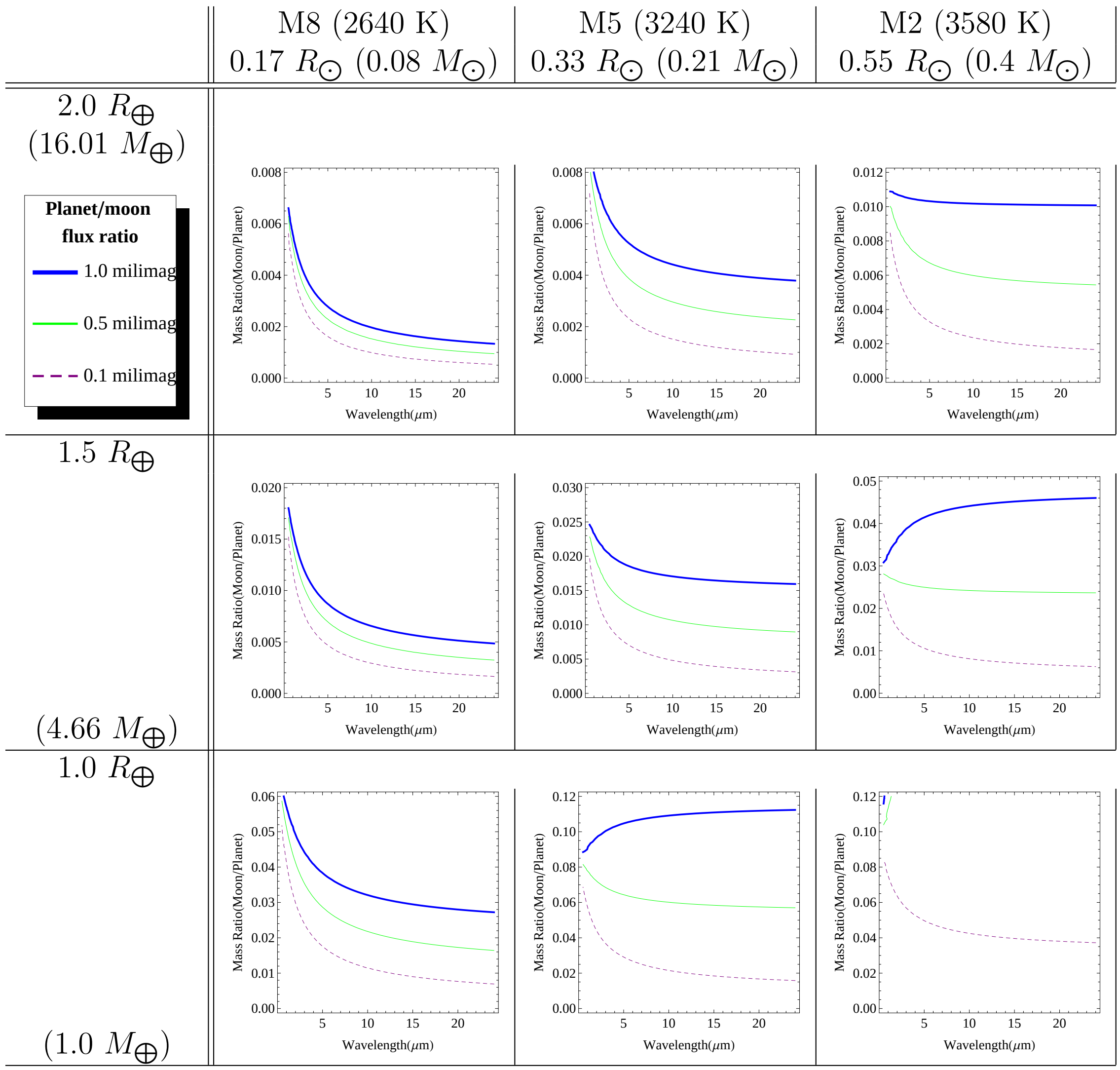

Figure 4.4: This figure shows the mass of the moon required to reach 0.1, 0.5, and 1.0 milimagnitude of planet/star flux ratio. The number next to the type of star is the effective temperature of the star. The numbers below of them are radius and mass of the star [11]. The mass of each plant is provided below the radius of the planet [20].

is difficult compared to ice and gas giant plants. When a moon hits a planet, the temperature of the planet increases by several hundreds to thousands Kelvin because all kinetic energy the moon has converts to heat energy. For simplicity, we assume that the initial temperature of the entire planet is zero Kelvin and that the planet has uniform temperature after the collision.

Our results show the longer the observing wavelength, the smaller the moon/planet mass ratio needs to be in order a certain planet/star flux ratio. This means that there is better chance to detect a planet if a longer wavelength is used. For example, consider a 1.0 $R_{\oplus}$ planet orbiting around a M8 type star case. In this case, if 5μm wavelength is used, then the mass of the moon needs to be at least 4% of the mass of the planet to be detectable. If 15μm wavelength is used, however, the minimum moon/planet mass ratio is 3%.

There are, however, three unusual cases in which using short wavelengths would be the better detection method: a M5 type star with 1.0 $R_{\oplus}$ planet, and a M2 type star with either 1.0 $R_{\oplus}$ or 1.5 $R_{\oplus}$ planets. For example, consider the M2 type star with a 1.5 $R_{\oplus}$ planet. In this case, when the 2μm wavelength is used, 0.036 is the minimum moon/planet mass ratio. However, when 15μm wavelength is used, the minimum moon/planet mass ratio becomes 0.045.

When the temperature of a star is low and the radius of a planet is large enough, the planet/star flux ratio reaches 1.0 milimagnitude even if the temperature of the planet is lower than that of the star. However, when the temperature of a star is high and the radius of a planet is small, the temperature of the planet must be hotter than that of the star in order to reach 1.0 milimagnitude planet/star flux ratio. Usually, stars are hotter than planets. However, when a large enough moon hits a planet, the temperature of the planet becomes hotter than that of the star.

4.5 Astrobiological Consequences

If a planet is in habitable zone, then life may exist on the planet. However, our results show that not all habitable zones are good for life. The habitable zones of the small mass stars are not ideal place for life, especially multicellular creatures. Imagine a planet-moon system forms in the habitable zone of a small mass star, 0.3 $M_{\odot}$ for example. Because the planet is in the habitable zone, life may exits on it. Our result shows that the moon of such a planet would collide with the planet within 5 Gyrs. If the planetary tidal dissipation value Q_p is 12, then this disaster happens within 1 Gyrs.

Or consider a stellar mass of 0.6 $M_{\odot}$ with Q_p=12. The planet-moon system on the far edge side of the habitable zone has a more than 5 Gyrs time limit; the planet-moon system on the near edge of the habitable zone has a less than 1 Gyrs time limit. These two planet-moon systems both formed inside the habitable zone of the same star, but the fates of respective moons, and thus life on the planet are completely different. The far side planet-moon system may have enough time for life to evolve to multicellular creatures. The near side planet-moon system, however, will experience a moon-collision, the catastrophic consequences of which would halt life evolution if not annihilate it completely.

When the stellar mass is 1.0 $M_{\odot}$, every planet-moon system in the habitable zone has more than 5 Gyrs before a planet-moon collision happens. Our Earth-Moon system is in this case. Our results suggest that the Earth-Moon system has about 81 Gyrs before the collision of the Moon.

When we consider the fate of stars, habitable planets around large mass stars are not good candidates for life because the lifetime of a large mass star is short. Small mass stars are, on the other hand, thought to be better parent stars of habitable planets because they stay in main sequence for a long time. Because planetary climate is sensitive, stars must be stable for planets to have a climate moderate enough to foster life. Our research shows, however, that stars that are too small of mass are *not* good parent stars because moons hit their planets with in 1 Gyrs. Stellar mass should be at least 0.8 $M_{\odot}$ to be good parent star.

4.6 Conclusion

Large moons stabilize planetary obliquity in some cases, and it has been suggested that large moons are necessary for the evolution of complex life [64]. Ward & Brownlee(2000)[65], Barnes & O'Brien(2002)[2], and Sasaki et al.(2012)[57] show, however, that if we consider a star-plant-moon system with both lunar and stellar tides, then the moon eventually either hits or escapes from the planet. Moons are important of the evolution of life, but cause the huge disaster for life on planets.

In this research, we apply their method to 0.3 - 1.0 $M_{\odot}$ star systems with 0.1 - 10 $M_{\oplus}$ terrestrial planets in habitable zone. For large mass star, the planets in habitable zone have more than 5 Gyrs before the moons hit their planets. In this case, life on the planets may have enough time to evolve complex form. On the other hand, for small mass star, the planets in habitable zone have less than 1 Gyrs before the moons hit the planets. Unfortunately, life on such planets may experience the catastrophic event in the middle of evolution. Our result suggest that stellar mass for a parent star should be at least 0.8 $M_{\odot}$ suitable for large moon stability.

A moon-planet collision would be a catastrophic for life on the planet. This event, however, gives an observer on the Earth a chance to detect the planet. Usually, rocky planets are hard to detect because they are small and dim. When a moon hits such a planet, however, the surface temperature of the planet increases dramatically, and thus brightness of the planet increases dramatically.

Our results suggest that observations at longer wavelengths are better for detecting rocky planets. If the planets are the size of Earth, the moon/planet mass ratio should be 0.026 or more to reach 1.0 milimagnitude of planet/star flux ratio. Compared to our Moon/Earth mass ratio, which is 0.0123, such a ratio is huge. If the planetary radii are 1.5 $R_{\oplus}$ and 2.0 $R_{\oplus}$, the moon/planet mass ratio should be 0.005 and 0.0018 or more, respectively. Because there are no rocky planets about of these sizes in our solar system, we are not sure if these moon/planet mass ratios are small or large. If a planet that changes its brightness suddenly is detected, then the planet may just be experienced a collision with its moon.

Chapter 5
Conclusion

In this thesis, I consider tidal evolution of extrasolar moons of star-planet-moon systems. I formulated tidal decay lifetimes for hypothetical moons with both lunar and stellar tides. I found four types of trajectories depending on the astronomical parameters and the initial conditions. For each type, I derive a formula that can calculate the lifetimes of moons. Our results allow us to find the type of system and the lifetime of the moon without the need to numerically solve a system of differential equations.

If the tidal torque on the planet from the moon is always greater than that from the star, then the star-planet-moon system will be Type I. Our Sun-Earth-Moon system is this type. On the other hand, if the tidal torque on the planet from the moon is always smaller than that form star, then the system will be Type III. When moons are small relative to their planets, then the system tends to be Type III. Sun-Jovian-planet-moon systems are in this type. Type II is between Type I and Type III. First, the planet and the star reach synchronized state, and then the planet and moon reach synchronized state. If the moon migrates outward more than 0.36 Hill's radius, then the system will be Type IV. The planet loses the moon to interplanetary space. The type of system depends on characteristics of the star, planet, and moon (masses, planetary radii, etc.) as well as the initial conditions of the planet and the moon, but is independent of Love number Q_p.

I applied our results to find the conditions for moons to have more than 5 Gyrs lifetimes. On our 4.6-billion-year-old Earth, life took about 3.8 billion years to evolve from single-celled organisms to multicellular. A long-term moderate climate is thought to be crucial for life to evolve into complex forms. Stable obliquity of the Earth is key for such a scenario, as Earth's obliquity is stabilized by the Moon [38]. If other habitable planets require moons to maintain obliquity, then the longevity of a planet's moon is also important for life to evolve there. I assume that 5 billion years is long enough for life on other planets to become multicellular.

When I considered our Sun-Earth-Moon system, I found that the Moon would survive more than 5 Gyrs even if the initial rotational rate were as slow as 20 hr/rev. According to the giant impact hypothesis, the initial rotational period is thought to have been be from 5 to 8 hr/rev. Under this condition, my results suggest that the Earth's Moon could easily survive more than 10 Gyrs under most scenarios.

I found that there is a minimum stellar mass below which moons of habitable planets cannot survive for more than 5 Gyrs. My result shows that for Q_p=10, the stellar mass needs to be larger than 0.55 $M_\odot$ for a rocky planet in the habitable distance to have a moon whose lifetime is longer than 5 Gyrs. For Q_p=100, the stellar mass would need to be larger than 0.42 $M_\odot$.

I calculated tidal decay lifetimes for hypothetical moons of Kepler-62e and f, which are in the habitable zone. I examined four possible compositions as well as Q_p=10 and 100. I found that Kepler-62e could only possess a moon whose lifetime is longer than 5 Gyrs if the planet is made of iron and has Q_p=100. On the other hand, there are a lot of situations in which Kepler-62f could have a moon whose lifetime is longer than 5 Gyrs. Especially for Q_p=100, Kepler-62f could have a 5-Gyr-lifetime-moon for any planetary compositions.

Because large moons stabilize planetary obliquity, long-lived large moons are an important condition for life on planets to evolve into complex forms. In Chapter 2, however, we showed that if we consider a star-plant-moon system with both lunar and stellar tides, then the moon eventually either hits or escapes from the planet. Moons are important for the evolution of life, but could cause a huge disaster for life on planets.

For a high mass star, planets in the habitable zone have more than 5 Gyrs before their moons hit the planets. On the other hand, for a low mass star, planets in the habitable zone have less than 1 Gyrs before the moons hit the planets. My results suggest that stellar mass for a parent star should be at least 0.8 $M_{\odot}$ to be suitable for large moon stability.

Rocky planets are hard to detect because they are small and dim. When a moon hits such a planet, however, the surface temperature of the planet, and thus the brightness, increases dramatically.

My results suggest that observations at longer wavelengths are better for detecting rocky planets. If a planet is the size of Earth, the moon/planet mass ratio should be 0.026 or more to reach 1.0 milimagnitude of planet/star flux ratio. Compared to our Moon/Earth mass ratio, which is 0.0123, such a ratio is huge. If the planetary radii are 1.5 $R_{\oplus}$ and 2.0 $R_{\oplus}$, the moon/planet mass ratio should be 0.005 and 0.0018 or more, respectively. Because there are no such rocky planets about of these sizes in our solar system, we are not sure if these moon/planet mass ratios are small or large. If a planet that changes its brightness suddenly is detected, then the planet may just be experienced a collision with its moon.

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Appendix A: Type I Solution

Derivation of the formula of the lifetime of the moon for Type I.

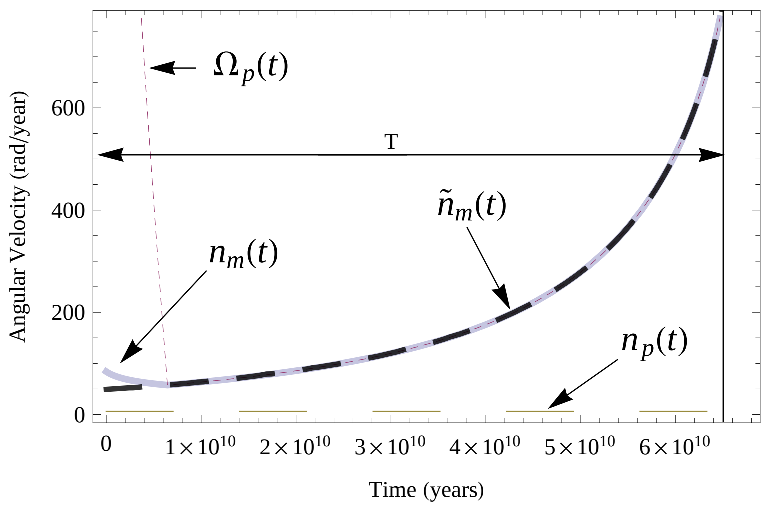

Figure 5.1: The thick dashed line is $\widetilde{n}_m(t)$. The thick light blue line is $n_m(t)$. As you can see, $\widetilde{n}_m(t)$ and n_m have the same maximum time. We use the present data of our Sun-Earth-Moon system. The initial conditions are $n_m(0)$=84 rad/year, $\Omega_p(0) = 730\pi$ rad/year, and $n_p(0) = 2\pi$ rad/year. The new initial conditions are $\widetilde{n}_m(0)$=$\Omega_p(0) =$ 48.5524 rad/year and $n_p(0) = 2\pi$ rad/year.

Suppose that the initial conditions $n_m(0)$, $\Omega_p(0)$, and $n_p(0)$ are known. From these initial conditions, we can calculate $n_m(t)$, $\Omega_p(t)$, and $n_p(t)$ by solving the equations (2.10) numerically (Fig.2.1).

For $0 \leq t < T1$, we can use equation (2.13) with $\text{sgn}(\Omega_p - n_m) = \text{sgn}(\Omega_p - n_p) = 1$ because the planet is not tidally locked with either the star nor the moon, and $\Omega_p > n_m > n_p$. For $T1 \leq t < T$, we can use (2.14) with $\text{sgn}(\Omega_p - n_p) = 1$ because the planet is tidally locked with the moon, and $\Omega_p > n_p$.

Define a function $\widetilde{n}_m(t)$ such that $\widetilde{n}_m(t)$ satisfies the following equation for $0 \leq t < T$;

$$\frac{M_m(GM_p)^{2/3}}{\widetilde{n}_m^{1/3}(t)} + \alpha R_p^2 M_p \widetilde{n}_m(t) + \frac{M_p(GM_s)^{2/3}}{n_p^{1/3}(t)} = L_0$$

where $L_0 = \frac{M_m(GM_p)^{2/3}}{n_m^{1/3}(0)} + \alpha R_p^2 M_p \Omega_p(0) + \frac{M_p(GM_s)^{2/3}}{n_p^{1/3}(0)}$ is the initial angular momentum of the system. In other words, $\widetilde{n}_m(t)$ represents situation in which the planet and the moon are tidally locked from beginning to end. Because $\widetilde{n}_m(t)$ and $n_m(t)$ are the same for $T1 \leq t < T$, we can calculate the maximum lifetime of the moon if we know the domain of $\widetilde{n}_m(t)$ (Fig.5.1).

For given t, set $x = \widetilde{n}_m$ and define

$$f(x) \equiv \frac{(GM_m)(GM_p)^{2/3}}{x^{1/3}} + \alpha R_p^2 (GM_p) x + \frac{(GM_p)(GM_s)^{2/3}}{n_p(t)^{1/3}} - GL_0.$$

The condition that $f(x)$ has at least one zero, i.e. equation (2.14b) has a real solution, is

$$\frac{4}{3^{3/4}} \left\{(GM_m)^3 (GM_p)^3 \alpha R_p^2\right\}^{1/4} + \left(\frac{(GM_p)(GM_s)^{2/3}}{n_p(t)^{1/3}} - GL_0 \right) \leq 0. \tag{5.1}$$

Since we know $n_p(t)$, we can plug in equation (2.14a) and solve for t:

$$\begin{aligned} t \leq{} & \tfrac{2}{39} \tfrac{Q_p}{k_{2p} R_p^5} (GM_p)(GM_s)^{2/3} \\ & \left[\left(\tfrac{3^{3/4} GL_0 - 4\{(GM_m)^3 (GM_p)^3 \alpha R_p^2\}^{1/4}}{3^{3/4} (GM_p)(GM_s)^{2/3}} \right)^{13} - \left(\tfrac{1}{n_p(0)} \right)^{13/3} \right]. \end{aligned} \tag{5.2}$$

Hence, the lifetime of the moons for Type I, T, is

$$\begin{aligned} T ={} & \tfrac{2}{39} \tfrac{Q_p}{k_{2p} R_p^5} (GM_p)(GM_s)^{2/3} \\ & \left[\left(\tfrac{3^{3/4} GL_0 - 4\{(GM_m)^3 (GM_p)^3 \alpha R_p^2\}^{1/4}}{3^{3/4} (GM_p)(GM_s)^{2/3}} \right)^{13} - \left(\tfrac{1}{n_p(0)} \right)^{13/3} \right]. \end{aligned} \tag{5.3}$$

Appendix B: Type II Solution

Derivation of the formula of the lifetime of the moon for Type II.

For Type II, there are three stages (Fig.2.3). We start by finding $T2$. Assume $n_m(T1)$ and $n_p(T1)$ are known. At the end of the planet-star synchronized state, the torque due to the star is equal to the torque due to the planet. Hence, $\tau_{p-s}(T1+T2) = \tau_{p-m}(T1+T2)$. From equation (5.39), equation (5.40) and Kepler's Law,

$$n_m(T1+T2) = \left(\frac{GM_p}{GM_m}\right)^{1/2} n_{pc} \tag{5.4}$$

where $n_{pc} = n_p(T1+T2)$ (Fig. 2.3).

For Stage 2, we can see that $\Omega_p - n_m < 0$ from Fig. 2.3. Hence, by the equation (2.15a) with $n_m(0) = n_m(T1)$

$$n_m(t) = \left(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{Gm_m}{(GM_p)^{8/3}}\, t + n_m^{-13/3}(T1)\right)^{-3/13}.$$

But in this equation, we measure the time t from $T1$. Measuring the time t from zero, for $T1 \leq t \leq T2$,

$$n_m(t) = \left(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{Gm_m}{(GM_p)^{8/3}}\, (t - T1) + n_m^{-13/3}(T1)\right)^{-3/13}.$$

Hence,

$$n_m(T1+T2) = \left(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{Gm_m}{(GM_p)^{8/3}}\, T2 + n_m^{-13/3}(T1)\right)^{-3/13}. \tag{5.5}$$

For stage 1, we can see $\Omega_p - n_m > 0$ from Fig. 2.3. Hence, by the equation (2.13a),

$$n_m(T1) = \left(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{Gm_m}{(GM_p)^{8/3}}\, T1 + n_m^{-13/3}(0)\right)^{-3/13}. \tag{5.6}$$

Combine equation (5.4), equation (5.5), and equation (5.6). Solve for $T2$.

$$T2 = T1 + \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\frac{(GM_p)^{8/3}}{(GM_m)}\left[n_m(0)^{-13/3} - \left\{\left(\frac{GM_p}{GM_m}\right)^{1/2} n_{pc}\right\}^{-13/3}\right]. \tag{5.7}$$

Next, we will find n_{pc} and $T3$. At $t = T1+T2$, we can see that $\Omega(t) = n_p(t) = n_{pc}$ from Fig.2.3. By the conservation of angular momentum and equation (5.4),

$$GL_0 = \frac{(GM_m)^{7/6}(GM_p)^{1/2} + (GM_p)(GM_s)^{2/3}}{n_{pc}^{1/3}} + \alpha R_p^2 (GM_p) n_{pc}. \tag{5.8}$$

The second term, $\alpha R_p^2(GM_p)n_{pc}$, is the spin angular momentum of the planet. We will ignore this term to approximate n_{pc} because we know the spin angular momentum is small compared to the total angular momentum.

Hence,

$$n_{pc} \approx \left\{ \frac{(GM_m)^{7/6}(GM_p)^{1/2} + (GM_p)(GM_s)^{2/3}}{GL_0} \right\}^3 . \tag{5.9}$$

When we use $\Omega_p(0) = n_p(0) = n_{pc}$ and $n_m(0) = n_m(T1+T2)$ as a initial condition, we can calculate $T3$ by using the formula for Type I. Hence,

$$\begin{aligned} T3 = {} & \tfrac{2}{39}\tfrac{Q_p}{k_{2p}R_p^5}(GM_p)(GM_s)^{2/3} \\ & \left[\left(\tfrac{3^{3/4}GL_0 - 4\{(GM_m)^3(GM_p)^3\alpha R_p^2\}^{1/4}}{3^{3/4}(GM_p)(GM_s)^{2/3}} \right)^{13} - \left(\tfrac{1}{n_{pc}} \right)^{13/3} \right]. \end{aligned} \tag{5.10}$$

From equation (5.7), equation (5.9), and the equation (5.10), the total life time, T, is

$$\begin{aligned} T &= T1 + T2 + T3 \\ &= 2T1 + \tfrac{2}{39}\tfrac{Q_p}{k_{2p}R_p^5} \left[\tfrac{(GM_p)^{8/3}}{(GM_m)} n_m^{-13/3}(0) \right. \\ &\quad + \frac{\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13}}{3^{39/4}(GM_p)^{12}(GM_s)^8} \\ &\quad \left. - \frac{(GL_0)^{13}}{\left\{(GM_p)^{1/2}(GM_m)^{7/6} + (GM_p)(GM_s)^{2/3}\right\}^{12}} \right]. \end{aligned} \tag{5.11}$$

Appendix C: Type III Solution

Derivation of the formula of the lifetime of the moon for Type III.

In Type III, the graph of $n_m(t)$ is comprised of two parts. The first part, $n_m^+(t)$, is from $0 \leq t \leq T1$ and the second part, $n_m^-(t)$ is from $T1 \leq t < T$. $\operatorname{sgn}(\Omega_p - n_m)$ is 1 for $0 \leq t \leq T1$ and -1 for $T1 \leq t < T$ because the planet and the moon are not in a synchronized state. To have $n_m^+(t)$, we can use equation (2.13) directly. To have $n_m^-(t)$, we set $n_m(0) = n_m(T1)$ and $t = t - T1$ because we need to shift the graph $T1$ in a positive direction. From equation (2.13),

$$n_m^+(t) = \left(\frac{39}{2} \frac{k_{2p} R_p^5}{Q_p} \frac{GM_m}{(GM_p)^{8/3}}\, t \, + n_m^{-13/3}(0) \right)^{-3/13} \tag{5.12}$$

for $0 < t < T1$ and

$$n_m^-(t) = \left(-\frac{39}{2} \frac{k_{2p} R_p^5}{Q_p} \frac{GM_m}{(GM_p)^{8/3}} (t - T1) + \left\{ n_m^+(T1) \right\}^{-13/3} \right)^{-3/13} \tag{5.13}$$

for $T1 < t < T$.

By the symmetry, $Tb = T1 + \frac{n_m^{-13/3}(0)}{a}$ where $a = \frac{39}{2} \frac{k_{2p} R_p^5}{Q_p} \frac{GM_m}{(GM_p)^{8/3}}$. Hence,

$$\begin{aligned} T &= T1 + Tb \\ &= 2T1 + \frac{n_m^{-13/3}(0)}{a} \\ &= 2T1 + \frac{2}{39} \frac{Q_p}{k_{2p} R_p^5} \frac{(GM_p)^{8/3}}{GM_m} n_m^{-13/3}(0). \end{aligned} \tag{5.14}$$

We can get the same result if we set the inside of the parenthesis of equation (5.13) equal to 0, and then solve for t.

Appendix D: Condition for Type I Case 1

Derivation of the formula of the condition for Type I Case 1.

The condition for Type I Case 1 is that the magnitude of the torque due to the moon is greater than the magnitude of the torque due to the star at $t = T1$ (Fig.2.1),

$$|\tau_{p-m}(T1)| \geq |\tau_{p-s}(T1)|. \tag{5.15}$$

By equation (5.39), equation (5.40), and Kepler's Law, equation (5.15) implies that

$$n_m(T1) \geq \left(\frac{GM_p}{GM_m}\right)^{\frac{1}{2}} n_p(T1). \tag{5.16}$$

For the time period $0 \leq t \leq T1$, the planet is not tidally locked with either the star or the moon. We can use equations (2.13) with sgn$(\Omega_p - n_m)$=1 and sgn$(\Omega_p - n_p)$=1.

For the time period $0 \leq t \leq T1$, the planet is not tidally locked with either the star or the moon. We can use equations (2.13) with sgn$(\Omega_p - n_m)$=1 and sgn$(\Omega_p - n_p)$=1.

Combine equation (5.16), equation (2.13a), and equation (2.13b), then solve for $T1$. We obtain

$$\begin{aligned} T1 \leq \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}&\frac{(GM_p)(GM_m)^{7/6}(GM_s)^{2/3}}{(GM_p)^{1/2}(GM_s)^{2/3} - (GM_m)^{7/6}} \\ &\times \left\{ n_p^{-13/3}(0) - \left(\frac{GM_p}{GM_m}\right)^{13/6} n_m^{-13/3}(0) \right\}. \end{aligned} \tag{5.17}$$

Appendix E: Condition for Type I Case 2

Derivation of the formula of the condition for Type I Case 2.

Assume $n_p(T1)$ and $n_m(T1)$ are known. Let t_* be the time from $T1$, when the magnitudes of two torques are equal (Fig.2.8).

The condition for Type I Case 2 is

$$\Omega_p(t_*) \geq n_p(t_*) \tag{5.18}$$

where t_* satisfies

$$|\tau_{p-m}(t_*)| = |\tau_{p-s}(t_*)|. \tag{5.19}$$

Equation (5.19) implies that

$$n_m(t_*) = \left(\frac{GM_p}{GM_m}\right)^{1/2} n_p(t_*). \tag{5.20}$$

We did a similar calculation to arrive at equation (5.16).

By conservation of angular momentum and equation (5.20), we derive

$$\begin{aligned}\Omega_p(t_*) = &\tfrac{1}{\alpha R_p^2 (GM_p)} \\ &\left[GL_0 - \tfrac{(GM_p)^{1/2}\{(GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\}}{n_p^{1/3}(t_*)}\right].\end{aligned} \tag{5.21}$$

After $T1$, the planet is not tidally locked with either the star or the moon for a time. We can use equations (2.13) with $\mathrm{sgn}(\Omega_p - n_m) = -1$ and $\mathrm{sgn}(\Omega_p - n_p) = 1$. We use equation (2.13a) and equation (2.13b) with initial conditions $n_m(0) = n_m(T1)$ and $n_p(0) = n_p(T1)$, which are

$$n_m(t) = \left(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\, t + n_m^{-13/3}(T1)\right)^{-3/13} \tag{5.22}$$

and

$$n_p(t) = \left(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\, t + n_p^{-13/3}(T1)\right)^{-3/13}. \tag{5.23}$$

Plug in equation (5.22) and equation (5.23) into the equation (5.20), and solve for

t_*. Then, we have

$$t_* = \frac{39}{2}\frac{Q_p}{k_{2p}R_p^5}\frac{(GM_p)(GM_m)^{7/6}(GM_s)^{2/3}}{\{(GM_m)^{7/6}+(GM_p)^{1/2}(GM_s)^{2/3}\}} \times \left\{\left(\frac{GM_p}{GM_m}\right)^{13/6} n_m^{-13/3}(T1) - n_p^{-13/3}(T1)\right\}. \tag{5.24}$$

By using this t_*, we have

$$\Omega_p(t_*) = c(GL_0 - a_1 b^{12} X) \tag{5.25}$$

$$n_p(t_*) = a_2 b^3 X^{-3} \tag{5.26}$$

where

$$c = \frac{1}{\alpha R_p^2 (GM_p)}$$
$$a_1 = (GM_p)^{1/2}(GM_m)^{7/78}$$
$$a_2 = \frac{1}{(GM_m)^{7/26}}$$
$$b = \left\{(GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right\}^{1/13}$$
$$X = \left\{\left(\frac{GM_p}{GM_m}\right)^{13/6} n_m^{-13/3}(T1) + \frac{(GM_p)^{1/2}(GM_s)^{2/3}}{(GM_m)^{7/6}} n_p^{-13/3}(T1)\right\}^{1/13}.$$

Applying equation (5.18), we have

$$a_1 b^{12} X^4 - GL_0 X^3 + \frac{a_2}{c} b^3 \leq 0. \tag{5.27}$$

Appendix F: Conditions for Type II and III

Derivation of the formula of the condition for Type II and III.

From Fig.2.9a and Fig.2.9b, the condition for Type II is $T5 > T6$ and the condition for Type III is $T5 \leq T6$.

For Stage 3, we know

$$\begin{cases} n_m^{'}(t) = \Big(-\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{GM_m}{(GM_p)^{8/3}}\,t \\ \qquad\qquad\qquad +n_m^{-13/3}(T1+T2)\Big)^{-3/13} \\ n_p(t) = \Big(\frac{39}{2}\frac{k_{2p}R_p^5}{Q_p}\frac{1}{(GM_p)(GM_s)^{2/3}}\,t \\ \qquad\qquad\qquad +n_p^{-13/3}(T1+T2)\Big)^{-3/13}. \end{cases} \tag{5.28}$$

At $t = T6$, $n_m^{'}(t) = \infty$. Hence,

$$T6 = \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\frac{(GM_p)^{8/3}}{GM_m}n_m^{-13/3}(T1+T2). \tag{5.29}$$

To find $T5$, we use the same formula for Type I, which is equation (2.21). We set $n_p(0) = n_p(T1+T2)$. $T5$ is

$$\begin{aligned} &T5 = \tfrac{2}{39}\tfrac{Q_p}{k_{2p}R_p^5}(GM_p)(GM_s)^{2/3}\times \\ &\left[\left(\tfrac{3^{3/4}GL_0-4\{(GM_m)^3(GM_p)^3\alpha R_p^2\}^{1/4}}{3^{3/4}(GM_p)(GM_s)^{2/3}}\right)^{13} - \left(\tfrac{1}{n_p(T1+T2)}\right)^{13/3}\right]. \end{aligned} \tag{5.30}$$

From Fig.2.3, $n_{pc} = n_p(T1+T2)$. And we know that $n_m(T1+T2) = (\frac{GM_p}{GM_m})^{1/2}n_{pc}$ from equation (5.4). By using this information, we have

$$\begin{cases} T5 = \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}(GM_p)(GM_s)^{2/3} \\ \qquad \times\left[\left(\frac{3^{3/4}GL_0-4\{(GM_m)^3(GM_p)^3\alpha R_p^2\}^{1/4}}{3^{3/4}(GM_p)(GM_s)^{2/3}}\right)^{13} - n_{pc}^{-13/3}\right] \\ T6 = \frac{2}{39}\frac{Q_p}{k_{2p}R_p^5}\frac{(GM_p)^{1/2}}{GM_m^{7/6}}n_{pc}^{-13/3}. \end{cases} \tag{5.31}$$

From equation (5.9), we also know that

$$n_{pc} = \left\{\frac{(GM_m)^{7/6}(GM_p)^{1/2} + (GM_p)(GM_s)^{2/3}}{GL_0}\right\}^3. \tag{5.32}$$

The condition for Type II, $T5 > T6$, implies

$$\begin{aligned}
&\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13}\times \\
&\left((GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right)^{12} \\
&> 3^{39/4}(GM_p)^6(GM_s)^8(GL_0)^{13}.
\end{aligned} \tag{5.33}$$

Similarly, the condition for Type III, $T5 \leq T6$, implies

$$\begin{aligned}
&\left(3^{3/4}GL_0 - 4\left\{(GM_m)^3(GM_p)^3\alpha R_p^2\right\}^{1/4}\right)^{13}\times \\
&\left((GM_m)^{7/6} + (GM_p)^{1/2}(GM_s)^{2/3}\right)^{12} \\
&\leq 3^{39/4}(GM_p)^6(GM_s)^8(GL_0)^{13}.
\end{aligned} \tag{5.34}$$

Appendix G: Love Numbers and Moment of Inertia Constants

Table of Love numbers and moment of inertia constants.

In tidal theory, the Love number, k_2, and the moment of inertia constant, α, are important numbers. These numbers depend on planet's mass and composition. The Love number is measure of how much a planet's surface moves in response to the gravitational pull of nearby bodies. This number can be between 0 and 1.5. The Love number is 0 and 1.5 if a planet is a rigid body and made of liquid, respectively. The moment of inertia constant tells us the mass distribution within a planet. For a uniform mass distribution planet, the moment of inertia constant is 0.4. The Earth's moment of inertia constant is 0.33. This is due to the fact that the Earth has a dense inner core surrounded by a less dense outer core and an even less dense mantle.

Table 5.1 shows the Love numbers and the moment of inertia constants that we used in this study. The structure of spherical symmetric planets in hydrostatic equilibrium obeys the following relations (Fortney et al. 2007)[20]. Equations (5.35) and (5.36) are called mass continuity and hydrostatic equilibrium, respectively:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{5.35}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \tag{5.36}$$

where r is the radius of a mass shell, m is the mass of a given shell, ρ is the local mass density, P is the pressure, and G is the gravitational constant. Fortney et al. (2007)[20] shows pressure-density relations for iron, rock, and water ice. By using Equations (5.35), (5.36) and pressure-density relations, we can find density-radius relations. Hence, we can calculate the moment inertia constant.

The Love number, k_2, is defined by Murray & Dermott (2000)[48]

$$k_2 = \frac{3/2}{1+\tilde{\mu}} \tag{5.37}$$

Planet Compositions		Planetary Mass ($M_\oplus$)			
		0.1	1	5	10
Ice (100%)	α	0.388	0.360	0.356	0.355
$\mu = 4.0 \times 10^9$ (N/m^2)	k_2	0.283	0.915	1.309	1.395
Iron-Rock (50-50%)	α	0.313	0.314	0.307	0.302
2.7×10^{10} (N/m^2)	k_2	0.085	0.426	0.958	1.169
Rock (100%)	α	0.397	0.383	0.360	0.349
5.0×10^{10} (N/m^2)	k_2	0.119	0.520	1.053	1.241
Earth-Like (67-33%)	α	0.335	0.318	0.296	0.273
1.4×10^{11} (N/m^2)	k_2	0.059	0.300	0.802	1.056
Iron (100%)	α	0.386	0.367	0.350	0.343
3.4×10^{11} (N/m^2)	k_2	0.068	0.352	0.891	1.137

Table 5.1:

where $\tilde{\mu}$ is the effective rigidity. The effective rigidity is given by [48]

$$\tilde{\mu} = \frac{19}{2} \frac{\mu}{\rho g_s R} \tag{5.38}$$

where μ is rigidity, g_s is the surface gravity and R is the radius of the planet. We obtain the rigidities of ice, rock, and the Earth form Murray & Dermott (2000)[48]. We choose the rigidity of ice-rock is the average of ice and rock, and iron is the linear extension of rock and Earth-Like.

Appendix H: The Minimum Lunar Mass

Derivation of the formula of the minimum lunar mass.

The minimum lunar mass required to stabilize a planet's obliquity is important but complicated. Here, we calculate a estimate of the minimum lunar mass necessary to affect a planet's axial precession. In order to affect planetary obliquity, the torque on a planet due from its moon must be comparable to that due to a star. The torque on the planet due to the moon τ_{p-m} is given by Barnes & O'Brien (2002) [2]; Goldreich & Soter (1966) [22]; Murray & Dermott (2000) [48] in Chapter 4.

$$\tau_{p-m} = -\frac{3}{2}\frac{k_{2p}GM_m^2R_p^5}{Q_pa_m^6}, \tag{5.39}$$

where k_{2p} is the tidal Love number of the planet, G is the gravitational constant, R_p is the radius of the planet, M_m is the mass of the moon, and a_m is the semimajor axis of the moon's orbit. Similarly, the torque on the planet due to the star τ_{p-s} is

$$\tau_{p-s} = -\frac{3}{2}\frac{k_{2p}GM_s^2R_p^5}{Q_pa_p^6}, \tag{5.40}$$

where M_s is the mass of the star, a_p is the semimajor axis of the planet's orbit. Set $|\tau_{p-m}| > |\tau_{p-s}|$ and simplify. We have

$$M_m > M_s\left(\frac{a_m}{a_p}\right)^3. \tag{5.41}$$

Let β be a constant such that

$$a_m = \beta R_H \tag{5.42}$$

where R_H is the radius of the Hill's sphere [15]

$$R_H = a_p\left(\frac{M_p}{3M_s}\right)^{1/3}. \tag{5.43}$$

Simplify Equation (5.41) by using Equations (5.42), and (5.43).

We have

$$\frac{M_m}{M_p} > \frac{\beta^3}{3}. \tag{5.44}$$