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Growth and Yield of Leucaena in the Philippines

JAMES W. GOUDIE[1] and JAMES A. MOORE[2]
[1]*Research Branch, British Columbia Ministry of Forests, Victoria, B.C. V8W 3E7 (Canada)*
[2]*College of Forestry, Wildlife and Range Sciences, University of Idaho, Moscow, ID 83843 (U.S.A.)*

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ABSTRACT

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Empirical yield tables for leucaena plantations in the Philippines were developed. Site index and height growth equations were developed as components of leucaena growth and yield. The relationships between volume and biomass per ha and top height were quantified. The yield tables and site index equation can be applied in the Philippines and perhaps other geographic areas where leucaena grows under similar edaphic and management conditions. The methodology described in this paper should be applicable to a variety of tree species grown in even-aged stands.

INTRODUCTION

Leucaena leucocephala (Lam.) deWit (hereafter: leucaena) is a commonly used exotic tree species for plantations in the tropical and sub-tropical regions of the world. Several studies of leucaena growth and yield for localized conditions have been reported (Kanazawa et al., 1981; Lu and Hu, 1981), but there is little growth and yield information available for a wide range of conditions within a region.

The Philippine National Dendrothermal Rural Electrification Project established leucaena plantations to provide fuel for potential wood-fired electrical generation plants at 51 locations throughout the country. These plantations are located on a variety of climatic, soil and topographic conditions. This paper describes the results of one approach for constructing regional yield tables for leucaena plantations in the Philippines. The methodology used to predict yields should be useful in other geographic regions or management conditions.

MODELING APPROACH

The strategy selected for modeling the leucaena plantations is a two-step process that has been used elsewhere around the world (Alder, 1980). First,

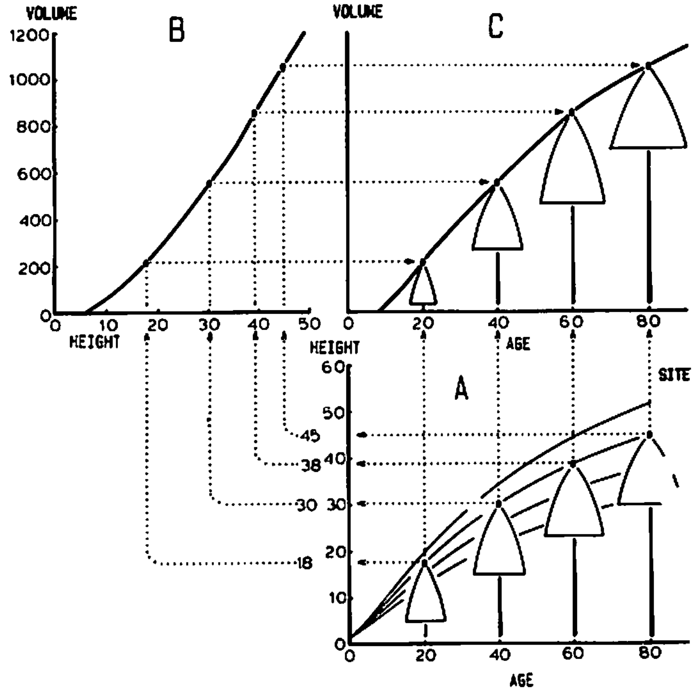

Fig. 1. Illustration of the two-step modelling strategy used in projecting development of leucaena plantations in the Philippines.

the relationship between dominant height and age is developed and then the patterns of other stand attributes, such as volume or biomass, are related to dominant height of the stand. Figure 1 (from Mitchell and Cameron, 1985) graphically depicts the technique. Note that the height–age curves (A) are combined with the volume–height (or biomass–height) relationship (B) and the classical sigmoid volume–age curve (C) results.

The major advantages to this approach are that (1) in developing the *ideal* height–age model, either individual trees or stands are tracked through time and thus a real growth series is followed; (2) functional forms of the height vs. age, and for other necessary relationships, are well known; and (3) realistic and usable first-approximation yields can be produced in a relatively short period of time. Resulting tables can be for either average stocking (empirical), as in this study, or full stocking (normal), depending on the sampling techniques. The major underlying assumptions of this process are:

(1) The height growth of a portion of the largest trees in the stand is a good reflection of the *site productivity* since it is the factor least affected by varying densities. This assumption is generally supported in the forestry literature, most recently by Pienarr and Shiver (1984) in South Africa for *Pinus elliotti*

Engelm. Notable exceptions to this rule do occur, however, such as in lodgepole pine stands that regenerate in excessively high densities on marginal sites in North America. This assumption is important because a measure of site productivity is integral in any yield projection system.

(2) The relationship between volume or biomass and dominant height is constant across a reasonably broad range of site qualities (Eichhorn's law, 1902, cited by Assman, 1970). Variation within the relationship is primarily due to differences in density. It has been observed that stands on extremely low-quality sites can depart from the overall trends.

Height-age and site index

A vast quantity of information about the growth patterns of dominant height with age is available. Early works used graphical or simple regression techniques to put a guide, or average, curve through the center of either height–age data from inventory records or single measurements of dominant tree height and age. A particular base age was arbitrarily selected (usually 50 or 100 years) and the development of stands of higher or lower site was estimated by a simple proportional scaling up or down over the length of the guide curve. These so-called 'anamorphic' curves assume that the inflection points occur at the same age on all sites, and that young trees or stands will grow to become older trees or stands in the sample. The site-index of a stand could be determined by locating the average dominant height and age and projecting forward or tracing backwards in time along a curve to the base age. The estimated height at the base is site index. A major problem pointed out most recently by Monserud (1985) is that the assumptions of regression are rarely met in single-measurement approaches. Primarily old high-site plots are rarely found, which results in an overestimate of early growth, and an underestimate of growth later in stand development. In addition, the assumption of anamorphism is usually not a realistic depiction of height growth across sites.

More recent works have relied on stem analysis data of individual trees (e.g. Monserud, 1984) or remeasured permanent plots (Alder, 1975) to develop height–age curves in which the shape changes across sites (so-called polymorphic). These types of the data are required because actual height at the chosen base age (site index) must be known or readily estimated for each plot or tree. Height (H) then becomes a function of both age (A) and site index (S).

Examples. Numerous model forms have been proposed to describe the development of height over age, ranging from relatively simple multiple linear regressions to very complex and highly nonlinear forms of sigmoid shape. One of the more common (but not necessarily the best) nonlinear forms which has parameters that are easily interpreted is the Chapman–Richards (CR) function, also known as the generalized Von Bertalanffy (Richards, 1959). It has three parameters of the form[1]:

$$DH = B1 * [1-\exp(-B2 * A)] ** B3 \qquad (1)$$

where DH is dominant height, A is age, and $B1$, $B2$, $B3$ are parameters.

Note that $B1$ is the asymptote, $B2$ is the rate, and $B3$ determines the vertical location of the inflection point, which is mathematically constrained to be between zero ($B3 < 1.0$) and 0.37 of the way to the asymptote ($0.37 = 1/e$, e = base of natural logarithm). The three-parameter form can provide a guide curve for anamorphic systems. Proportional site curves are generated by dividing $B1$ by the height predicted at index age by the guide curve and substituting $B1' * S$ for $B1$. This results in:

$$DH = B1' * S * [1-\exp(-B2 * A)] ** B3 \qquad (2)$$

Polymorphic curves have been developed using the Chapman-Richards formulation by making one or more of the parameters functions of site index and/or age. Alder (1975) used the following equation form and permanent sample plot data to model *Pinus patula* in Africa:

$$DH = B1 * \underbrace{[1-\exp(-B2*S)]}_{f1} * [1-\exp(\underbrace{-B3*A)]}_{f2} ** [\underbrace{\exp(B4+B5/S}_{f3}] \qquad (3)$$

where $f1$, $f2$, and $f3$ are functions corresponding to $B1$, $B2$, and $B3$, respectively, in Eqn. (1). Another form used by numerous researchers is a five-parameter Chapman-Richards (see Monserud and Ek, 1976) where $f1$ and $f3$ are power functions in S.

$$DH = (B1*S**Bs) * [1-\exp(-B3*A)] ** (B4*S**B5) \qquad (4)$$

From many results, the function that governs the location of the inflection point ($f3$) often decreases in value with decreasing site. That is, the inflection point becomes a lower proportion of the distance to the asymptote for the site curve determined by $f1$. Caution should be exercised here because, as seen in Alder (1975), the value of $f3$ can be estimated as less than the critical value of 1.0. When $f3$ is less than 1.0, the low-site curves can become concave down and cross other site curves at young ages. Rather than real growth patterns, this is likely due to either weak data for low sites at young ages, or an overextrapolation of the curves.

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[1] Standard FORTRAN symbology is used in equations throughout this paper. For example, * = multiply, ** = exponentiate and exp = e, base of natural logarithms.

Prediction of site index

All of the above models predict height as a function of site index and age. However, a need has arisen for equations to predict site index as a function of height and age, not only for efficiency in computer applications but also since Curtis et al. (1974) demonstrated the statistical considerations of changing the dependent variable from height to site index. In anamorphic forms, site index is solved for mathematically or a 'table-lookup' approach is used since true site index is unknown. Curtis et al. (1974) showed that this is an incorrect statistical procedure but necessary for systems derived from single-measurement data. As an example, Eqn. (2) can be solved for S and the site index prediction equation is:

$$S = \mathrm{DH}/B1' * [1 - \exp(B2 * A)] ** B3 \qquad (5)$$

In polymorphic forms, however, separate site index prediction equations are necessary, both for statistical considerations and because explicit solutions for site index are rarely possible (although numerical routines can resolve the latter problem). Experience shows that adequate site index prediction equations are much more difficult to develop than height–age models (Monserud, 1984). They usually are multiple regressions of site index on various transformations and combinations of height and age.

Other stand attributes

After the development of an adequate height growth model, other stand attributes can be related to dominant height because site quality is not usually an important independent variable in these other relationships. A single curve can then depict the average development of stands at a particular dominant height. The age at which the stand reaches that height is then determined by the site curve. Alder (1980) suggests the power or allometric relationship to relate volume (V) or biomass (B) to dominant height because it is usually upward-sweeping and theoretically passes through 1.3 m at breast height age of 0. That is,

$$V \text{ or } \mathrm{B} = B1 * (\mathrm{DH} - 1.3) ** B2 \qquad (6)$$

Fitting Eqn. (6) to data commonly accounts for about 55–70% of the variation in the dependent variable. Most of the residual variation can be accounted for by including a density measure such as basal area. A measure of density is not included in this study because density is difficult to project over time since it is strongly influenced by mortality. Adequate mortality modeling usually requires substantial permanent sample-plot data measured over time to develop sophisticated dynamic modeling procedures. The yield curves in this paper are thus the projection of average-density plots.

MODEL DEVELOPMENT

Data collection

Twenty plantations were sampled using a stratified design based on site productivity and stand (or 'module') age. Areas were classified in each leucaena plantation based on three subjectively defined productivity classes; poor, average and good. A 10×10-m permanent plot was installed in each commonly occurring productivity class and each age class represented within a plantation. A total of 87 plots in 20 National Electrification Administration (NEA) or Farming System Development Corporation (FSDC) plantations were sampled across a wide area at the Philippines. The plantations were all established with trees at 1×1-m spacing and ranged in age from 14 to 46 months.

On each plot, the largest-diameter tree was cut down so that its total height could be precisely determined. This tree served as the top-height tree for the plot. The diameter at breast height (DBH) was measured for all trees in the plot. Total height was recorded for a subset of 20 trees which were identified with tags or paint for future remeasurements. We were subsequently able to obtain additional data for use in this study collected from other leucaena plantations in the Philippines (by A.V. Ravila and M.C. Gregorio, College of Forestry, University of Philippines at Los Baños, undated). This data included 157 plots established in plantations ranging in age from 6 months to 10 years. Thus, these new data would help assess the effect of a wider range in ages on the shape of the site index curves and consequent yields to be examined. Their definition of dominant height (average height of dominants) was somewhat different than that previously described for this study (top height), therefore we compared dominant height versus age for their 157 plots to the top height versus age of the 87 plots established for this study. No apparent difference existed between the two data sets in the common range of age data, even though a different portion of the population was used as an indication of dominant height. No statistical comparison was attempted because of the different ranges of data. Note that growth trends of different measures of dominant height are often parallel for much of the range in ages (see Burkhart and Tennet, 1977). It follows that biases in curve shape introduced by combining the two data sets here are probably trivial.

Plot summary calculations

The tree raw data were keypunched and screened for obvious errors. A relationship was developed to predict height (H) of trees for which only diameter was measured. Linear least-squares regression was used to fit the natural logarithm of the following model for each plot:

$$H = \exp[B1 + B2/(\mathrm{DBH}+1)] \tag{7}$$

where H is total height, and DBH = diameter breast height (at 1.3 m).

The height–tree data were well distributed across the range of diameters in each plot so the above model performed well and explained from 68 to 92% of the variation in height with negligible residual loss.

Equations were required to predict volume (V) for each tree. No trees were measured for volume in this study so the following modification of the volume equation of Kanazawa et al. (1982) was used:

$$\text{Total stem volume} = 0.5 * (\mathrm{DBH} ** 2) * H \tag{8}$$

with total stem volume is given as m^3 and both DBH and H in m.

Kanazawa et al. (1982) originally developed the volume equation for leucaena using plot-summary data from northern Mindanao Island in the Philippines. It was of the form:

$$V = 0.5(\mathrm{DBH}^2)\bar{H} \tag{8a}$$

where V is total plot volume (m^3 ha^{-1}), DBH^2, the sum of squared DBH's (m^2) on the plot, and $\bar{H}$ the average height (m).

It was modified for use as an individual-tree volume equation here to minimize the possible effect of different distributions of diameter and height in this study. Note that the estimate of the form factor portion of the equation (0.5) closely coincides with the value of 0.48 reported by Lu and Hu (1981) for trees planted at an initial density of 40 000 per ha in Taiwan. Most of the plantations sampled in this study were established at that spacing. Lu and Hu also found form factors of 0.52, 0.52, 0.50, and 0.55 for initial densities of 2500, 5000, 20 000 and 40 000 trees per ha, respectively. The parameters should be validated from future data taken in the Philippines but Eqn. (8) should provide adequate estimates until such additional information becomes available. Total biomass (B; kg) was measured on 223 trees, representing most of the plantations. The data were combined and a single equation was used for all trees. The model selected was the following commonly-used 'allometric' relationship:

$$\mathrm{B} = 0.1189 * \mathrm{DBH} ** 2.4452 * H ** 0.3002 \tag{9}$$

with DBH given in cm and H in m. Nonlinear regression was used to estimate the parameters and the model explained 97% of the variation in total biomass (standard error of estimate = 3.185 kg). The results of the powers of DBH and H were surprising as they are normally closer to 2.0 and 1.0, respectively. The inclusion of leaves and small branches in the relationships may have changed the expected results.

Top height

The first growth relationship needed was one expressing dominant height as a function of age and site index. Several measures of dominant height are suggested in the literature but the one gaining most attention currently is top height. This measure is defined as the average height of the 100 largest-diameter trees per ha. The use of diameter lessens the subjective classification of dominant trees but does increase the possibility of including damaged trees. In this study, top height is the height of the largest-DBH tree on the 0.01-ha plot.

An anamorphic system of site curves was the only possibility for this study, since remeasurement of the plots has not occurred and true site index is unknown. This recognizes the potential bias that can occur in temporary plot height-growth analysis as recently demonstrated by Monserud (1985) for interior Douglas-fir in North America. As mentioned previously, he found that temporary plot curves are often too steep early and too flat beyond index age because average site index usually declines with age. He attributed the negative correlation between age and site index to the early harvesting of the better sites. Hopefully this potential bias is minimal here because no harvesting had occurred in the leucaena plantations prior to collecting the data.

The Chapman–Richards function was used in the analysis. Nonlinear regression was used to estimate parameters for the guide curve through the data. The results were:

$$\mathrm{DH}=13.786*[1-\exp(-0.337*A)]**0.9615 \tag{10}$$

About 57% of the variation was explained, but what is more important is that the coefficient that governs the inflection point (0.9615) was less than 1.0, causing a slightly concave-down shape and no inflection point. This result is at variance with common knowledge and, in addition, the predictions at early ages were higher than those shown by Cadiz (1976). We feel that these results were due to the small amount of age data below 10 months. Insufficient data for young ages can have an extreme effect on the $B3$ parameter. The results of Alder (1975) suggested that the value of this parameter is often approximately 1.4. The model was refitted to the data with $B3$ fixed at 1.4 and the coefficients became:

$$\mathrm{DH}=12.862*[1-\exp(-0.0502*A)]**1.4 \tag{11}$$

The r^2 values are 57.2 and 56.6% for models (10) and (11) respectively, and the corresponding standard errors of estimate are 1.97 and 1.98 m. The predicted height at selected base age of 36 months is 10.0 m. The inclusion of Eqn. (11) into the form of Eqn (2) results in:

$$\mathrm{DH}=1.2862*S*[1-\exp(-0.0502*A)]**1.4 \tag{11a}$$

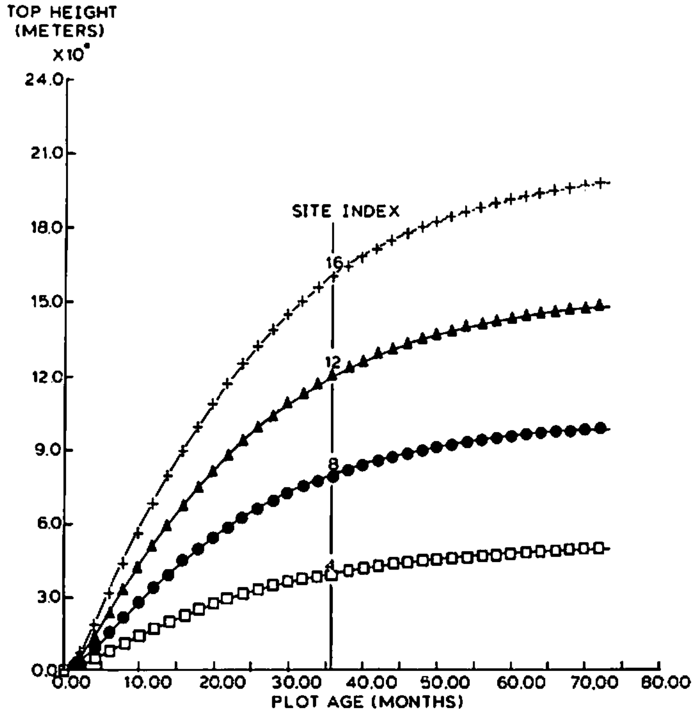

Fig. 2. Site index curves for leucaena in The Philippines. Site indexes 4, 8, 12 and 16 (base age 3 years) are shown. Curves are predicted by Eqn. (11a) in the text.

The shapes of four selected site curves derived from the combined data are shown in Fig. 2.

Prediction of site index

To predict site index of a plot, Eqn. (11a) is solved for S. This results in the equation:

$$S = \mathrm{DH}/1.2862 * [1 - \exp(-0.0502 * A)] ** 1.4 \qquad (12)$$

The overall average site index for the combined data set is 10.05 ± 2.26 m.

Volume and biomass

The allometric relationship (Eqn. 5) was first investigated to relate volume per ha and biomass per ha to top height. The model fitted very well at low heights but was biased high for top height above 10 m. Correspondingly, it extrapolated on a very steep projection judged to be unrealistic. The following

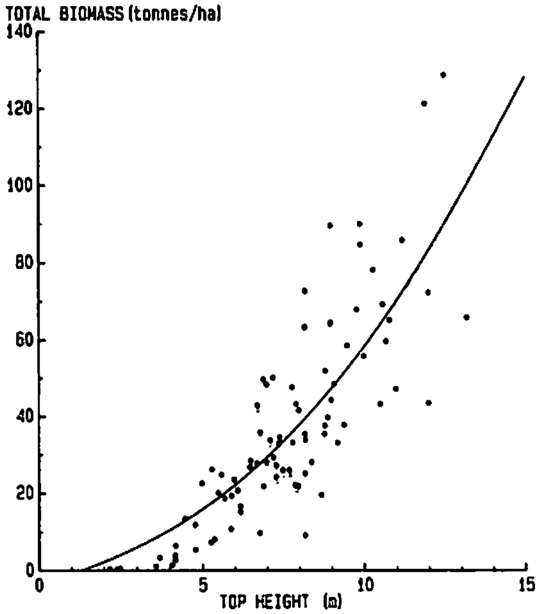

Fig. 3. Biomass vs. top height for leucaena plantations. Constrained logistic fit is also displayed.

logistic model was fitted to the data and explained significantly more of the variation in both dependent variables:

$$V \text{ or } B = B0 * [1 - \exp(-B2 * (DH - B3))] / [1 + B1 * \exp(-B2 * (DH - B3))] \tag{13}$$

This model is constrained to pass through the point (B3,0.0). The value of $B3$ can either be estimated or fixed at a theoretical value such as 1.3 m (breast height) for total volume. Normally the logistic model is sigmoidal with an inflection point at one-half the distance to the asymptote, $B0$. Yield/top height relationships are usually upward-sweeping (exponential) and thus display no evidence of an inflection point. However, the inflection point will occur beyond the end of the data, if $B0$ is fixed at a value about double the average yield at the upper range of the data, and extrapolations will be conservative because the curve will begin to bend over. Conservative extrapolation in most situations is a desirable characteristic. Figures 3 and 4 show the results of the fits of Eqn. (13) for biomass and volume, respectively. Several values of $B0$ were investigated, with little change in location of the curve within the range of the data. Extrapolations did, however, change with the various asymptotes. Values of $B0$ ultimately selected were 250 m^3 ha^{-1} for total volume and 300 tonnes ha^{-1} for biomass. The parameter estimates for the two independent variables

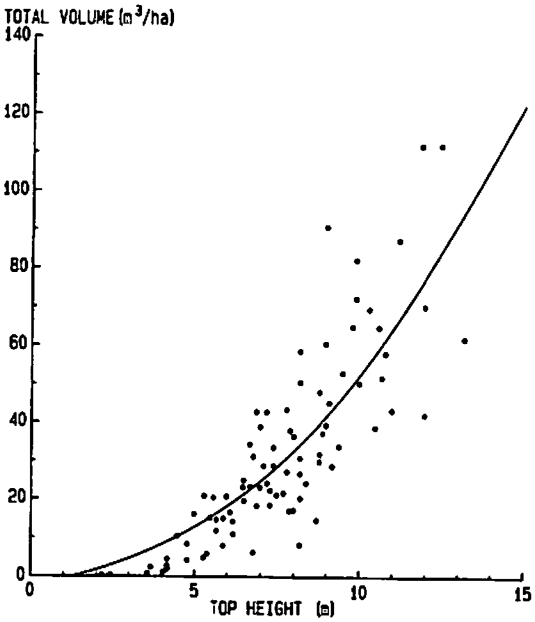

Fig. 4. Cubic volume vs. top height for leucaena plantations. Constrained logistic fit is also displayed.

are given in Table 1. This formulation provided a significantly better fit to the data than the allometric relationship (Eqn. 5).

Yield versus age

The volume-height and biomass-height functions were used with the site curves (Eqn. 11a) to generate the final yield. Table 2 summarizes the yields for three sites. The culmination of total volume MAI occurs at approximately 35 months. Total biomass culminates at 34 months. Note that on the highest sites extrapolations of the volume-height curve were necessary to reach cul-

TABLE 1

Parameter estimates for volume per ha and biomass per ha versus top height

Independent variable	Units	B0*	B1	B2	B3*	Percent explained variation	Standard error
Volume	m^3 ha^{-1}	250	25.2760	0.2383	1.3	68.6	13.9
Biomass	t ha^{-1}	300	18.7370	0.2011	1.3	67.2	15.1

*Fixed.

TABLE 2

Growth and yield of leucaena in the Philippines for site indexes 5, 10 and 15

Age[1]	Dominant height[2]	Biomass[3]		Total volume[4]	
		Cumulative	MAI[5]	Cumulative	MAI
Site index 5					
0	0.00	0.00	0.00	0.00	0.00
5	0.78	0.00	0.00	0.00	0.00
10	1.75	1.43	1.71	1.07	1.28
15	2.63	4.61	3.69	3.51	2.81
20	3.39	7.74	4.65	6.00	3.60
25	4.02	10.66	5.12	8.37	4.02
30	4.52	13.25	5.30	10.53	4.21
35	4.93	15.49	5.31	12.43	4.26
40	5.25	17.38	5.21	14.05	4.21
45	5.51	18.94	5.05	15.40	4.11
50	5.71	20.22	4.85	16.52	3.96
55	5.87	21.25	4.64	17.42	3.80
60	5.99	22.07	4.41	18.15	3.63
Site index 10					
0	0.00	0.00	0.00	0.00	0.00
5	1.56	0.82	1.97	0.61	1.47
10	3.50	8.21	9.85	6.38	7.65
15	5.27	17.49	13.99	14.14	11.31
20	6.78	27.76	16.65	23.25	13.95
25	8.03	38.13	18.30	32.86	15.77
30	9.05	47.89	19.15	42.19	16.88
35	9.86	56.61	19.41	50.72	17.39
40	10.50	64.12	19.24	58.15	17.45
45	11.01	70.41	18.78	64.43	17.18
50	11.42	75.57	18.14	69.71	16.71
55	11.73	79.75	17.40	73.82	16.11
60	11.98	83.10	16.62	77.19	15.44
Site index 15					
0	0.00	0.00	0.00	0.00	0.00
5	2.34	3.51	8.41	2.66	6.37
10	5.24	17.34	20.80	14.01	16.81
15	7.90	36.97	29.58	31.77	25.42
20	10.18	60.24	36.14	54.29	32.58
25	12.05	84.10	40.37	78.20	37.53
30	13.57	106.06	42.42	100.34	40.14
35	14.79	124.83	42.80	119.04	40.81
40	15.76	140.14	42.04	133.97	40.19
45	16.52	152.28	40.61	145.53	38.81
50	17.12	161.76	38.82	154.36	37.05
55	17.60	169.11	36.90	161.07	35.14
60	17.97	174.79	34.96	166.16	33.23

[1] Age in months.
[2] Average height of dominants and codominants (m).
[3] Metric tonnes per ha.
[4] Total volume of all stems (m^3 ha^{-1}).
[5] MAI = Mean annual increment.

mination. Due to the contrained logistic volume–height model, extrapolations are conservative and begin to decline after reaching half the asymptote (125 $m^3 ha^{-1}$). As a result, the culmination ages may be early but seem 'reasonable'. Varying culmination ages across sites for total volume and biomass are likely to occur with site curves based on permanent sample data.

SUMMARY AND RECOMMENDATIONS

The results reported here provide useful growth and yield information for the management of leucaena in the Philippines, and perhaps the site index equation (Eqn. 12) and yield tables (Table 2) could be applied directly in other geographic areas where leucaena grows in similar edaphic and management conditions. The methodology outlined in this paper should be widely applicable to a variety of tree species grown in even-aged stands. We suggest one change in the methodology: remeasurement data should be collected from permanent sample plots. These data would allow the tracking of real growth trajectories through time and provide for the development of polymorphic site index curves. Potential differences in stand development over time for varying site productivity classes could thus be expressed.

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