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SPATIAL VARIATION IN CONIFER FOLIAR NUTRIENT CONCENTRATIONS

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ABSTRACT

The spatial distribution of foliar nutrient concentrations in Douglas-fir and grand fir species was investigated in a study conducted in northern Idaho. Mensurational data were collected from 55 Douglas-fir and 69 grand fir individual trees from 14 one-half acre plots. Foliage samples from the third whorl of each tree were taken, and tree positions were mapped using a Global Positional System (GPS). A stochastic modeling method, the sequential conditional Gaussian simulation, was used to study tree volume and foliar nutrient flow dynamics. Spatial variations were analyzed between individual tree measurements and nitrogen, potassium and boron concentrations under different fertilization treatments. The study showed that there are species differences in the distribution of foliage nutrient concentrations and tree biomass. Potassium and boron concentrations were lower in plots treated with high nitrogen rates. Boron concentration values were higher on the untreated plot.

INTRODUCTION

Nutrient deficiency in Inland Northwest forests has been demonstrated in past studies (Zasoski, 1979; Lavender and Walker, 1979). Nitrogen is commonly the limiting nutrient for tree growth. According to results of the Intermountain Forest Tree Nutrition Cooperative (IFTNC), foliar nitrogen concentrations in Douglas-fir trees were below the adequate level of 1.6% in the Inland Northwest region (Mika et al, 1994). Without adequate nitrogen, tree growth is significantly reduced.

IFTNC has been working in the Inland Northwest since 1980. Its primary focus has been fertilization of forest tree species using nitrogen and potassium fertilizers. Study sites are located throughout the region. Data on growth response has been collected extensively for Douglas-fir (*Pseudotsuga menziessi* var. *glauca*). Other tree species such as ponderosa pine (*Pinus ponderosa*), grand fir (*Abies grandis*), western larch (*Larix occidentalis*), and lodgepole pine (*Pinus contorta*) also are being studied (Moore et al, 1991). According to IFTNC tree growth response data, nitrogen fertilization has produced growth increases of trees across all regions in the area (Mika et al, 1992).

One way to examine tree growth response is through foliage nutrient analysis. Substantial numbers of foliage tree samples have been collected in most of the IFTNC trial plots under different fertilizer treatment regimes. Nutrient spatial distributions for different tree species is being investigated as a means to assist in tree nutrition management decisions. This paper presents the results of a study to evaluate the spatial variation in foliar nutrient concentrations of Douglas-fir and grand fir, as well as tree biomass distribution. We also evaluated nutrient distribution maps by overlaying data points not analyzed in the training data set. The research site is one of the IFTNC installations located near Bovill, Idaho. Preliminary spatial analysis results will help relate tree biomass and nutrient distributions for this particular site that can be expanded in the future for developing operational foliage sampling designs.

METHODS AND MATERIALS

Study area geology and vegetation

The study site is located in Latah County, 1 kilometer east of Bovill, Idaho. The area is 600 by 600 meters square, or 36 hectares (89 acres), that is managged by Idaho Department of Lands. Topography is gently sloping with elevations ranging from 849 to 1052 meters (2785 to 3451 feet). Geology is characterized by a mixture of parent materials: colluvium, stream and lake deposits overlaying granite bedrock (Johnson and Raines, 1995). Figure 1 shows the study area and geology.

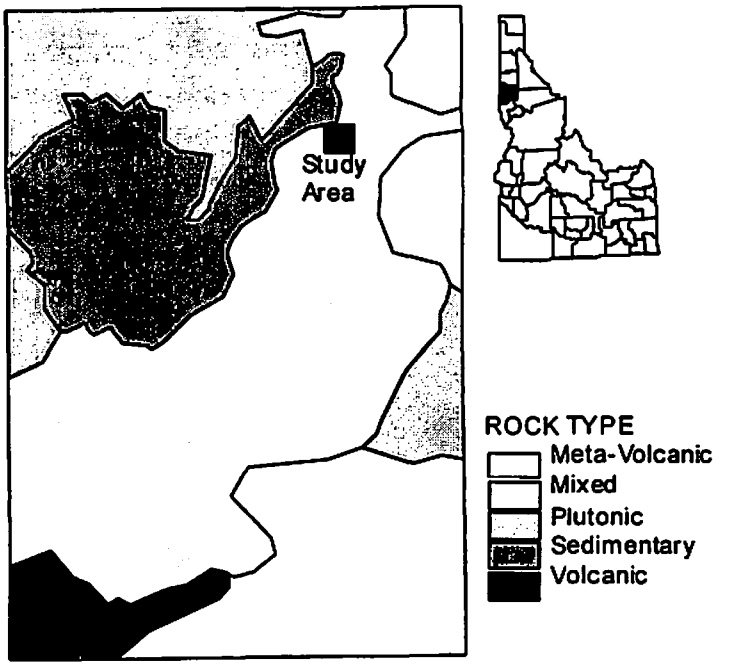


Figure 1. Map of the study area and geology near Bovill, Idaho

The stand is comprised mainly of Douglas-fir and grand fir. Western larch and white pine are minor species in the study area. The research site was established in 1994 as a nitrogen and potassium multi-rate study trial composed of 14 one-half acre plots. A description of the different plot treatments is provided in Table 1. Figure 2 shows the location of the plots within the study area.

Table 1. Fertilizer treatment rates for nitrogen (N) and potassium (K) in the study area

Plot number	Treatment
Plot 1	512.1 pounds of N & 341.4 pounds of K
Plot 2	600 pounds of N & 200 pounds of K
Plot 3	512.1 pounds of N & 58.6 pounds of K
Plots 4, 8, 10, 11, and 12	300 pounds of N & 200 pounds of K
Plot 5	Control
Plot 6	300 pounds of N & no K
Plot 7	87.9 pounds of N & 58.6 pounds of K
Plot 9	300 pounds of N & 400 pounds of K
Plot 13	87.9 pounds of N & 341.4 pounds of K
Plot 14	No N & 200 Pounds of K

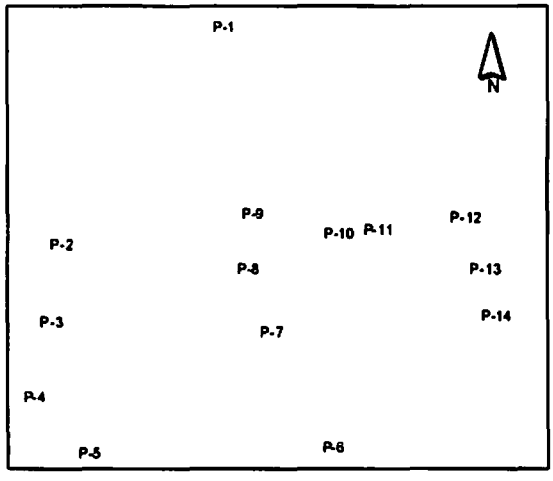


Figure 2. Sample plot locations within the study area (installation 335)

Data collection

Tree measurement. Individual tree measurements were taken in 1994 for all trees in the plots. The tree data included: diameter at breast height, basal area, height, crown closure, age; and physical site characteristics: slope, aspect, elevation, and habitat type. The DF trees ranged from 56 to 89 years old, diameter between 10.75 and 24.94 inches, and height between 72.6 and 108.6 feet. For GF trees ages ranged from 52 to 100 years old, diameter between 10.46 and 21.70 inches, and height between 73.7 and 117.4 feet.

Foliage. In 1995, samples of foliage were collected for Douglas-fir and grand fir to assess foliar nutrient concentrations. The samples were taken from the third internode (whorl) down from the top of the tree from five selected dominant or co-dominant trees for each of the two species in each of the 14 plots of the study area (IFTNC, 1995). A total of 55 DF and 69 GF sample trees were sampled at this time (the requirement of five DF per plot was not always met at some locations). Foliage samples were dried and sent to Scotts laboratory for tissue analysis. The foliage analysis included aluminum, boron, calcium, copper, iron, potassium, magnesium, manganese, molybdenum, nitrogen, sodium, phosphorus, and zinc concentrations (Scotts, 1996).

Tree location. To map the location of each tree on the ground, a Global Positional System was utilized (Trimble, 1992). The receiver was placed at the base of each sampled tree. Coordinates were expressed as longitude and latitude. These then were converted to Albers metric projection coordinates.

Data analysis

Statistics. The data set for each species was analyzed independently. Variability within

the stand was analyzed. The first step was to conduct a multivariate statistical analysis in order to identify the more important nutrient variables for subsequent modeling. Principal component analysis and factor analysis are statistical procedures dealing with situations where a number of variables have a joint multivariate distribution and lead to a reduction in the number of variables selected for further study (Johnson and Wichern, 1992; Laar, 1987). The multivariate statistical analysis was achieved using SAS (1993). Each data set of 13 nutrients was analyzed. For the DF data set, the first principal component, nitrogen, produced one of the largest positive values (scores) among the other nutrients, 0.327; and potassium produced the largest negative score -0.404. Thus, the large difference in these scores indicates that nitrogen and potassium are important nutrients in this multivariate data set. In this manner each component was analyzed. For the DF data set, seven important nutrients were identified: nitrogen, potassium, molybdenum, sodium, zinc, iron, and boron. For the GF data set, seven nutrients also were identified: nitrogen, potassium, iron, sodium, boron, zinc, and phosphorus. The results are shown in Tables 2 and 3 below.

Table 2. Principal component analysis for Douglas-fir

	PRIN1	PRIN2	PRIN3	PRIN4	PRIN5
Al	-0.147	0.287	0.253	0.451	-0.210
B	0.138	-0.041	0.144	0.340	0.713
Ca	0.241	0.364	0.119	-0.279	0.001
Fu	0.184	-0.176	0.524	0.039	0.121
Fe	-0.210	0.198	0.268	0.486	-0.155
K	-0.404	0.115	0.216	-0.320	0.003
Mg	0.354	0.363	0.007	-0.036	0.216
Mn	0.327	0.390	-0.019	-0.162	0.010
Mo	0.106	0.506	-0.055	0.004	-0.168
N	0.324	-0.236	0.116	-0.047	-0.141
Na	-0.194	0.253	-0.107	0.204	0.268
P	-0.207	0.041	0.356	-0.359	0.386
Zn	0.161	-0.063	0.589	-0.070	-0.302

Table 3. Principal component analysis for grand fir

	PRIN1	PRIN2	PRIN3	PRIN4	PRIN5
Al	0.325	-0.171	0.319	0.158	0.294
B	0.114	0.084	0.498	-0.019	-0.325
Ca	0.270	0.229	0.387	-0.128	-0.218
Fu	-0.006	0.560	0.003	0.026	0.249
Fe	0.144	-0.294	-0.192	-0.095	0.499
K	0.314	-0.146	-0.310	0.367	-0.068
Mg	0.306	0.441	-0.174	-0.022	-0.039
Mn	0.293	-0.143	0.205	0.035	0.264
Mo	0.434	0.018	0.224	0.098	0.195
N	-0.284	0.024	0.135	0.632	0.122
Na	0.052	-0.268	-0.068	-0.282	-0.292
P	0.202	-0.057	-0.161	0.539	-0.438
Zn	0.154	0.424	-0.345	-0.008	0.105

Once these variables were identified, univariate statistics were calculated as shown in Tables 4 and 5. For subsequent spatial analysis purposes, just nitrogen, potassium, and boron were selected for this research study. Their frequency distribution plots were generated for both species (Figure 3).

Table 4. Univariate statistics for Douglas-fir data set

Statistics	B	Fe	K	Mo	N	Na	Zn
Mean	25.372	36.338	6643.400	0.322	14284.400	42.360	28.302
Standard Error	1.077	1.003	278.100	0.014	526.352	2.197	2.036
Median	25.550	34.800	6300.000	0.302	13150.000	38.100	24.900
Mode	32.700	34.300	6520.000	0.330	13100.000	34.400	29.300
Standard Dev.	7.618	7.093	1966.461	0.097	3721.874	15.535	14.398
Sample Variance	58.038	50.311	3866969.837	0.009	13852347.592	241.327	207.314
Kurtosis	-0.683	1.184	8.958	0.250	-0.138	-0.258	15.163
Skewness	0.031	0.750	2.218	0.554	0.573	0.767	3.249
Range	30.300	36.100	12130.000	0.481	16340.000	61.900	91.900
Minimum	10.000	23.000	3770.000	0.127	7360.000	22.800	12.100
Maximum	40.300	59.100	15900.000	0.608	23700.000	84.700	104.000
Sum	1268.600	1816.900	332170.000	16.085	714220.000	2118.000	1415.100
Count	50.000	50.000	50.000	50.000	50.000	50.000	50.000
Con. Level(95.0%)	2.165	2.016	558.862	0.028	1057.744	4.415	4.092

Table 5. Univariate statistics for Grand fir data set

Statistics	B	Fe	K	N	Na	P	Zn
Mean	36.205	40.537	11209.839	13207.903	49.756	1760.806	22.343
Standard Error	1.307	0.862	334.144	439.963	2.203	47.272	1.101
Median	35.250	39.700	11050.000	12700.000	49.500	1700.000	22.400
Mode	37.600	37.200	14200.000	11700.000	28.300	1700.000	15.500
Standard Dev.	10.291	6.786	2631.051	3464.274	17.344	372.221	8.667
Sample Variance	105.895	46.049	6922427.842	12001193.892	300.807	138548.519	75.115
Kurtosis	0.976	0.169	-0.357	0.882	0.207	-0.004	0.934
Skewness	0.518	0.675	0.356	0.801	0.593	0.772	0.698
Range	55.100	29.300	10800.000	17350.000	79.000	1610.000	43.810
Minimum	13.000	27.600	6800.000	7750.000	18.600	1260.000	8.290
Maximum	68.100	56.900	17400.000	25100.000	97.600	2870.000	52.100
Sum	2244.700	2513.300	695010.000	818890.000	3084.900	109170.000	1385.250
Count	62.000	62.000	62.000	62.000	62.000	62.000	62.000
Con. Level(95.0%)	2.613	1.723	668.162	879.761	4.405	94.526	2.201

Geostatistics. Geostatistical methods were used to analyze spatial dependency in each data set. The common tool used to describe spatial dependence is the variogram function, which relates the difference in paired data values to the spatial distance that separates the two sampling locations (see Appendix). Variograms are characterized by: 1) sill, the variogram value to which the function rises then levels off at a plateau (equivalent to the sample variance); 2) range, the separation distance at which the sill is reached; 3) nugget, the variogram value at zero separation distance (Isaaks and Srivastava, 1989).

Once variograms were calculated and modeled, a spatial simulation procedure, sequential Gaussian conditional simulation (sgCS), was used to map the nutrients by estimating values at unsampled locations (Deutsch and Journel, 1992). Such a "conditional" simulation will honor (reproduce) the known data values if they happen to coincide with simulation grid nodes. To describe the uncertainty in the potential nutrient levels at unsampled locations, 100 simulation passes (realizations) were generated. Each realization is intended to produce one possible map of nutrients. Further, to evaluate

simulation validity, four DF and six GF samples were reserved for testing and validation.

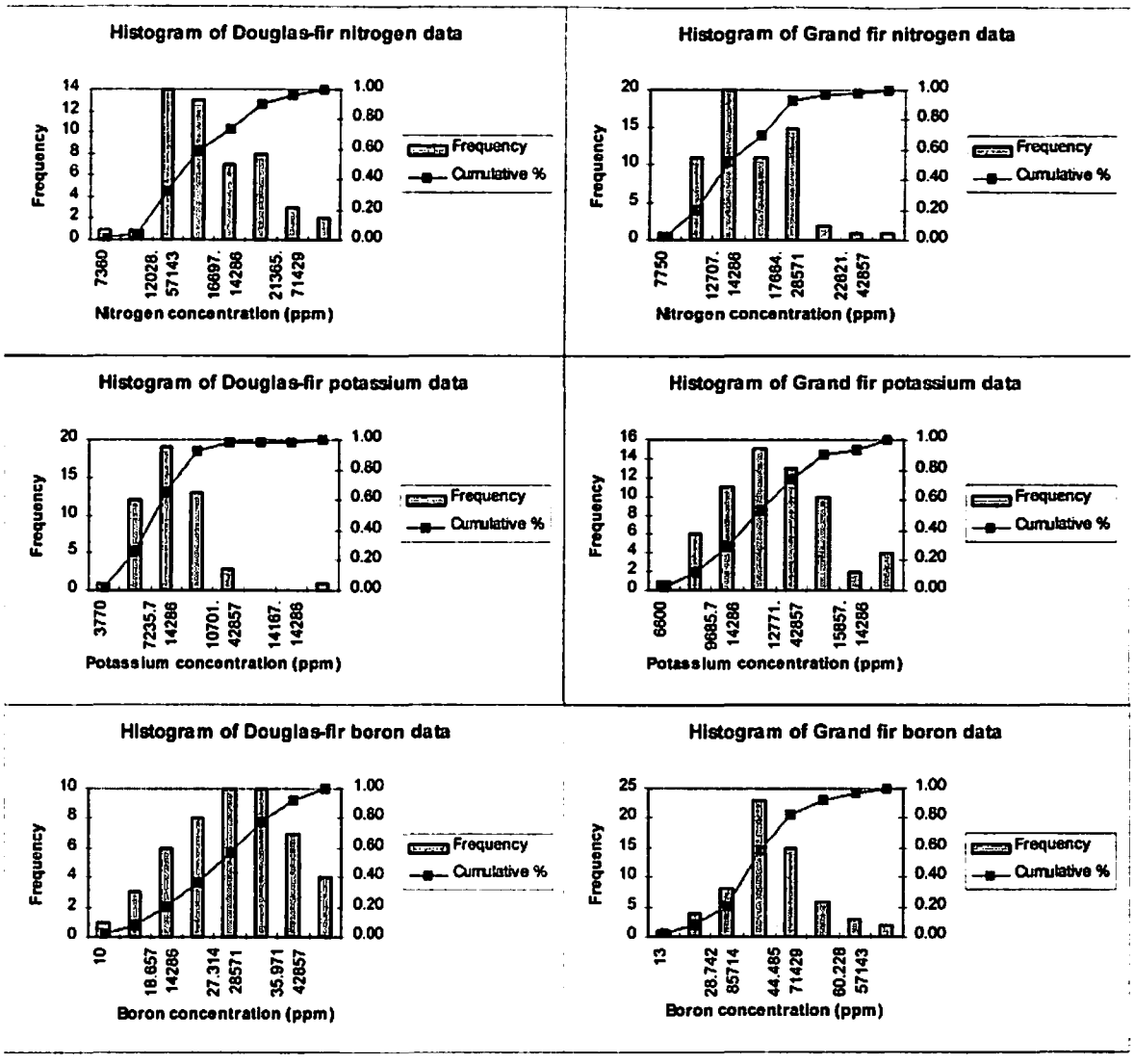
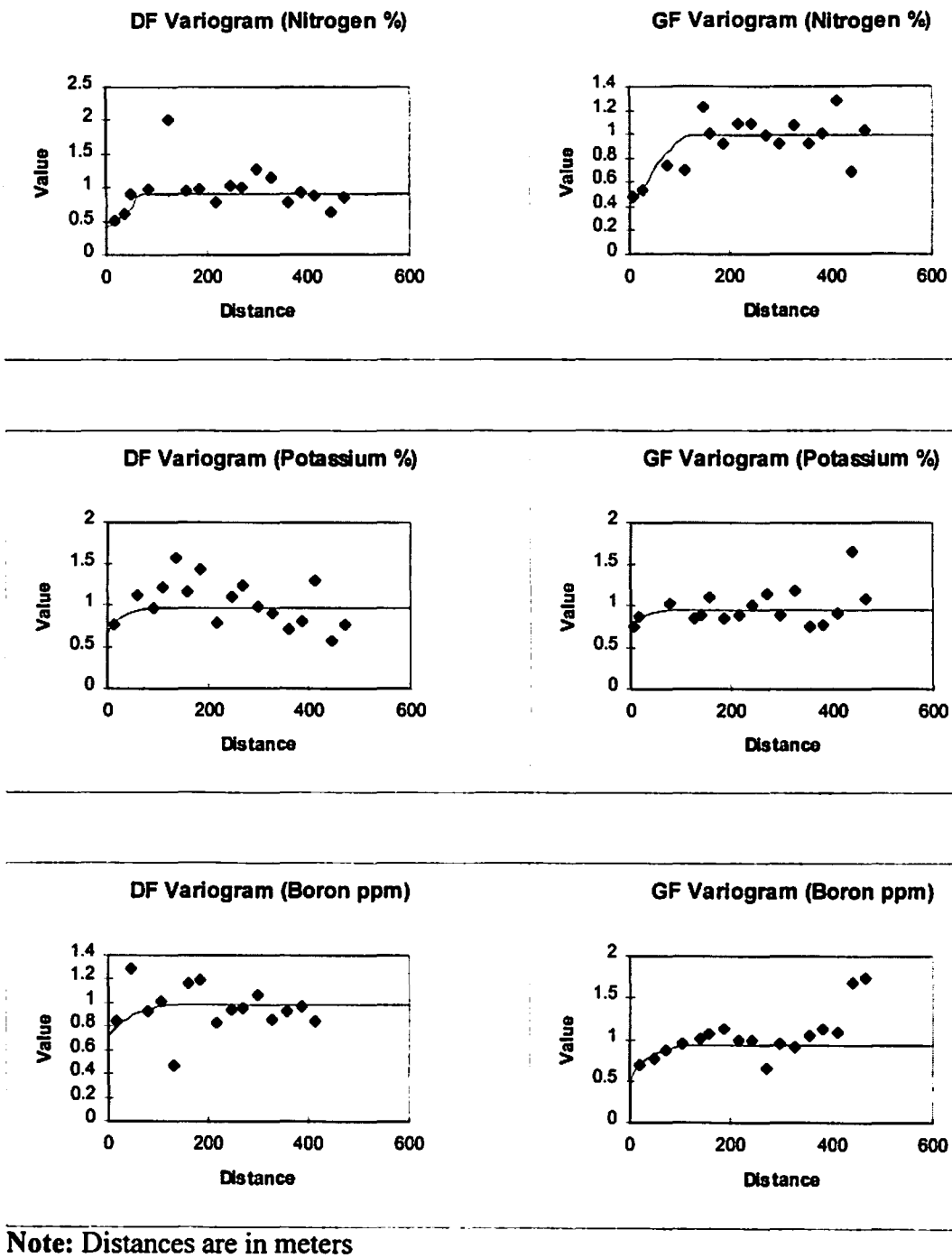


Figure 3. Histograms of nitrogen, potassium, and boron data for Douglas-fir and grand fir.

Two computer packages, one by Englund and Sparks (1991) GEO-EAS, and one by Deutsh and Journel (1992) GSLIB, were used to estimate variograms, then conduct the conditional simulations, respectively. The simulation area was 600 by 600 meters producing a grid map of 40 by 40 nodes at a regular spacing of 15 meters. Once the realization values were obtained, they were mapped using the grid module in ArcInfo (ESRI, 1995), converted to vector mode, then displayed as the simulated realization maps in ARCVIEW 2.1 (ESRI, 1993). Figure 4 presents sample variograms for the variables under study. These variograms were based on normal score transforms, the type of input needed for the sgCS routine in GSLIB (Deutsh and Journel, 1992).



Note: Distances are in meters

Figure 4. Sample variograms for Douglas-fir and grand fir data sets.

The variograms above differ in terms of nugget, sill, and range values (Table 6 is a summary of those results).

Table 6. Nugget, sill, and range values for the Douglas-fir and grand fir variograms

Species	Nutrient	Model	Nugget	Sill	Range (m)
Douglas-fir	N	Gaussian	0.45	0.525	90
"	K	Spherical	0.7	0.275	100
"	B	Spherical	0.7	0.275	100
Grand fir	N	Gaussian	0.45	0.53	130
"	K	Exponential	0.7	0.28	80
"	B	Exponential	0.5	0.48	125

RESULTS

The simulation mean and standard deviation for the 100 realizations of nitrogen, potassium, and boron for DF and GF are shown in Figures 5 and 6, respectively. The probability of exceedance for selected cutoff values of the nutrients under study is shown in Figure 7. Figure 8 is a representation of volume mean for the simulated values of both species.

We interpreted the simulated maps from a pure spatial point of view; that is, we did not use statistical correlation to make the comparisons.

Douglas-fir data. Nitrogen mean values are high toward the west, northeast, southeast, and north side of the research site (where high nitrogen fertilization rates occurred), while lowest values are toward the northwest, northeast, southwest (where control and low nitrogen fertilization rates occur), and at the center, even though plot 9 has a high nitrogen rate, (see Figure 2 and Table 1). High standard deviation values are scattered throughout the area, while pronounced low values trend from northeast to southwest. Highest potassium mean values are toward the center and in minor areas of the northeast and southwest. Low values are in those areas where high nitrogen mean values are found, while the highest standard deviation values occur throughout the area. Boron shows the highest mean values toward the west, south, and northeast sides; and its standard deviation values tend to be higher in the south portion of the study area.

Grand fir data. High nitrogen mean values are found toward the west, southeast, and northeast portions of the study area and lowest values from the center toward the southwest zone. This area of low means is more spatially continous for the GF values compared to the DF values. Highest standard deviations are toward the southeast and toward the east and north; low values occur from the center to the southwest portion of the study area. Potassium shows high values toward the southwest zone and in the center of the area. Again, its standard deviations are scattered throughout. Boron shows its highest values toward the southwest corner and to a lesser extent in the northeast corner.

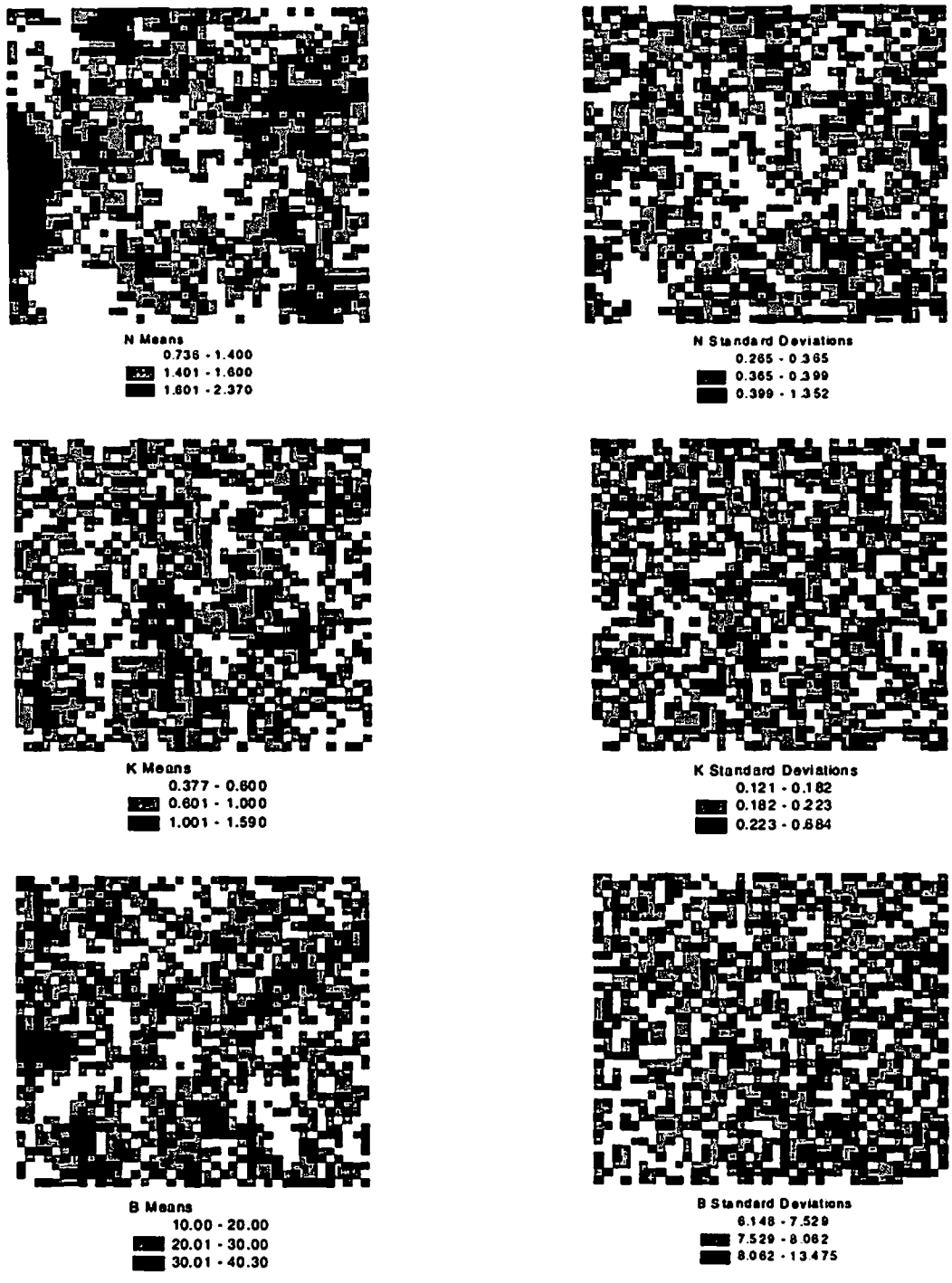


Figure 5. Douglas-fir simulated mean and standard deviation values for nitrogen (N), potassium (K), and boron (B) foliar concentrations.

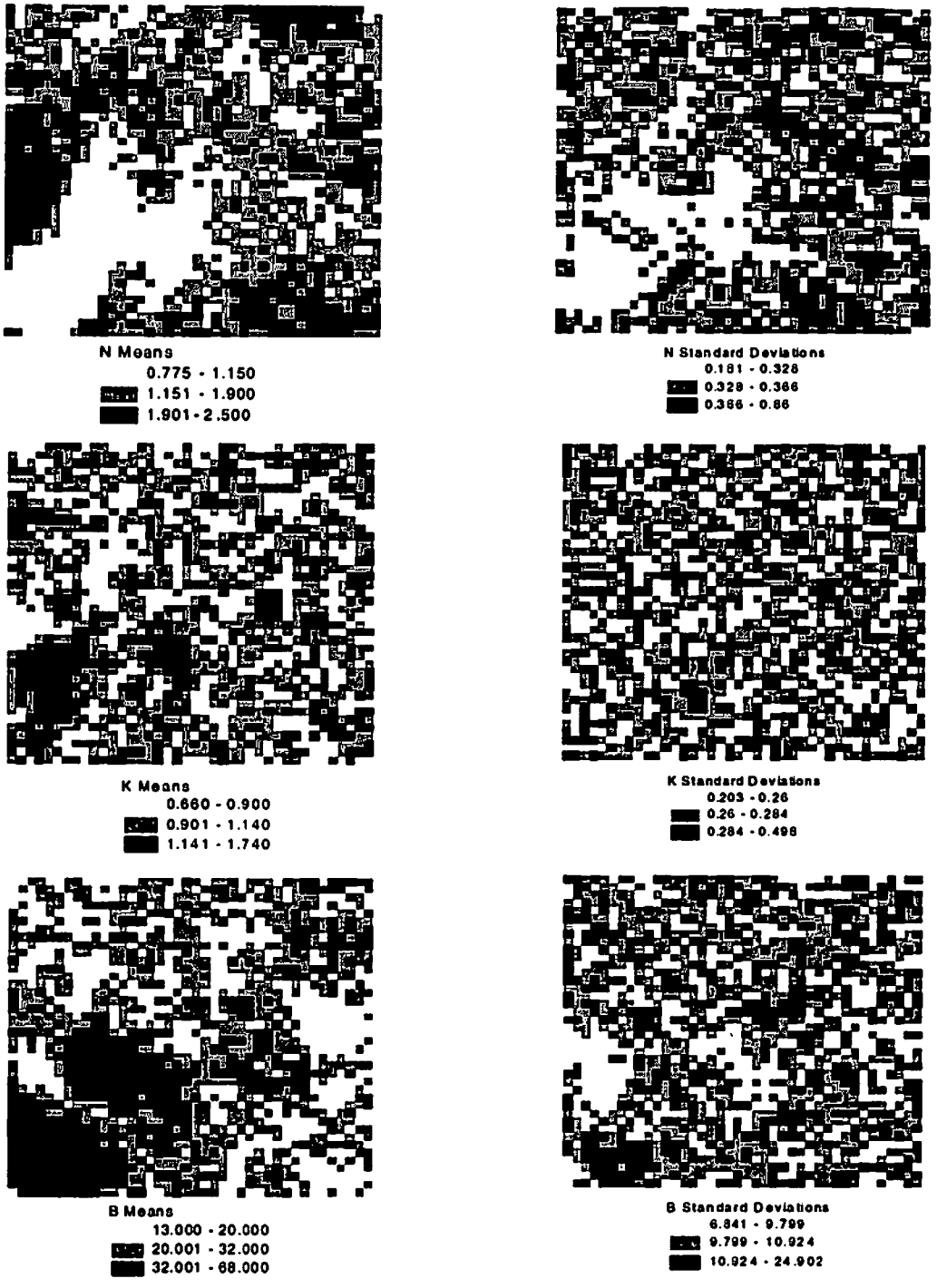


Figure 6. Grand fir simulated mean and standard deviation values for Nitrogen (N), potassium (K), and boron (B) foliar concentrations.

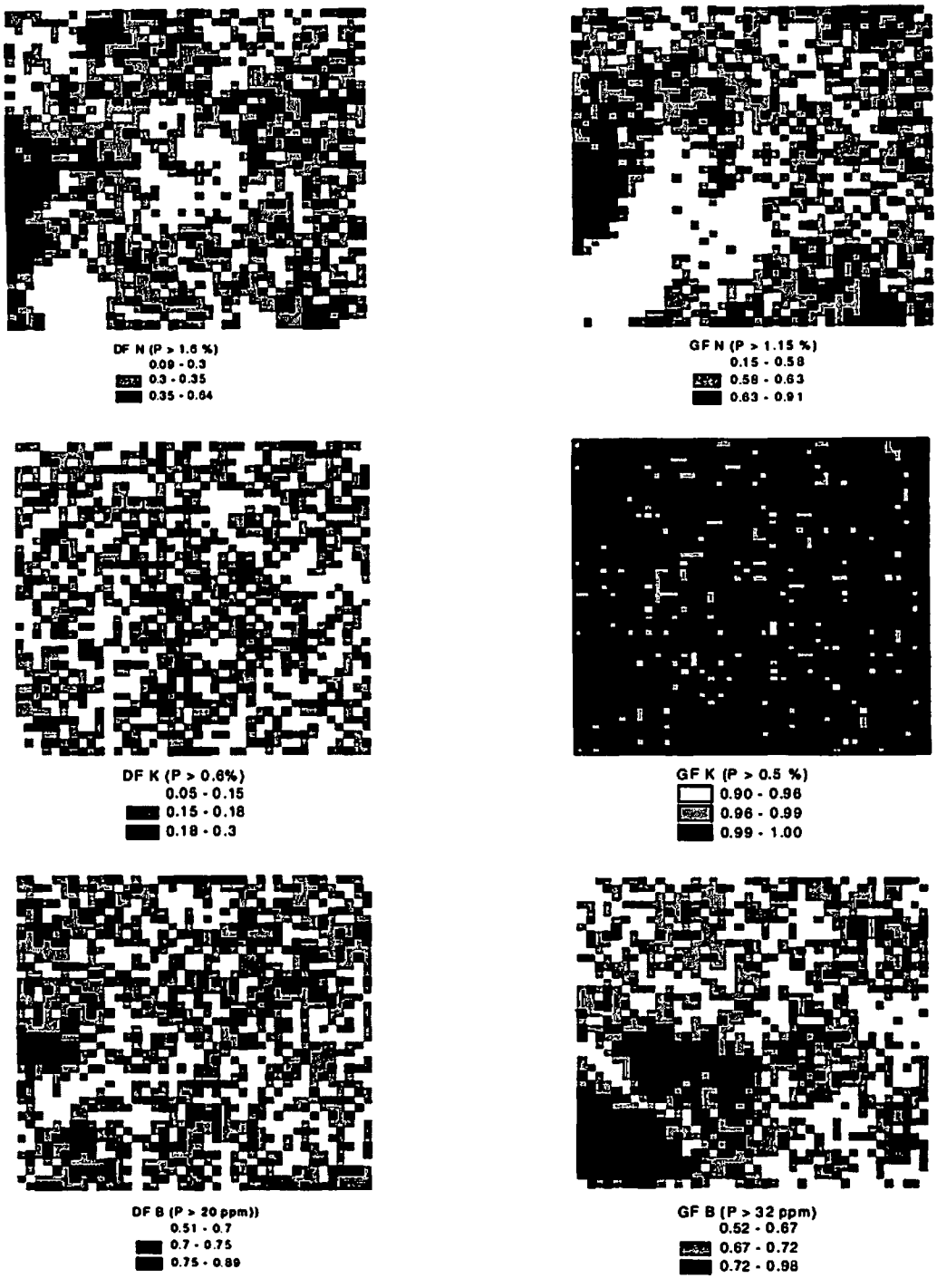


Figure 7. Douglas-fir and grand fir probability maps based on cutoff values of adequate nutrient levels for nitrogen (N), potassium (K), and boron (B) foliar concentrations.

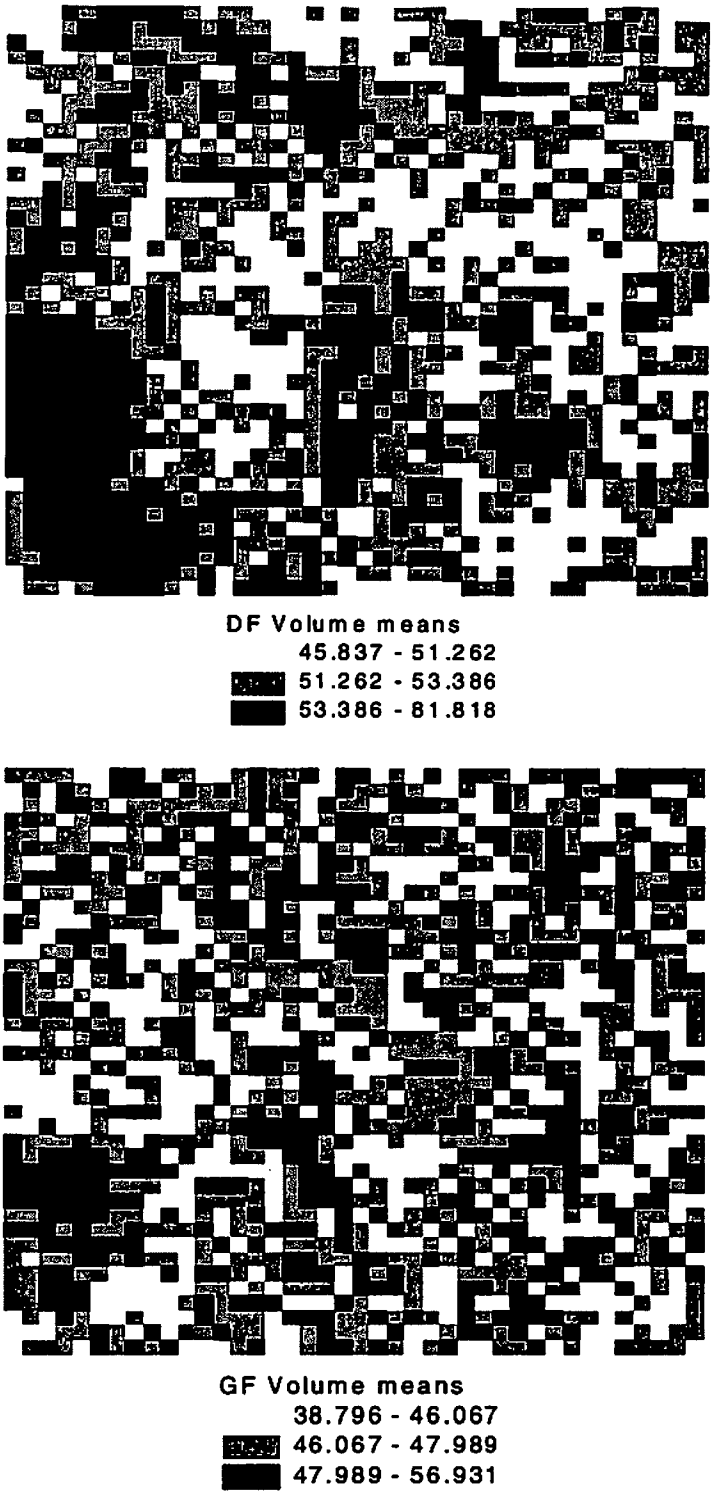


Figure 8. Douglas-fir and Grand fir volume mean values (CuFt).

In general, the DF spatial maps show that the lowest potassium mean values are found in areas having the highest nitrogen mean values, however the association was not as strong for boron. By the same token, GF data behave similarly but the distribution is wider than those of the DF data set. Also, the highest boron mean values are found near the control plot area and the lowest concentrations are found at the high nitrogen treatments.

Figure 7 represents exceedance probability values for selected cutoffs of nitrogen, potassium, and boron considerated to be adequate foliar nutrient levels (Webster and Dobkowski, 1983; Ingestad, 1979; Powers, 1983). For nitrogen DF data the highest probability that the stand will be above 1.6 % is toward the east, southwest and northeast. The lowest probability is in the center and southeast zone. This same pattern holds for nitrogen GF data. Exceedance probability values for potassium are more pronounced for the GF data set than that of the DF data. The potassium cutoff value of 0.5 % for GF is low compared to most of the simulated values, which reflect the high original data values (the minimum measured K value was 0.66%). Thus, the map of GF K ($P > 0.5$ %) shows most of the area to have a very high probability of exceeding a K concentration of 0.5%. Boron exceedance probability values are lowest in the southwest quadrant for DF, but highest for GF.

Figure 8 shows the spatial distribution of tree volumes for the study area. Volumes are highest toward the southwest, center, and some parts toward the north for the DF data. For the GF, data volume is evenly distributed across the whole area with some volume concentration toward the center and southwest zones. Comparing tree biomass and nutrient concentrations, it seems that the west side and particularly the southwest side have adequate levels of nitrogen and potassium and abundant amounts of boron for both species. Also, by field observations of plots 2, 3, and 4, where most of the highest nutrient values occur, the stand is doing well in terms of tree health and biomass accumulation.

To validate the simulation models, we left out test samples to determine how well the simulated nutrient maps agree with actual observations. Figure 9 portrays results for the four DF and six GF trees used as controls for testing the simulated model against the actual foliar nutrient concentrations. The point data set for these testing trees was overlaid on the simulated nitrogen, potassium, and boron mean values. There were some differences, but in general the simulation models performed quite well. The GF potassium true and simulated means graph shows the closest comparison of all the data sets. For example, for tree 24 the true and simulated mean values are the same. Also, the same is true for tree 20 for the GF boron comparisons. The largest differences were shown by the DF potassium true and simulated means comparison, but even here the simulated means tend to be well within one standard deviation of the true value (compare Figure 9 to Figure 5).

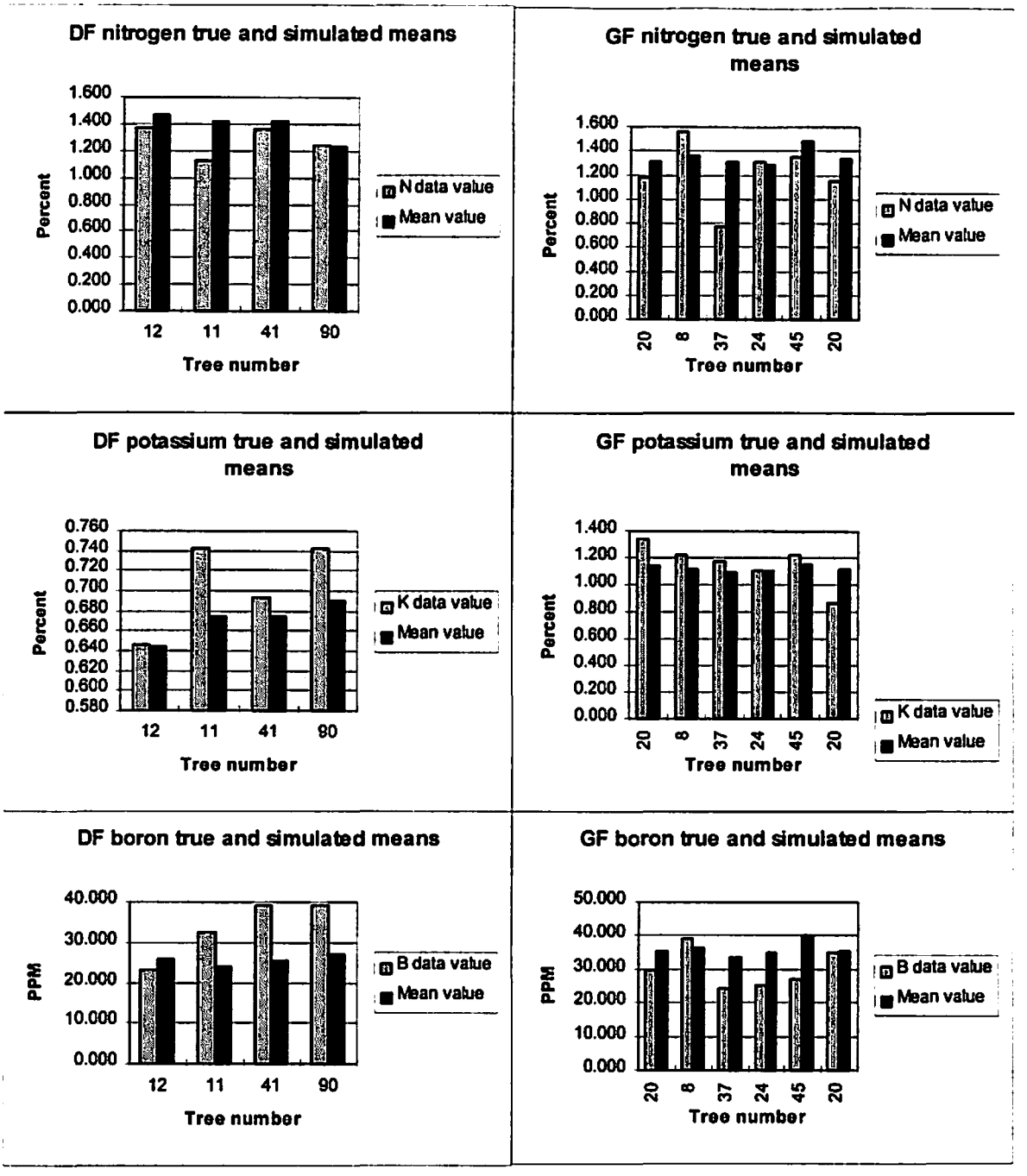


Figure 9. Douglas-fir and grand fir true and simulated mean values for nitrogen, potassium, and boron foliar concentrations.

CONCLUSIONS

A spatial analysis of Douglas-fir and grand fir foliar concentrations in the study area shows similar spatial distribution patterns, but with generally wider distributions for the GF data than for those of the DF data. When developing a foliage experimental

design, the spatial range of influence for each nutrient under study should by considered, as well as the nugget effect. For example, GF nitrogen has a longer range of influence and lower nugget effect (more spatial dependency at shorter distances) than DF nitrogen for this particular study area (See Figure 4 and Table 6). Thus, the long-term sampling plan for GF nitrogen can use more widely spaced samples (i.e., less sampling density) than that used for DF nitrogen. Typically, for a range of influence r, the minimum sample spacing can be set to about 0.3r to 0.5r and still provide adequate sampling for reliable spatial modeling. Sampling plans with minimum spacings wider than this distance will not provide sufficient spatial continuity information.

This assumes that an initial phase of sampling already has provided adequate data to compute and model variograms. Also, this study is based on one type of geological parent material, which certainly can be different at other sites across the landscape. Research should be expanded to other parent materials to understand the spatial dependency in soils derived from other types of rocks.

Spatial simulations provide a means to probabilistically describe spatial patterns and distributions of foliar nutrients. This methodology can be useful in designing nutrient treatment programs and sampling layouts, and in assisting with related timber management decisions.

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Appendix

The following formula provides the traditional basis for computing the data-based variogram, $\gamma(h)$, for a given data set (for example, refer to Isaacs and Srivastava, 1989):

$$\hat{\gamma}(h) = \frac{1}{2n_h} \sum_{i=1}^{n_h} (x_i - x_{i+h})^2 \qquad \text{(A1)}$$

where: n_h = number of data pairs referenced to lag h; x_i = data value at spatial location i; and x_{i+h} = data value at spatial location i+h . In practice, when the data are irregularly spaced, the data pairs must be grouped into lag cells (say, one cell might be all pairs with h = 0-10 m, and another with h = 10-20 m, etc.). Then, the averaging of Eq. A1 is done cell by cell, and the mean h within each cell is used as the h for the variogram value in that cell.

Valid variogram models for describing smooth fits through data-based variogram plots include the following:

1. Spherical variogram model

$$\gamma(h) = d + (s - d)\left(\frac{3h}{2r} - \frac{h^3}{2r^3}\right), \text{ for } 0 \le h < r \qquad \text{(A2)}$$
$$= s, \text{ for } h \ge r$$

where: h = separation distance (i.e., the spatial lag); d = nugget value (i.e., discontinuity at h = 0); s = sill value (usually set equal to sample variance); and r = range of influence.

2. Exponential variogram model

$$\gamma(h) = d + (s - d)\left(1 - e^{-h/c}\right), \text{ for } h \ge 0 \qquad \text{(A3)}$$

where: h = separation distance (i.e., the spatial lag); d = nugget value (i.e., discontinuity at h = 0); s = sill value (usually set equal to sample variance); and c = range parameter defined as 3c = effective range (i.e., the lag distance where γ equals 0.95s).

3. Gaussian variogram model

$$\gamma(h) = d + (s - d)\left(1 - e^{-(h/c)^2}\right), \text{ for } h \ge 0 \qquad \text{(A4)}$$

where: h = separation distance (i.e., the spatial lag); d = nugget value (i.e., discontinuity at h = 0); s = sill value (usually set equal to sample variance); and c = range parameter defined as $c\sqrt{3}$ = effective range (i.e., the lag distance where γ equals 0.95s).