Effects of intermediate silvicultural treatments on the distribution of within-stand growth[1]

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The distribution of within-stand basal area growth following silvicultural treatments was investigated using a relative size – relative growth (RSG) function. The effects of thinning on the distribution of tree basal area, including changes in location or scale, can be incorporated into the estimation of the RSG function parameters. Additional stand growth due to fertilization can also be allocated to individual trees using the same RSG function, since the contribution of a tree's response to total stand treatment response depends on its relative size in the stand. Statistical tests and validation of the RSG function indicated that thinning and fertilization do not alter the characteristic relationships between tree size, stand density, stand structure, and the relative distribution of growth across size classes within a stand. Therefore, silvicultural treatment growth responses predicted at a whole-stand level of resolution can be disaggregated to a list of individual trees using the RSG function developed from untreated plots.

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La répartition de la croissance en surface terrière à l'intérieur d'un peuplement à la suite de travaux sylvicoles a été étudiée à l'aide d'une fonction « taille relative-croissance relative » (RSG). Les effets de l'éclaircie sur la répartition de la surface terrière, y compris des modifications de localisation ou d'échelle peuvent être intégrées dans l'estimation des paramètres de la fonction RSG. La croissance supplémentaire attribuable à la fertilisation peut aussi être allouée aux arbres individuels en utilisant la même fonction RSG puisque la contribution de la réaction d'un arbre à celle du peuplement en entier dépend de sa taille relative à l'intérieur du peuplement. Les tests statistiques et la validation de la fonction RSG indiquent que l'éclaircie et la fertilisation ne modifient pas les relations caractéristiques entre la dimension des arbres, la densité du peuplement, sa structure, et la distribution relative de la croissance entre les différentes catégories de taille à l'intérieur d'un peuplement. Ainsi, les prédictions de réaction de croissance aux traitements sylvicoles pour l'ensemble du peuplement peuvent être ramenées à une liste d'arbres individuels en utilisant la fonction RSG développée à partir de parcelles témoins.

[Traduit par la rédaction]

Introduction

Thinning and fertilization are important intermediate silvicultural practices. Efforts to quantify growth response to these treatments for inclusion in growth and yield simulation models have focussed at both the whole-stand (e.g., Heath and Chappell 1989; Matney and Sullivan 1982) and individual-tree (e.g., Daniels and Burkhart 1975; Shafii et al. 1990) levels. Other investigators have used statistical distributions to model silvicultural treatment effects on tree diameter distributions (Bailey et al. 1981, 1989; Bailey and Da Silva 1987; Cao et al. 1982). Murray and Von Gadow (1991) recently proposed equations to predict the changes in mean and variance of the diameter distribution after thinning. The level of resolution in analysis of response to intermediate silvicultural treatments has corresponded to whole-stand or individual-tree resolution of the simulation models.

Zhang et al. (1993*a*) developed a disaggregation function, the relative size–growth (RSG) function, to distribute stand volume growth predicted from a whole-stand model to individual trees. We believe that the RSG function would also be a useful approach for quantifying the effects of thinning or fertilization treatments on within-stand distribution of growth. The objective of this study was to test the ability of the RSG function to represent the effects of thinning and fertilization treatments on the distribution of growth among trees within a stand. Coincidentally, parameterizing the RSG function would allow these silvicultural treatment effects on individual trees to be represented in a whole-stand simulation model (Zhang et al. 1993b).

Data

Data used in this portion of the study represent even-aged, managed and single-species Douglas-fir (*Pseudotsuga menziesii* var. *glauca* (Beissn.) Franco) stands in the Inland Northwest United States. The study area covers northern Idaho, western Montana, northeastern Oregon, and central and northeastern Washington. The data were primarily obtained from 94 installations established by the Intermountain Forest Tree Nutrition Cooperative (IFTNC) for nitrogen fertilization experiments (Fig. 1). Each installation consists of six square

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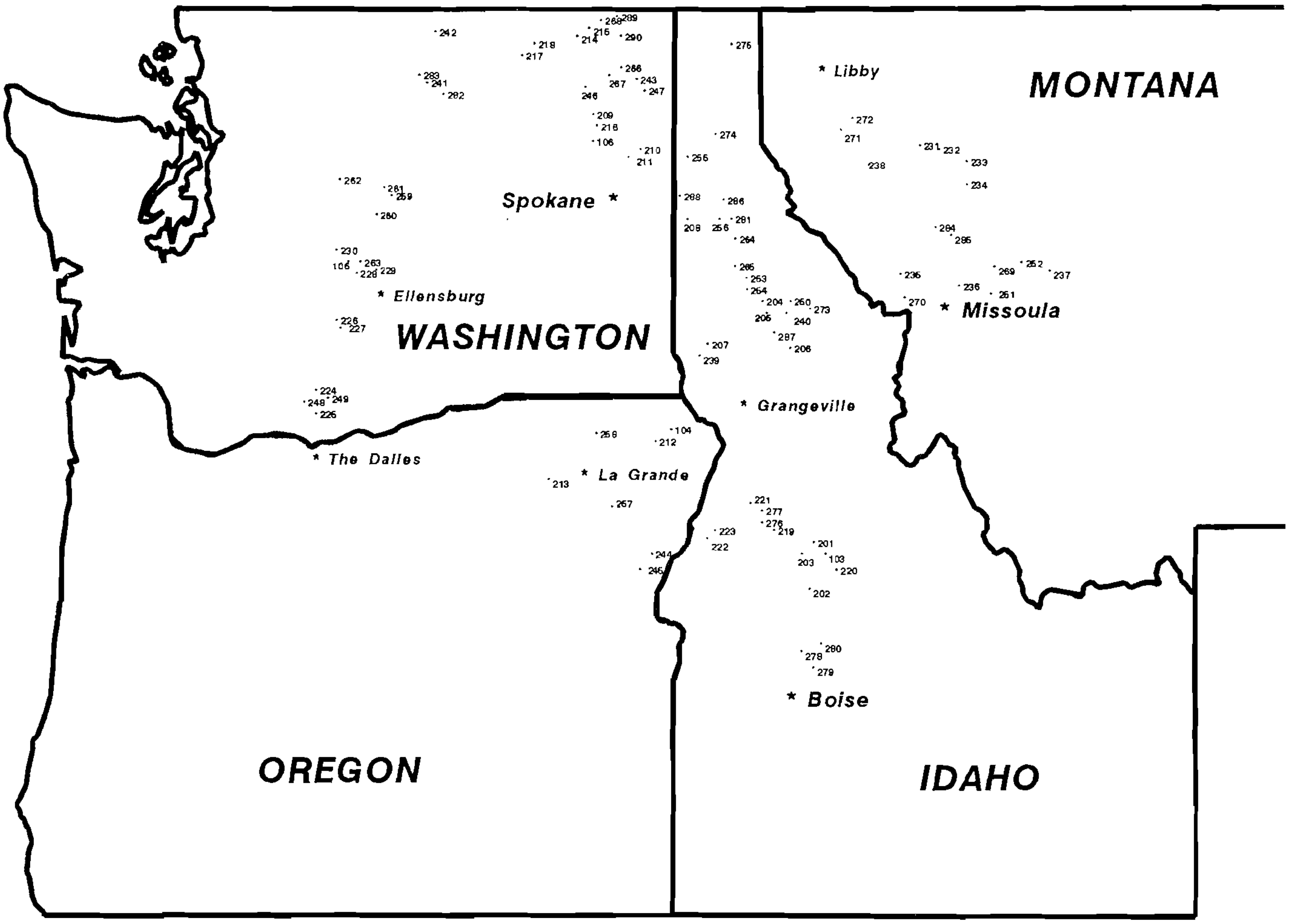

FIG. 1. Intermountain Forest Tree Nutrition Cooperative Douglas-fir installation locations.

plots from 0.1 to 0.2 acre in size (1 acre = 0.405 ha). The plots each contain at least 10 Douglas-fir sample trees and were selected to minimize among-plot variation in terrain, vegetation composition, tree stocking, and tree size. Plots were grouped into two blocks of three plots based on similarity of these features to further reduce variation. Three fertilizer treatments (0, 200, and 400 lb/acre of nitrogen) (1 lb = 0.454 kg) were randomly assigned to the plots within each block. Nitrogen in the form of urea was applied in the late fall utilizing handheld spreaders. All live trees were measured every 2 years for diameter (to the nearest 0.01 in.) (1 in. = 2.54 cm) for a 6-year growth period. Most of these stands had been thinned 5–12 years prior to fertilizer treatments between 1980 and 1982; other stands were unthinned, but naturally well spaced. The data cover a wide range of stand densities, tree ages and sizes, and site productivities (Table 1).

TABLE 1. Averages and ranges of stand characteristics at the beginning of a 6-year growth period for Douglas-fir control plots

Characteristic	Mean	Min.	Max.
Site index (ft @ 50 year)	69	39	105
Stand total age (years at breast height)	63	12	100
Trees per acre	309	90	1640
Top height (ft)	73	33	118
Basal area (ft^2/acre)	141	24	267
Quadratic mean diameter (in.)	10.0	2.5	16.9
Coefficient of variation of basal area distribution	56	23	129

Model development

The RSG function for tree basal area distribution

Since silvicultural treatments may influence tree stem profile, we decided to work with basal area relative size – relative growth (RSG) relationships rather than volume RSG function. Relative basal area growth (RBAG) is defined as the ratio of individual tree basal area growth to stand total basal area growth (i.e., the sum of the individual trees) on a unit area. Relative basal area (RBA) is defined as the ratio of individual tree basal area to stand total basal area on a unit area. Since the frequency distributions of tree volume and basal area have similar patterns, but different degrees of skewness (Hara 1984*a*, 1984*b*), the RSG relationships for tree volume and basal area should be similar. Thus, the RSG function for tree basal area distribution was expressed as a quadratic equation relating RBAG to RBA:

$$[1]\quad \mathrm{RBAG} = \beta_0 + \beta_1\mathrm{RBA} + \beta_2\mathrm{RBA}^2$$

where β_0, β_1, and β_2 are coefficients to be estimated for each plot. This form is analogous to the RSG function for volume in Zhang et al. (1993*a*).

The prediction models for the three coefficients (β_0, β_1, and β_2) of the RSG function were developed as a recursive system (see Zhang et al. (1993*a*) for more statistical detail regarding parameter estimation methodology). Stand attributes representing stand growth stages, tree competition, and variability of tree size were used as predictor variables in the recursive system:

$$\beta_1 = f(N, \text{QMD}, \text{CV})$$

$$\beta_2 = f(\beta_1, N, \text{QMD}, \text{CV})$$

$$\beta_0 = f(\beta_1, \beta_2, N, \text{QMD}, \text{CV})$$

where N is initial stand density (trees per acre), QMD is initial quadratic mean diameter (in.), and CV is the coefficient of variation (%) of initial tree basal area distribution for a plot.

Characteristics of the unfertilized plot RSG function

The RSG function (eq. 1) was fitted to the 6-year growth data from each of 188 unfertilized plots. The Pearson's correlation coefficients between 188 pairs of the three coefficients were −0.86 (β_0 vs. β_1), 0.58 (β_0 vs. β_2), and −0.88 (β_1 vs. β_2), respectively. Generally, if β_1 equals one, both β_0 and β_2 are nearly equal to zero; if β_1 is less than one, both β_0 and β_2 are positive; and in contrast, if β_1 is larger than one, both β_0 and β_2 have negative values. The above relationships represent three basic shapes of the RSG function for tree basal area distribution: linear, convex, and concave, determining the future frequency distribution of tree basal area to be symmetric, positively skewed, and negatively skewed, respectively (Zhang et al. 1993*a*).

Prediction models for the RSG function coefficients

Prediction model for β_1

The linear coefficient β_1 of the RSG function was related to three stand variables (N, QMD, and CV) using nonlinear least squares regression, resulting in

$$[2]\quad \beta_1 = N\left[1 - \exp\left(\frac{-0.0249QMD}{\text{CV}}\right)\right]$$

The asymptotic standard error of the estimated parameter was 0.000 831 4. The R^2 of the model was 0.84 (R^2 for this nonlinear model is defined as $1 - (SS_{error}/SS_{total})$). Local minimum problems were not found in parameter estimation. Residual analysis showed no significant violation of nonlinear least squares assumptions.

Prediction model for β_2

The prediction model for the quadratic coefficient β_2 was developed using the linear β_1, N, and QMD as independent variables. The CV was tested and found statistically nonsignificant in the model. The final prediction equation was

$$[3]\quad \beta_2 = 6.7224 - 10.6910\beta_1 + 0.002\,213\,\text{N} + 0.2526\text{QMD}$$

All independent variables were statistically significant ($\alpha = 0.05$). The R^2 of the model was 0.78 and the root mean squared error was 3.02. We could not detect significant violations of ordinary least squares assumptions. The diagnostics indicated that no multicollinearity problems existed among the independent variables since the largest condition number was 16.8 (Meyers 1986).

Prediction model for β_0

The coefficient β_0 was related to both the linear coefficient β_1 and quadratic coefficient β_2 of the RSG function, as well as to the three stand variables. However, the CV was also statistically nonsignificant in this equation. The final prediction model for β_0 was

$$[4]\quad \beta_0 = 0.032\,85 - 0.039\,26\,\beta_1 - 0.001\,655\,\beta_2 + 0.000\,006\,392\,N + 0.000\,649\,6\,\text{QMD}$$

All independent variables were statistically significant ($\alpha = 0.05$). The R^2 of the model was 0.90 and the root mean squared error was 0.0041. The largest condition number was 18.3 indicating that no multicollinearity problems among the independent variables were diagnosed. Residual analysis showed no significant violations of ordinary least squares assumptions.

Model assessment

Fertilization effects

Nitrogen has been identified as a growth-limiting nutrient for interior Douglas-fir. Nitrogen fertilization can significantly increase basal area and volume growth over a 6-year period following treatments (Moore et al. 1991). Shafii et al. (1990) found that nitrogen fertilization impacts the absolute change in diameter growth distribution across tree size classes within a stand. Initial tree size and initial stand density produce significant interactions on an individual tree's response to fertilization. Thus, one objective of this study was to investigate the potential RSG function differences between unfertilized and fertilized plots and between levels of fertilization treatments.

A total of 350 nitrogen treatment plots from the IFTNC data were used to test and evaluate the RSG function, 175 plots for each of 200 and 400 lb N/acre treatments, respectively. Mensurational characteristics of the three fertilizer treatment plots (0, 200, and 400 lb N/acre) are essentially identical since the data were derived from a silvicultural experiment designed to make the plots similar. The RSG function (eq. 1) was fitted to the 6-year growth data from each fertilized plot. The characteristics of the RSG function and relationships between the three coefficients for the fertilized plots were compared with those for the unfertilized plots.

The Pearson's correlations between the three coefficients of the RSG function for 200 and 400 lb N/acre treatment plots were nearly identical to those of the control plots. Correlations between β_0 and β_1, and between β_1 and β_2 are strongly negative. The positive correlation between β_0 and β_2 is relatively weak. Importantly, the relationships between the RSG coefficients for 0, 200, and 400 lb N/acre plots were also nearly identical.

Statistical analyses were conducted to test the effects of fertilization treatments on the RSG function. Multivariate analysis of variance was performed on the three coefficients of the RSG function from all unfertilized and fertilized plots. The p-values for F approximations of Wilks' λ, Pillai's Trace and Hotelling–Lawley Trace were all approximately 0.63, thus the treatments were not significantly different. Analysis of covariance (test for heterogeneity of slopes) was also conducted to test the differences of the RSG relationships among the three fertilization treatments. The RSG function (eq. 1) was fitted to combined observations of three plots within a block. Fertilization treatment was coded as an indicator variable. For 92% of the installations, F-tests were statistically insignificant ($\alpha = 0.05$) for the three coefficients of the RSG function.

TABLE 2. Stand characteristics and the predicted parameters of the RSG function (eq. 1) for simulated thinnings

Characteristic	Unthinned	Simulated thinnings: Thinning below	Thinning above
Basal area (ft^2/acre)	207	152	152
Trees per acre	610	260	540
Quadratic mean diameter (in.)	7.9	10.4	7.2
Coefficient of variation	71	25	67
Relative density index	0.55	0.35	0.42
β_0	−0.0112	−0.0334	−0.0062
β_1	1.6863	2.6995	1.4421
β_2	−7.9652	−18.943	−5.6837
RBA			
Min.	0.0035	0.0239	0.0047
Max.	0.0450	0.0610	0.0429
RBAG			
Min.	0.0003	0.0154	0.0005
Max.	0.0682	0.0797	0.0634

NOTE: RBA, relative basal area; RBAG, relative basal area growth.

The statistical analyses indicated that fertilization treatments had no significant effects on the relationships between relative tree basal area growth and relative tree basal area. Therefore, it appears that the RSG function developed for the unfertilized plots can be used for fertilization treatments within single species, even-aged stands. Fertilization increases stand total growth and accelerates tree differentiation, but does not affect the distribution of tree basal area growth within a stand. The contribution of an individual tree's growth to stand growth is a function of its relative size in the stand. Thus, the additional growth due to fertilization can be disaggregated to individual trees using the previously developed RSG function. However, the characteristic patterns of within-stand growth depend on stand conditions such as density and structure.

Thinning effects

Thinning is also an important intermediate silvicultural practice. The type and intensity of thinning directly alter mean tree size, stand density and structure, and, depending on application, stimulates growth of the remaining trees. For example, thinning from below reduces stand density, increases mean tree size and truncates the frequency distribution of tree size from the left. Changes in tree basal area distribution due to thinning can be reflected in the shape and curvature of the RSG function, since the prediction models for the three coefficients (eqs. 2, 3, and 4) were developed as functions of initial stand density, mean tree size, and variability of tree size. Thus, the RSG function is formulated to include the effects of density management manipulations on stand dynamics and tree growth within a stand.

Long-term growth data from thinned plots where the trees were measured before and after treatment would be highly desirable for testing the RSG function's ability to represent thinning effects on the distribution of growth within a stand. Unfortunately, such data were not available. Thus, we have chosen to assess the behavior of the RSG function in representing thinning effects in two ways: (*i*) comparison of before and after thinning stand structure and consequent predicted RSG function and (*ii*) comparison of actual and predicted growth from thinned stands first measured after the thinnings were applied.

TABLE 3. Observed tree relative basal area and predicted relative basal area growth from eq. 1 for two example trees in the simulated thinnings

Attribute	Unthinned	Simulated thinnings: Thinning below	Thinning above
Tree 234			
RBA	0.0176	0.0239	0.0239
RBAG	0.0161	0.0203	0.0250
E^a	0.9124	0.8492	1.0453
Tree 235			
RBA	0.0316	0.0429	0.0429
RBAG	0.0342	0.0476	0.0452
E^a	1.0817	1.1087	1.0528

NOTE: RBA, relative basal area; RBAG, relative basal area growth; *E*, relative growth efficiency.
[a] E = RBAG/RBA.

The following example illustrates the behavior of the RSG function in representing thinning effects on within-stand distribution of growth. We took an actual tree list from an unthinned plot and removed trees both from above and below such that the residual total basal area was nearly identical. Stand variables before and after simulated thinnings are provided in Table 2. The appropriate resultant stand variables were then input to eqs. 2, 3, and 4 to obtain predicted RSG function (eq. 1) parameters for the unthinned plot and the simulated thinnings from above and below. The predicted parameters are also provided in Table 2, and the resultant RSG functions are illustrated in Fig. 2. Thinning from below has shifted the range of RBA such that the trees now span from 0.024 to 0.060 versus from 0.004 to 0.045 for the unthinned and thinned from above treatments. For illustration, two trees were selected from the actual tree list. Tree 235 was one of the larger trees (RBA = 0.032) in the unthinned stand (Table 3, Fig. 2). After thinning (both from above and below) its RBA changed to the same value (0.043).

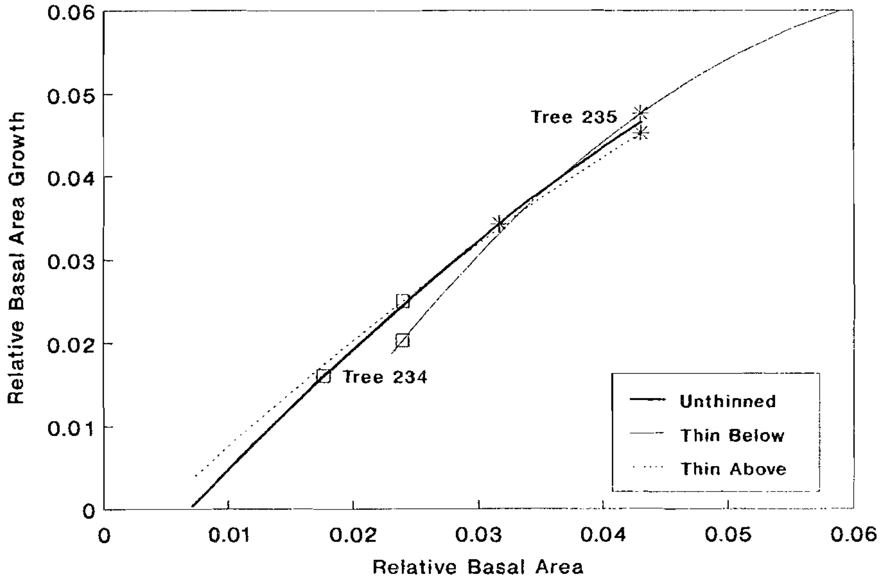

FIG. 2. Simulated thinning treatments (unthinned, thinning from below, and thinning from above) for illustrating the RSG function behavior in representing thinning effects. *, tree 235; □, tree 234.

TABLE 4. Silvicultural treatments and stand variables of validation plots used for testing the parameter prediction models (eqs. 2, 3, and 4) of the relative size-growth function

Silvicultural treatment	No. of plots	Trees per acre		Quadratic mean diameter (in.)		Coefficient of variation (%)	
		Mean	Range	Mean	Range	Mean	Range
Control	6	622	360–1080	8.5	6.1–10.1	77	56–98
100 lb N/acre	4	753	590–1090	7.0	5.4–8.6	86	69–113
200 lb N/acre	11	1132	290–2280	6.7	2.2–10.8	102	63–223
400 lb N/acre	3	777	730–830	6.3	6.0–6.7	87	83–94
Thinned only	14	339	180–630	7.5	1.1–12.7	59	30–98
Thinned + 200 lb N/acre	12	330	180–590	7.1	1.0–11.8	63	40–103

TABLE 5. Means and medians of the observed individual tree 6-year basal area growth and prediction error for each silvicultural treatment

Silvicultural treatment	BA growth (ft^2)		Prediction error (ft^2)			
	Mean	Median	Mean	% BAG	Median	% BAG
Control	0.0604	0.0400	−0.000 013	0.02	−0.0056	14.0
100 lb N/acre	0.0567	0.0359	0.000 018	0.03	−0.0031	8.6
200 lb N/acre	0.0532	0.0318	−0.000 015	0.03	−0.0042	13.2
400 lb N/acre	0.0533	0.0218	−0.000 040	0.08	−0.0057	26.1
Thinned only	0.0952	0.0876	−0.000 033	0.03	−0.0051	5.8
Thinned + 200 lb N/acre	0.0990	0.0922	−0.000 027	0.03	−0.0027	2.9
Overall mean	0.0696	0.0516	−0.000 018	0.03	−0.0044	8.5

NOTE: % BAG, percentage of basal area growth for prediction errors.

After thinning from above, tree 235 is the largest residual tree; however, in thinning from below it is a medium sized residual tree even though the RBA values are identical. In contrast, tree 234 was smaller than average (RBA = 0.018) in the unthinned stand (Table 3, Fig. 2). After both simulated thinnings its RBA changed to 0.024. After thinning from above, tree 234 is a moderate sized tree, but after thinning from below it is the smallest residual tree. If we define RBAG/RBA as an index of relative growth efficiency (E) in the model, then comparison of E for the two trees and the three simulated thinnings contrasts predicted effects on within-stand distribution of growth. The E-value for tree 235 remains about the same (1.05–1.11) for all three situations; however, E decreases for tree 234 after thinning from below (it is the smallest residual tree) and E increases for tree 234 after simulated thinning from above (Table 3). These examples are consistent with our understanding of competitive effects among trees within stands. Larger trees are

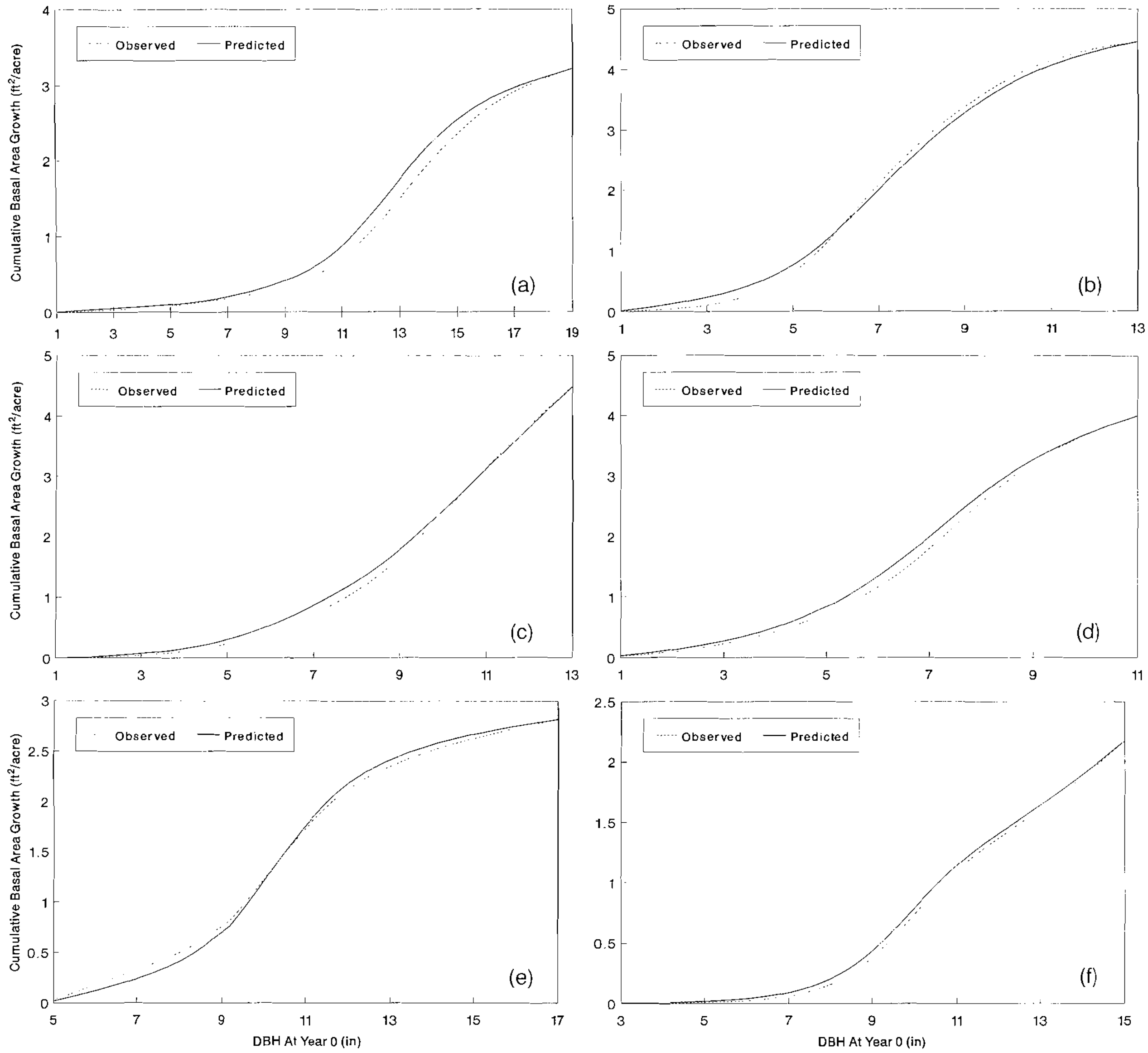

FIG. 3. Cumulative frequency distribution of the observed and predicted 6-year tree basal area growth by initial diameter class for (*a*) control, (*b*) fertilized with 100 lb N/acre, (*c*) fertilized with 200 lb N/acre, (*d*) fertilized with 400 lb N/acre, (*e*) thinned only, and (*f*) thinned and fertilized with 200 lb N/acre.

relatively less affected by thinnings than smaller trees. The narrow range of E-values for tree 235 and the broad range of E-values for tree 234 illustrate this point (Table 3). The value of E for different trees in a stand is determined by the stand density, structure, and size, as predicted by the RSG function. In this stand all three RSG functions have concave shapes (Fig. 2).

Model validation

Independent validation data with a 6-year growth period were also obtained from three separate sources to test the RSG function behavior, including various thinning and fertilization treatment combinations. A total of 50 plots consisted of six silvicultural treatments: control (unfertilized); fertilized with nitrogen at 3 different rates (100, 200, and 400 lb N/acre); thinned only; and thinned and fertilized with 200 lb N/acre. These testing plots covered a broad range of tree sizes, stand densities and structures, including thinning from below and mechanical thinning. In addition, the plots with thinning treatments were thinned immediately prior to the growth period under study in contrast with the development data, wherein the thinnings occurred at least 5 years prior to the analysis growth period. This provides an additional test of the "robustness" of the RSG function. However, actual tree lists prior to the thinning treatments are not available from these data. Certain mensurational characteristics for each of the silvicultural treatments are shown in Table 4.

The three coefficients of the RSG function were estimated for each plot based on the stand variables at the beginning of the 6-year growth period using eqs. 2, 3, and 4 to obtain predicted RBAG for each tree. These predicted individual tree RBAGs were applied to actual 6-year stand basal area growth to obtain predicted 6-year individual tree basal area increments. This predicted tree basal area growth was then compared with corresponding observed tree basal area growth. Prediction error is the difference between observation and prediction. Positive error values are underprediction and negative values are overprediction.

The prediction error was calculated for each tree and then averaged for each plot and treatment. Since the distributions of the observed tree basal area growth and prediction error for some plots were asymmetric, both the means and medians of the observed 6-year basal area growth and prediction error for each treatment and across all treatments are provided in Table 5. The validation results indicated that the RSG function produced very small prediction errors across all silvicultural treatments (less than 0.1% of the observed mean basal area growth). The percentages of the prediction error for the treated plots were very similar to those for the control plots, and there was no particular pattern for different treatments. If median terms were used, the RSG function overpredicted tree basal area growth by 0.0044 ft^2 (1 ft^2 = 0.09 m^2) (8.5% of the observed median tree basal area growth across all treatments).

The performance of the RSG function across diameter classes within a stand was also examined for all plots. To illustrate the RSG function prediction behavior, one plot was selected for each of the silvicultural treatments such that the prediction error of the selected plot was similar to the mean prediction error for the corresponding treatment. For each of the six plots, the observed and predicted 6-year basal area growth of individual trees were categorized into 2-inch diameter classes for initial tree diameter. A cumulative frequency distribution of the predicted basal area growth was plotted and compared with that for the observed basal area growth (Fig. 3). The graphs show that the distributions of the predicted basal area growth are similar to those of the observed basal area growth in both levels and shapes. The observed and predicted curve shapes vary according to stand density and structure for each plot.

Conclusions

Validation results indicated that the RSG function performed well for distributing stand basal area growth to individual trees following silvicultural treatments. The similarities in shapes between the observed and predicted basal area growth showed that the RSG function reasonably represented within-stand basal area growth across diameter classes. These silvicultural treatments did not change the relationship between relative tree basal area growth and relative tree basal area, and did not alter the characteristic relationships between average tree size, stand density and structure represented in eqs. 2, 3, and 4. There is no need to develop treatment specific RSG functions.

Thinning immediately alters stand density, mean tree size, and stand structure. Artificial reductions in stand density influence competitive status of individuals and enable the remaining trees to accelerate their occupancy of growing space and their diameter growth. Changes in these stand variables are directly incorporated into estimating the coefficients of the RSG function. Fertilization treatments do not directly change tree basal area distribution within a stand, but accelerate tree growth and thus speed up the rate of crown differentiation. A tree's response to fertilization depends on its initial size, since larger trees in a stand produce more absolute growth response than smaller trees (Shafii et al. 1990). Our results suggest that if a tree is growing relatively well prior to fertilization, its response after treatment will be proportional to its prior relative growth. The absolute growth effects of these silvicultural treatments would need to be first estimated at the stand level. Growth could subsequently be distributed to a list of individual trees using the RSG function.

Acknowledgements

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