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**Effects of Intermediate Silvicultural Treatments  
on the Distributions of Within Stand Growth**

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**EFFECTS OF INTERMEDIATE SILVICULTURAL TREATMENTS  
ON THE DISTRIBUTION OF WITHIN STAND GROWTH**

**BY**

**LIANJUN ZHANG**

**JAMES A. MOORE**

**JAMES D. NEWBERRY**

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The authors are, respectively, Research Associate and Professor, Department of Forest Resources, University of Idaho, Moscow, ID 83843, and Forest Biometrician, Potlatch Corporation, P.O. Box 1016, Lewiston, ID 83501. The support of the Intermountain Forest Tree Nutrition Cooperative is greatly appreciated. College of Forestry, Wildlife and Range Experiment Station contribution No. XXXX.

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**ABSTRACT**

A disaggregation function, the relative size-growth (RSG) function, was developed to distribute stand basal area growth to individual trees following thinning and fertilization treatments. The RSG function is formulated as a quadratic equation relating relative tree basal area growth to relative tree basal area. The shape and curvature of the RSG function are determined by initial mean tree size, stand density and structure. The effects of thinning on tree basal area distribution, including changes in location and/or scale, can be incorporated into the estimation of the RSG function parameters. The additional stand growth due to fertilization can also be allocated to individual trees using the same RSG function since the contribution of a tree's response to stand fertilization response depends on its relative size in the stand. Validation of the model indicated that the relationships between relative tree size and relative tree growth in untreated stands were not changed by thinning and fertilization treatments. This means that when using this approach to disaggregate stand growth predicted from a whole-stand model to a list of individual trees, only treatment response at the stand level must be predicted. Thinning and fertilization do not alter the characteristic relationships between tree size, stand density,

stand structure, and the relative distribution of growth across size classes within a stand.

**KEY WORDS:** relative tree size, relative tree growth, stand structure and development, thinning and fertilization

### INTRODUCTION

Thinning and fertilization are important intermediate silvicultural practices. To date, analyses for these experiments have primarily focussed on stand and tree growth and yield following the treatments. Prediction models for treatment response have also been developed at both whole-stand (Heath and Chappell, 1989; Matney and Sullivan, 1982) and individual tree (Daniels and Burkhart, 1975; Shafii et al., 1990) levels. The Weibull distribution was employed for modeling diameter distribution in thinned (Bailey et al., 1981; Cao et al., 1982) and fertilized (Bailey and Da Silva, 1987) stands. Murray and Gadow (1991) recently proposed equations to predict the changes in mean and variance of the diameter distribution after thinning. The equations were derived from statistical consistency conditions to ensure that the thinned and the residual tree distributions add up exactly to the before thinning distribution. However, no research has been published to investigate the distribution of within stand growth and develop the links between

whole-stand and individual tree models following silvicultural treatments.

Zhang et al. (199?) proposed a disaggregation function, the relative size-growth (RSG) function, to distribute stand volume growth to individual trees. The advantages of this approach are (1) to avoid the assumption that tree size follows a particular statistical distribution, (2) to reflect effects of competition among individual trees on dynamics of stand structure, and (3) to predict the relationship between tree growth and tree size based on current stand condition and structure. We believe that this disaggregation function has the capability to incorporate the changes of tree growth and stand structure due to silvicultural treatments such as thinning and fertilization.

The purpose of this paper is to investigate the effects of thinning and fertilization treatments on the characteristics and applications of the RSG function. Since these silvicultural treatments may influence tree stem profiles we decided to work with basal area relative size-growth (RSG) relationships rather than volume RSG's as in Zhang et al. (199?).

#### DATA

Data used in this study represent even-aged, predominately thinned (at least 5 years prior to the growth periods used in the analysis), single-species, Douglas-fir (*Pseudotsuga menziesii* var. *glauca* (Beissn) Franco) stands (65% Douglas-fir by basal

area) in the inland Northwest. The study area covers six geographic regions: northern and central Idaho, western Montana, northeast Oregon, central and northeast Washington. The data were mostly collected from 94 installations established by the Intermountain Forest Tree Nutrition Cooperative (IFTNC) for nitrogen fertilization experiments. Each installation consisted of two control (i.e. unfertilized) plots, two plots with 200 lb N/acre treatment, two plots with 400 lb N/acre treatment. Plot size ranged from 0.1 to 0.2 acre. All trees were measured for both height (to the nearest 1 foot) and diameter (to the nearest 0.01 inch) for a 6-year growth period. Additional ten control plots with a 6-year growth period were used from a University of Idaho McIntire-Stennis study, which was established for thinning and fertilization experiments throughout northern Idaho in the early 1970s. The data base covers a wide range of stand densities, tree ages and sizes, and site productivities. Selected characteristics for the Douglas-fir database at the beginning of the 6-year growth period are summarized in Table 1.

### MODEL DEVELOPMENT

#### **I. The RSG Function for Stand Basal Area Distribution**

Relative basal area growth (RBAG) was defined as the ratio of individual tree basal area growth to stand total basal area growth (i.e. the sum of the individual trees) on a unit area.

Relative basal area (RBA) was defined as the ratio of individual tree basal area to stand total basal area on a unit area. The relative size-growth (RSG) function was expressed as a quadratic equation relating relative basal area growth (RBAG) to relative basal area (RBA) as follows:

$$RBAG = \beta_0 + \beta_1 * RBA + \beta_2 * RBA^2, \quad (1)$$

where  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  are coefficients to be estimated for each plot.

The prediction models for the three coefficients ( $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ) of the RSG function were developed as a recursive system. Since the frequency distributions of tree volume and basal area have similar patterns, but different degrees of skewness (Zhang et al., 199?), perhaps due to different average stem profiles, the RSG functions developed for tree volume and basal area are expected to have similar relationships. Stand attributes representing stand growth stages, tree competition, and variability of tree size were used as predictor variables in the recursive system:

$$\beta_1 = f(N, QMD, CV),$$

$$\beta_2 = f(\beta_1, N, QMD, CV),$$

$$\beta_0 = f(\beta_1, \beta_2, N, QMD, CV),$$

where  $N$  is initial stand density (trees per acre),  $QMD$  is initial stand quadratic mean diameter (inches), and  $CV$  is the coefficient of variation of initial tree basal area distribution for a plot. The formulations of the prediction models and the estimation

procedures basically followed Zhang et al., (199?).

## II. Characteristics of the RSG Function for Douglas-fir Control Plots

The relative size-growth (RSG) function for stand basal area distribution (Equation (1)) was fit to the 6-year growth data from each of the Douglas-fir control plots using ordinary least squares regression. Pearson's correlation coefficients between pairs of the three coefficients were  $-0.86$  ( $\beta_0$  versus  $\beta_1$ ),  $0.58$  ( $\beta_0$  versus  $\beta_2$ ), and  $-0.88$  ( $\beta_1$  versus  $\beta_2$ ), respectively. Generally, if  $\beta_1$  equals one, both  $\beta_0$  and  $\beta_2$  are nearly equal to zero; if the linear coefficient ( $\beta_1$ ) is less than one, both  $\beta_0$  and  $\beta_2$  are positive; and in contrast, if  $\beta_1$  is larger than one, both  $\beta_0$  and  $\beta_2$  have negative values (Figure 1). The above relationships represent three basic shapes of the relative size-growth (RSG) function for stand basal area distribution: linear, convex, and concave, determining the future frequency distribution of tree basal area to be normal, positively skewed, and negatively skewed, respectively.

## III. Prediction Models for the Coefficients of the RSG Function

(1). Prediction model for the coefficient  $\beta_1$ . The linear coefficient  $\beta_1$  of the RSG function (Equation (1)) was related to three stand variables ( $N$ ,  $QMD$ ,  $CV$ ) using nonlinear least squares



regression, resulting in:

$$\beta_1 = N * \left( 1 - e^{\frac{-0.0249 * QMD}{CV}} \right) . \quad (2)$$

The asymptotic error of the estimated parameter was 0.0008314. Local minimum problems were not found in parameter estimation. There was no detectable trend in residual analysis.

(2). Prediction model for the coefficient  $\beta_2$ . The prediction model for the quadratic coefficient  $\beta_2$  was developed using the coefficient  $\beta_1$  and number of trees (N) and quadratic mean diameter (QMD) as independent variables. The coefficient of variation of tree basal area distribution (CV) was tested and found statistically nonsignificant in the model. The resulting model was as follows:

$$\begin{aligned} \beta_2 = & 6.7224 - 10.6910 * \beta_1 + 0.002213 * N \\ & + 0.2526 * QMD. \end{aligned} \quad (3)$$

All independent variables were statistically significant ( $\alpha=0.05$ ). The  $R^2$  of the model was 0.78 and the root mean squared error was 3.02.  $\beta_1$  explained most of the variation accounted for by Equation (3) (the partial  $r^2$  of  $\beta_1$  was 0.77). The collinearity diagnostics indicated that no multicollinearity problems among the independent variables were found since the largest condition number was 16.8. The plot of residuals against the predicted values of  $\beta_2$  showed a random pattern around zero with no detectable trend.

(3). Prediction model for the coefficient  $\beta_0$ . The coefficient  $\beta_0$  was related to both the linear coefficient  $\beta_1$  and quadratic coefficient  $\beta_2$  of the RSG function, as well as to the three stand variables. However, the coefficient of variation of tree basal area distribution (CV) was also statistically nonsignificant in the model. The prediction model for the coefficient  $\beta_0$  was:

$$\begin{aligned} \beta_0 = & 0.03285 - 0.03926 * \beta_1 - 0.001655 * \beta_2 \\ & + 0.000006392 * N + 0.0006496 * QMD. \end{aligned} \quad (4)$$

All independent variables were statistically significant ( $\alpha=0.05$ ). The  $R^2$  of the model was 0.90 and the root mean squared error was 0.0041. The largest condition number was 18.3 indicating that no multicollinearity problems among the independent variables were diagnosed. The residual analysis did not show a detectable pattern.

### MODEL VALIDATION

#### **I. Effects of Thinning Treatments on the RSG Function**

Thinning is an important intermediate silvicultural practice. The type and intensity of thinning directly alters mean tree size, stand density and structure, and, if applied properly, stimulates growth of the remaining trees. For example, thinning from below reduces stand density, increases the mean of tree size

and truncates the frequency distribution of tree size from the left. Changes in tree basal area distribution due to thinning can be reflected in the shape and curvature of the RSG function, since the prediction models for the three coefficients ( $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ) were developed as functions of initial stand density, mean tree size and the coefficient of variation of tree basal area distribution. Additionally, the data used for model development in this study were substantially composed of thinned stands. Therefore, the RSG function is theoretically formulated to represent the effects of density management manipulations on stand dynamics and tree growth within a stand. Thinning effects on the behavior of the RSG function will be evaluated using independent validation data later in this paper.

## **II. Effects of Fertilization Treatments on the RSG Function**

Nitrogen has been identified as a major growth-limiting nutrient for interior Douglas-fir. Nitrogen fertilization can significantly increase basal area and volume growth over a 6-year period following treatment (Moore et al., 1991). Shafii et al. (1990) found that nitrogen fertilization impacts the absolute change in diameter growth distribution across tree size classes within a stand. Initial tree size and initial stand density produce significant interactions on individual tree's response to fertilization. One objective of this study was to investigate the potential RSG function differences between control and fertilized

plots and between levels of fertilization treatments.

A total of 350 nitrogen treatment plots from the IFTNC database were used to test and evaluate the RSG function, 175 plots for each of 200 lb and 400 lb N/acre treatments, respectively. Ordinary least squares regression was used to fit the RSG function (Equation 1) with the 6-year growth data from each treatment plot. The characteristics of the RSG function and relationships between the three coefficients ( $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ) for the fertilized plots were compared with those for the control plots.

The analysis showed similar Pearson's correlations between the three coefficients of the RSG function for control, 200 lb and 400 lb N/acre treatments (Table 2). Correlations between  $\beta_0$  and  $\beta_1$ , and between  $\beta_1$  and  $\beta_2$  are strongly negative. The positive correlation between  $\beta_0$  and  $\beta_2$  is relative weak. In addition, not only are the magnitudes and signs of Pearson's correlations between the three coefficients similar, but also are the patterns of the relationships between the coefficients (Figures 2 and 3).

The statistical tests for each of the three coefficients between control, 200 lb and 400 lb N/acre treatments are displayed in Table 3. The results indicated that differences between means (T-test) of the coefficients were not statistically significant. Analysis of variance (ANOVA) and multivariate analysis of variance (MANOVA) were also conducted to simultaneously test statistical significance among the coefficients, resulting in the same conclusion.

Therefore, it appears that the RSG function developed for the Douglas-fir control plots can be used for fertilization treatments. Fertilization increases stand total growth and accelerates tree differentiation, but does not affect the distribution of tree growth within a stand. The contribution of an individual tree's growth to stand growth is a function of its relative size in the stand. Thus, the additional growth due to fertilization can be disaggregated to individual trees using the previously developed RSG function (Equations 2, 3, and 4). Wherein the characteristic patterns of within stand growth vary according to stand conditions such as density and structure.

### **III. Validation of the RSG Function**

Independent validation data with a 6-year growth period were obtained from three separate sources: (1) University of Idaho McIntire-Stennis study supplied five installations. Each installation contained eight plots (square 0.1 acre in size). Four of the eight plots, selected at random, were thinned to approximately a 15x15 ft spacing and four were left unthinned. Nitrogen fertilization then was applied randomly at the rate of 200 lb N/acre to two thinned and two unthinned plots, thereby providing two replicates of four treatments within each installation: (i) control (i.e. unthinned and unfertilized), (ii) fertilized only with 200 lb N/acre, (iii) thinned only, and (iv) thinned and fertilized with 200 lb N/acre. Control plots had been

used for model development in this study, therefore, were not included in model validation. (2) Three installations were supplied from a cooperative study between the University of Idaho and the Forest Service, USDA, Intermountain Forest and Range Experiment Station, Intensive Timber Culture. This study was established in 1974-1977 and the treatments included (i) thinned only (low thinning to a residual spacing from 10x10 to 15x15 ft), and (ii) thinned in the same manner and also fertilized with 200 lb N/acre. (3) Potlatch Corporation provided four installations with a range of thinning and fertilization treatments. Various methods of thinning (such as mechanical, chemical, and commercial) were used with residual spacing from 12x12 to 15x15 ft. Silvicultural treatments were as follows: (i) control, (ii) fertilized only with 100 lb N/acre, (iii) fertilized only with 200 lb N/acre, (iv) fertilized only with 400 lb N/acre, (v) thinned only, and (vi) thinned and fertilized with 200 lb N/acre. These plots used in the validation were thinned immediately prior to the growth period under study in contrast to the development data, wherein the thinnings occurred at least 5 years prior to the analysis growth period. This provides an additional test of the "robustness" of the RSG function.

In total, the validation data consisted of 6 silvicultural treatments, i.e. no treatment, fertilized with nitrogen at 3 different rates, thinned only, thinned and fertilized with 200 lb N/acre. A total of 50 Douglas-fir (> 65% Douglas-fir by basal area) plots were used to evaluate the RSG prediction models

developed for the Douglas-fir control plots. These testing plots covered a broad range of tree sizes, stand densities and structures. Certain mensurational characteristics for each of the silvicultural treatments are shown in Table 4. The three coefficients of the RSG function were estimated for each plot based on the stand variables at the beginning of the 6-year growth period using Equations 2, 3, and 4. Actually observed 6-year stand basal area growth was disaggregated to individual trees applying the estimated plot-specific RSG function. Predicted tree basal area growth was compared with corresponding observed tree basal area growth. Prediction error is the difference between observation and prediction. Positive error values are under-prediction and negative values are over-prediction.

The prediction error was calculated for each tree and then averaged for each plot and treatment. Since the distributions of the observed tree basal area growth and prediction error for some plots were asymmetric, median prediction error may be a better measure for central tendency. The means and medians of the observed 6-year basal area growth and prediction error for each treatment and across all treatments are provided in Table 5. The validation results indicated that the RSG function produced very small prediction errors across all silvicultural treatments (less than 0.1% of the observed mean basal area growth), and there was no particular pattern for different treatments. If median terms were used, the RSG function over-predicted tree basal area growth

by 0.0044 ft<sup>2</sup> (8.5% of the observed median tree basal area growth across all treatments).

The performance of the RSG function across diameter classes within a stand was examined for all plots. To illustrate the RSG function prediction behavior, one plot was selected for each of the silvicultural treatments such that the prediction error of the selected plot was similar to the mean prediction error for the corresponding treatment. For each of the six plots, the observed and predicted 6-year basal area growth of individual trees were categorized into 2-inch diameter classes for initial tree diameter. A cumulative frequency distribution of the predicted basal area growth was plotted and compared with that for the observed basal area growth (Figure 4). The graphs show that the distributions of the predicted basal area growth are similar to those of the observed basal area growth in both levels and shapes. The observed and predicted curve shapes vary according to stand density and structure for each plot.

In summary, the validation results indicate that the relative size-growth (RSG) function performed well for distributing stand basal area growth to individual trees following silvicultural treatments. The similarities in shapes between the observed and predicted basal area growth showed that the RSG function reasonably represented within-stand basal area growth across diameter classes. These silvicultural treatments did not change the characteristic relationships between relative tree size and relative tree growth and average tree size, stand



density and structure represented in Equations 2 through 4. There is no need to develop treatment specific RSG function.

### CONCLUSIONS

Thinning immediately alters stand density, mean tree size, and stand structure. Artificial reductions in stand density influence competitive status of individuals and enable the remaining trees to accelerate their occupancy of growing space and their diameter growth. Changes in these stand variables are directly incorporated into estimating the coefficients of the RSG function. Fertilization treatments do not directly change tree basal area distribution within a stand, but accelerate tree growth and thus speed up the rate of crown differentiation. A tree's response to fertilization depends on its initial size, since larger trees in a stand produce more absolute growth response than smaller trees (Shafii et al., 1990). The growth effects of these silvicultural treatments would need to be first estimated at the stand level. Growth could subsequently be distributed to a list of individual trees using the RSG function.

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Table 1. Averages and ranges of stand characteristics at the beginning of a 6-year growth period for each silvicultural treatment\*

Characteristic	Silvicultural Treatment		
	Control	200 lb N/a	400 lb N/a
DF site index (ft @ 50 year)	69 (39 - 105)	68 ( 41 - 96)	69 (41 - 97)
Stand total age (year)	63 (12 - 100)	65 ( 27 - 100)	65 (27 - 100)
Trees per acre	309 (90 -1640)	269 (105 - 745)	269 (88 - 650)
Top height (ft)	73 (33 - 118)	73 ( 39 - 114)	74 (42 - 119)
Basal area (ft <sup>2</sup> /acre)	141 (24 - 267)	141 ( 44 - 283)	142 (42 - 275)
Quadratic mean diameter (in)	10 ( 3 - 17)	10 ( 6 - 16)	10 ( 6 - 18)
Coefficient of variation of basal area distribution	56 (23 - 129)	53 ( 18 - 103)	52 (14 - 100)

\* Numbers in parentheses represent the ranges of the stand variables.

Table 2. Pearson correlations between the three coefficients of the relative size-growth function (Equation 1) for each silvicultural treatment

	Silvicultural Treatment		
	Control	200 lb N/a	400 lb N/a
$\beta_0$ vs. $\beta_1$	-0.86	-0.89	-0.87
$\beta_0$ vs. $\beta_2$	+0.58	+0.55	+0.54
$\beta_1$ vs. $\beta_2$	-0.88	-0.85	-0.85

Table 3. Statistical test for equal means of the three coefficients of the relative size-growth function (Equation 1) for silvicultural treatments

	Treatment	Mean	Standard Error	Treatment Comparisons	T-Test	p
$\beta_0$	Control	-0.0039	0.0009	Control vs. 200 lb N/a	-1.27	0.21
	200 lb N/a	-0.0020	0.0012	Control vs. 400 lb N/a	-0.53	0.60
	400 lb N/a	-0.0031	0.0013	200 lb vs. 400 lb N/a	+0.63	0.53
$\beta_1$	Control	+1.3479	0.0390	Control vs. 200 lb N/a	+1.41	0.16
	200 lb N/a	+1.2669	0.0420	Control vs. 400 lb N/a	+0.43	0.67
	400 lb N/a	+1.3227	0.0437	200 lb vs. 400 lb N/a	-0.92	0.36
$\beta_2$	Control	-4.5750	0.4829	Control vs. 200 lb N/a	-0.96	0.34
	200 lb N/a	-3.9522	0.4335	Control vs. 400 lb N/a	+0.22	0.82
	400 lb N/a	-4.7260	0.4736	200 lb vs. 400 lb N/a	+1.21	0.23

Table 4. Silvicultural treatments and averages of stand variables of Douglas-fir plots used for testing the parameter prediction models (Equations 2, 3, and 4) of the relative size-growth function for basal area distribution\*

Treatment	Number of Plots	Trees per Acre	Quadratic Mean Tree Diameter (in)	Coefficient of Variation of BA Dist.
Control	6	622 (360-1080)	8.5 (6.1-10.1)	77 (56- 98)
100 lb N/a	4	753 (590-1090)	7.0 (5.4- 8.6)	86 (69-113)
200 lb N/a	11	1132 (290-2280)	6.7 (2.2-10.8)	102 (63-223)
400 lb N/a	3	777 (730- 830)	6.3 (6.0- 6.7)	87 (83- 94)
Thinned	14	339 (180- 630)	7.5 (1.1-12.7)	59 (30- 98)
Thinned + 200 lb N/a	12	330 (180- 590)	7.1 (1.0-11.8)	63 (40-103)

\* Numbers in parentheses represent the ranges of the stand variables.

Table 5. Means and medians of the observed 6-year basal area growth and prediction error for each silvicultural treatment\*

Treatment	BA Growth (ft <sup>2</sup> )		Prediction Error (ft <sup>2</sup> )			
	Mean	Median	Mean	(BAG%)	Median	(BAG%)
Control	0.0604	0.0400	-0.000013	(0.02%)	-0.0056	(14.0%)
100 lb N/a	0.0567	0.0359	+0.000018	(0.03%)	-0.0031	( 8.6%)
200 lb N/a	0.0532	0.0318	-0.000015	(0.03%)	-0.0042	(13.2%)
400 lb N/a	0.0533	0.0218	-0.000040	(0.08%)	-0.0057	(26.1%)
Thinned	0.0952	0.0876	-0.000033	(0.03%)	-0.0051	( 5.8%)
Thinned + 200 lb N/a	0.0990	0.0922	-0.000027	(0.03%)	-0.0027	( 2.9%)
Overall mean	0.0696	0.0516	-0.000018	(0.03%)	-0.0044	( 8.5%)

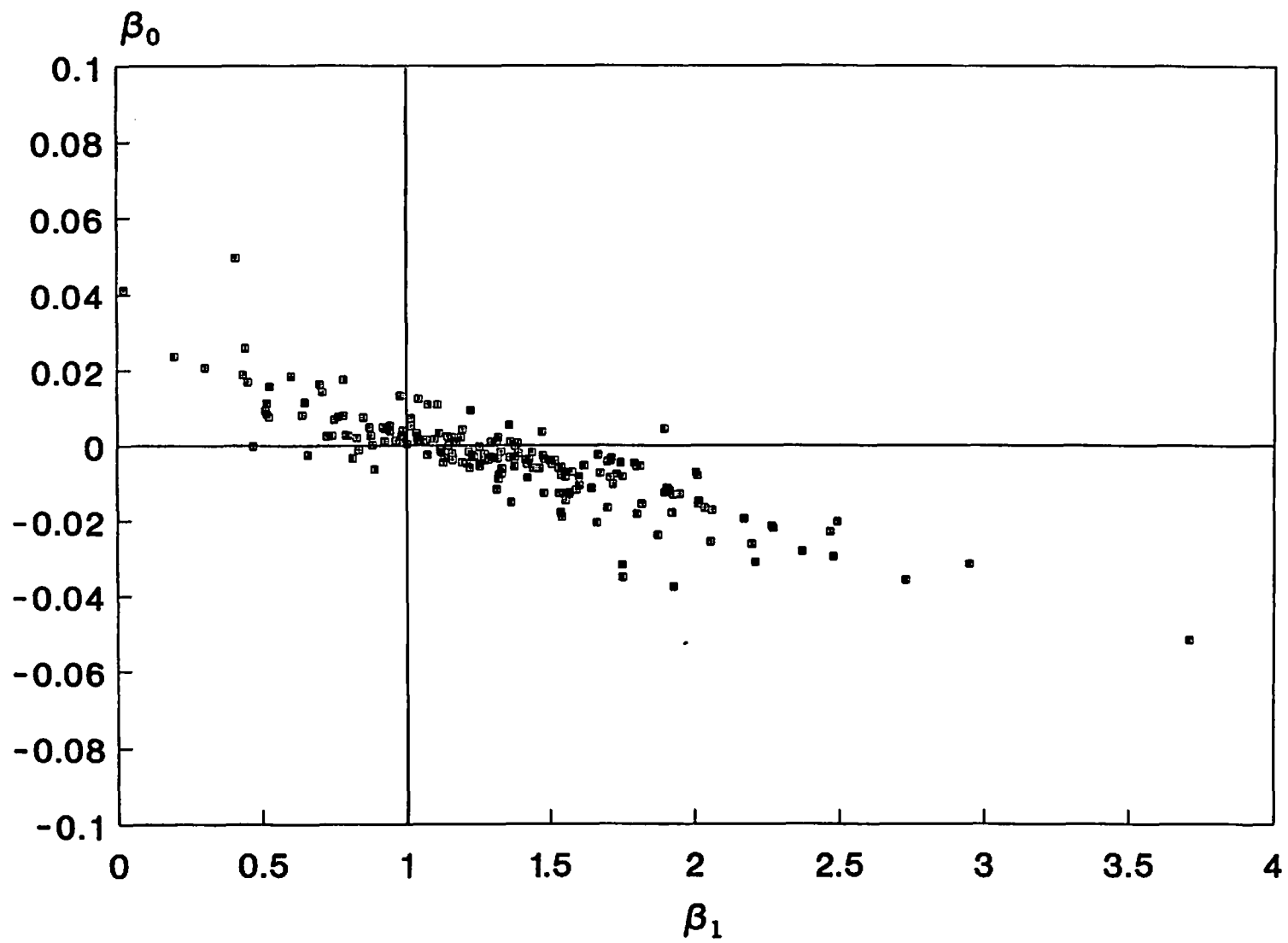
\* Numbers in parentheses represent the percentages of basal area growth for prediction errors.

## LIST OF FIGURES

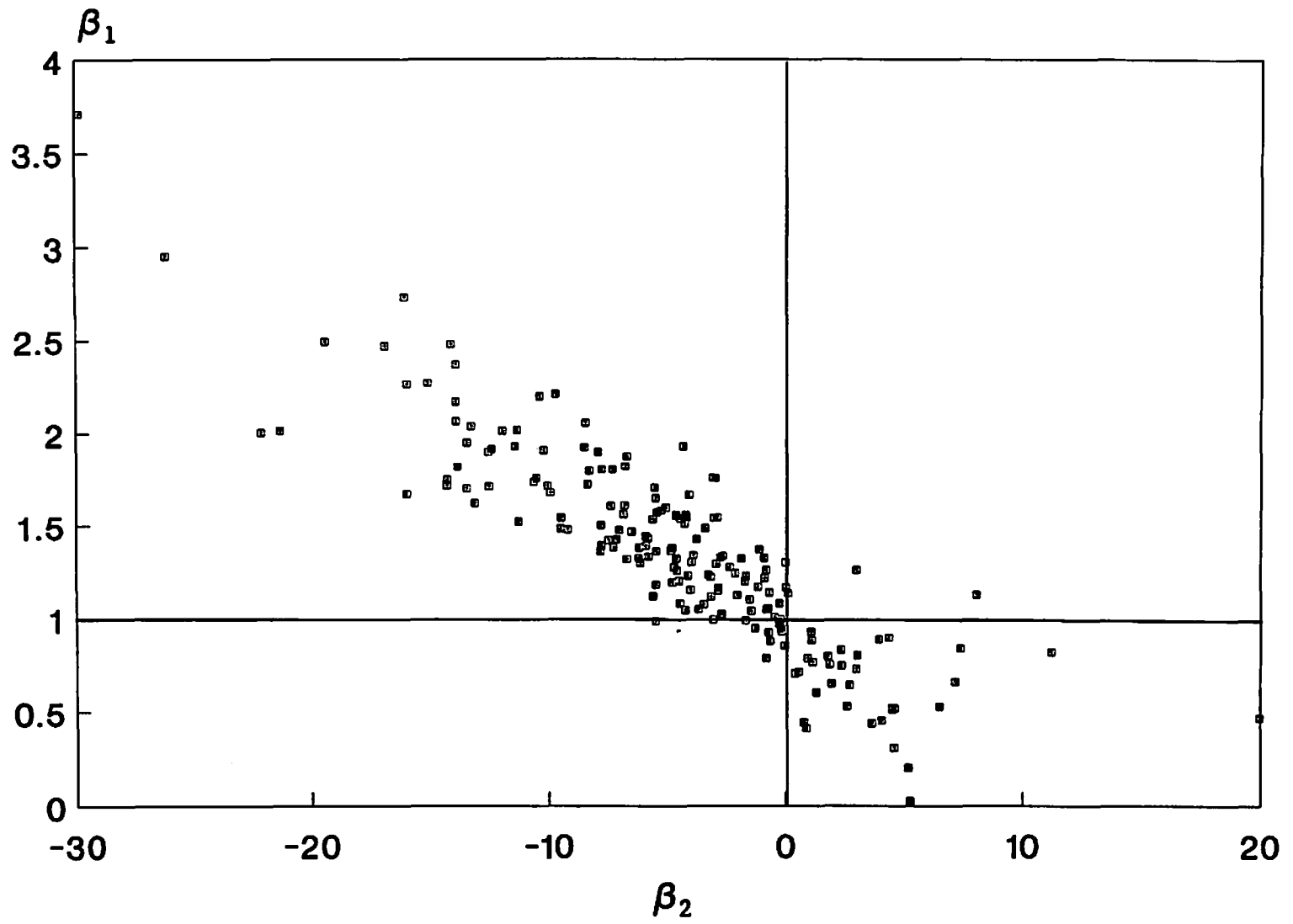
- Figure 1: Relationships between the coefficients of the relative size-growth function (Equation 1) for the Douglas-fir control plots: (a)  $\beta_0$  versus  $\beta_1$ , (b)  $\beta_1$  versus  $\beta_2$ .
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- Figure 3: Relationships between the coefficients of the relative size-growth function (Equation 1) for the Douglas-fir 400 lb N/acre treatment plots: (a)  $\beta_0$  versus  $\beta_1$ , (b)  $\beta_1$  versus  $\beta_2$ .
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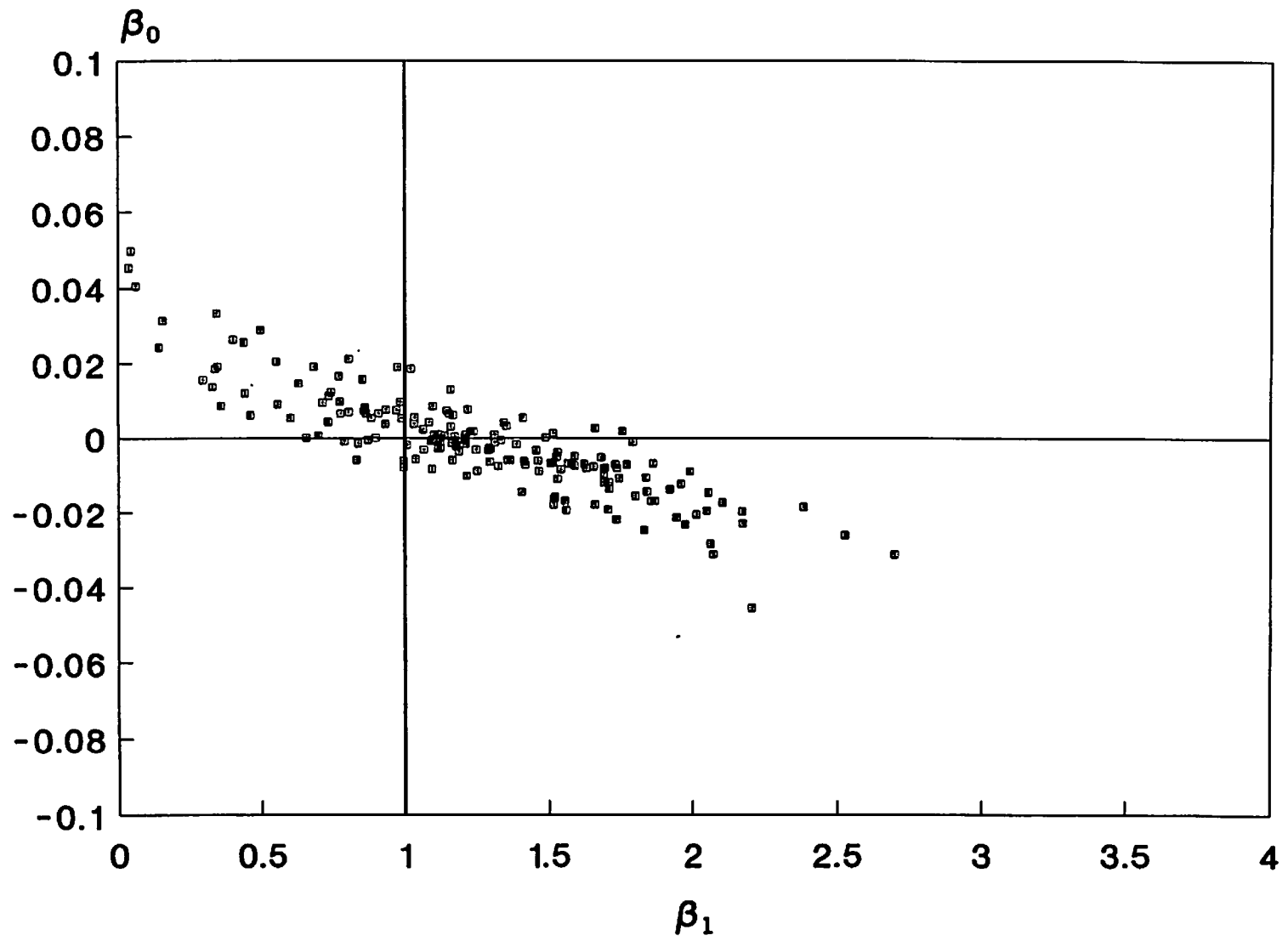
(a) Control



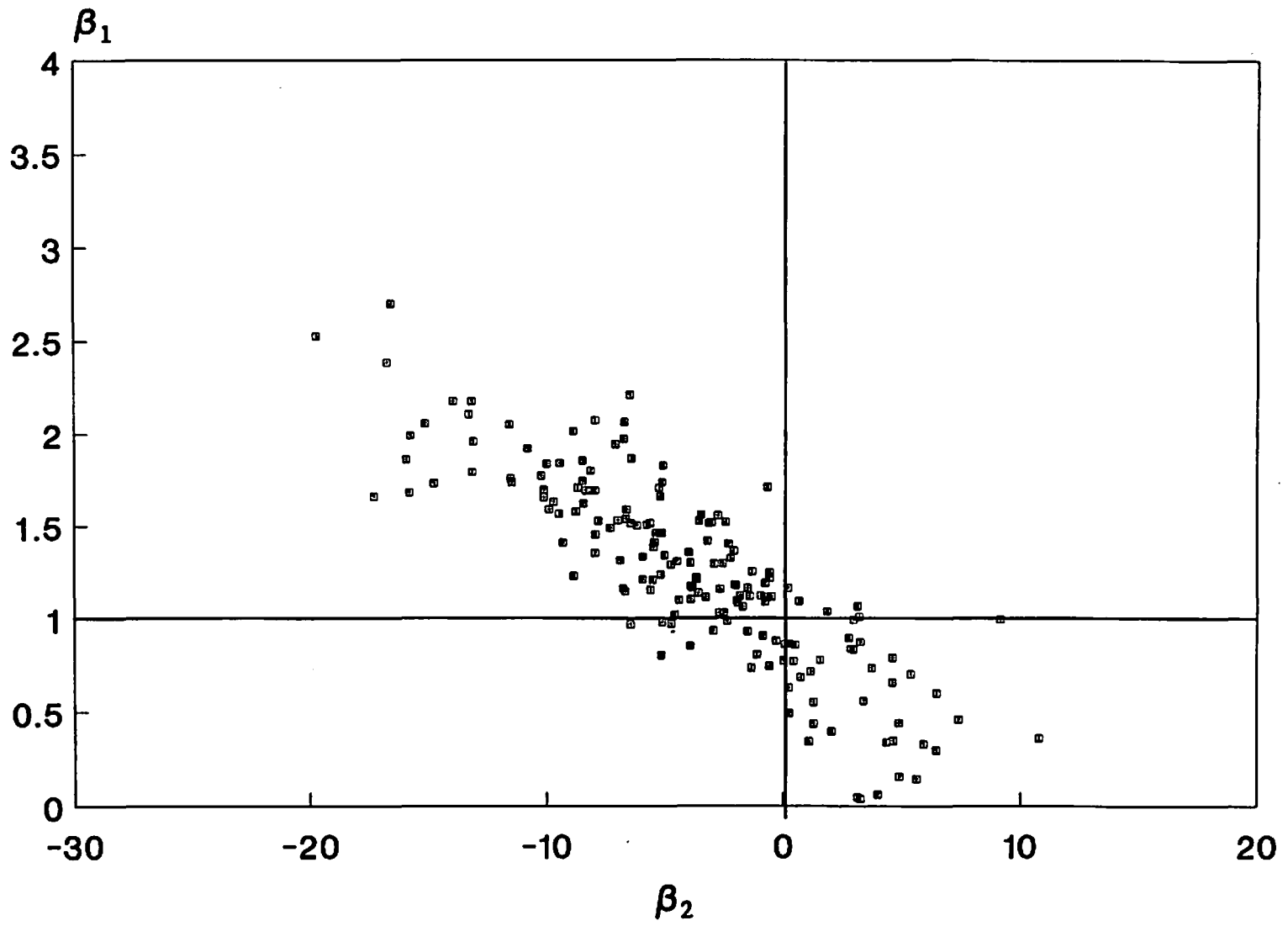
(b) Control



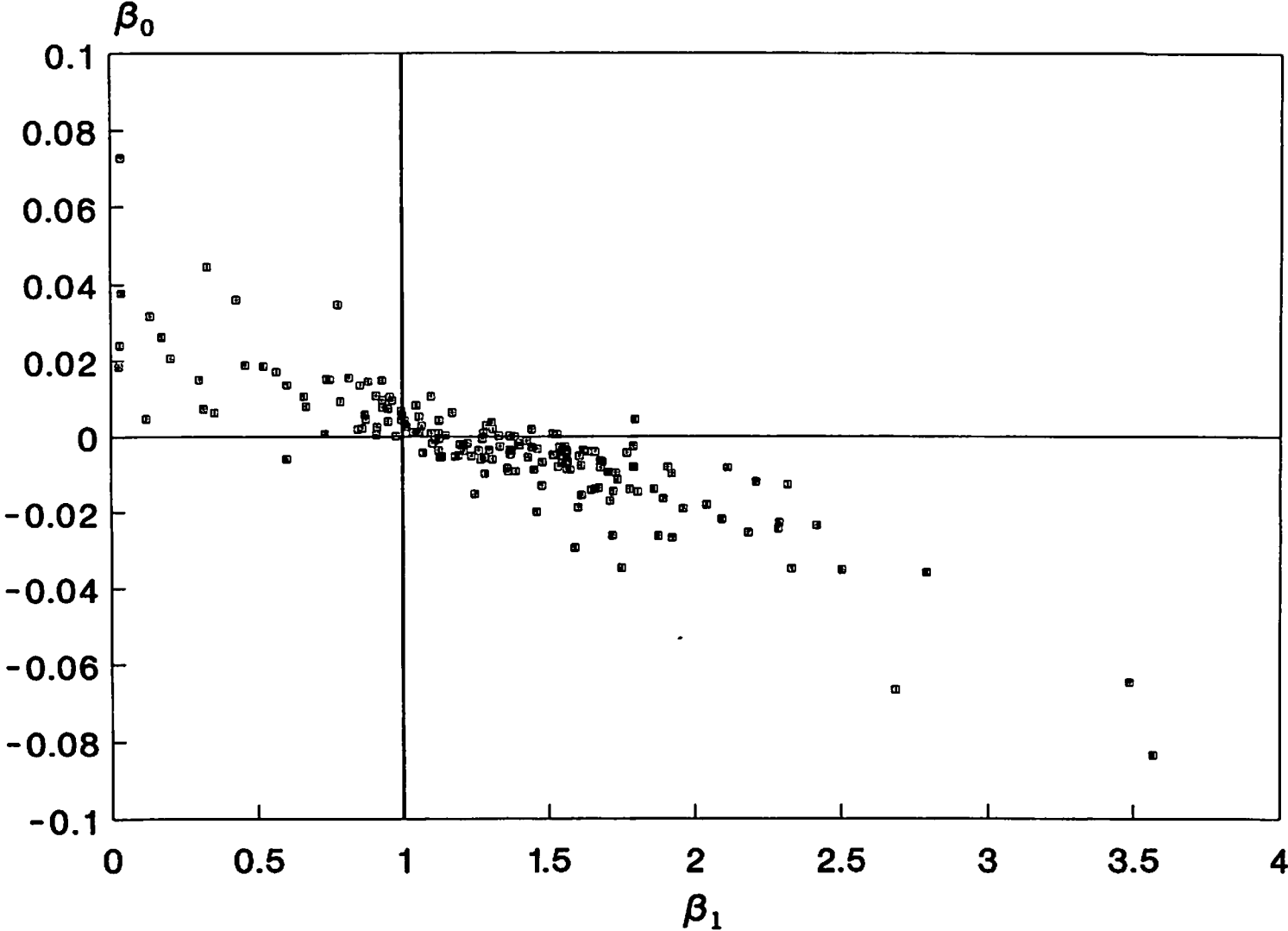
(a) 200 lb N/acre



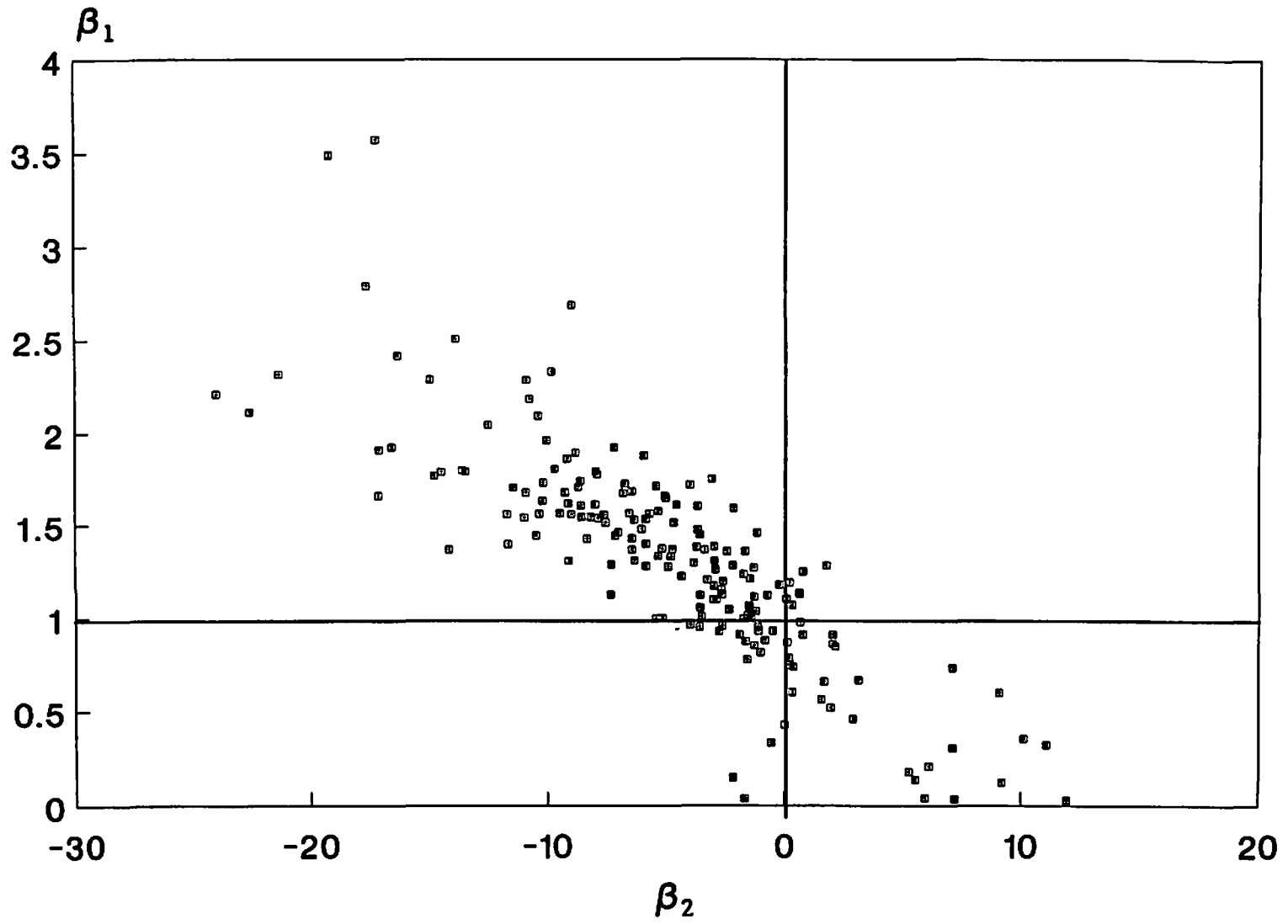
(b) 200 lb N/acre



(a) 400 lb N/acre

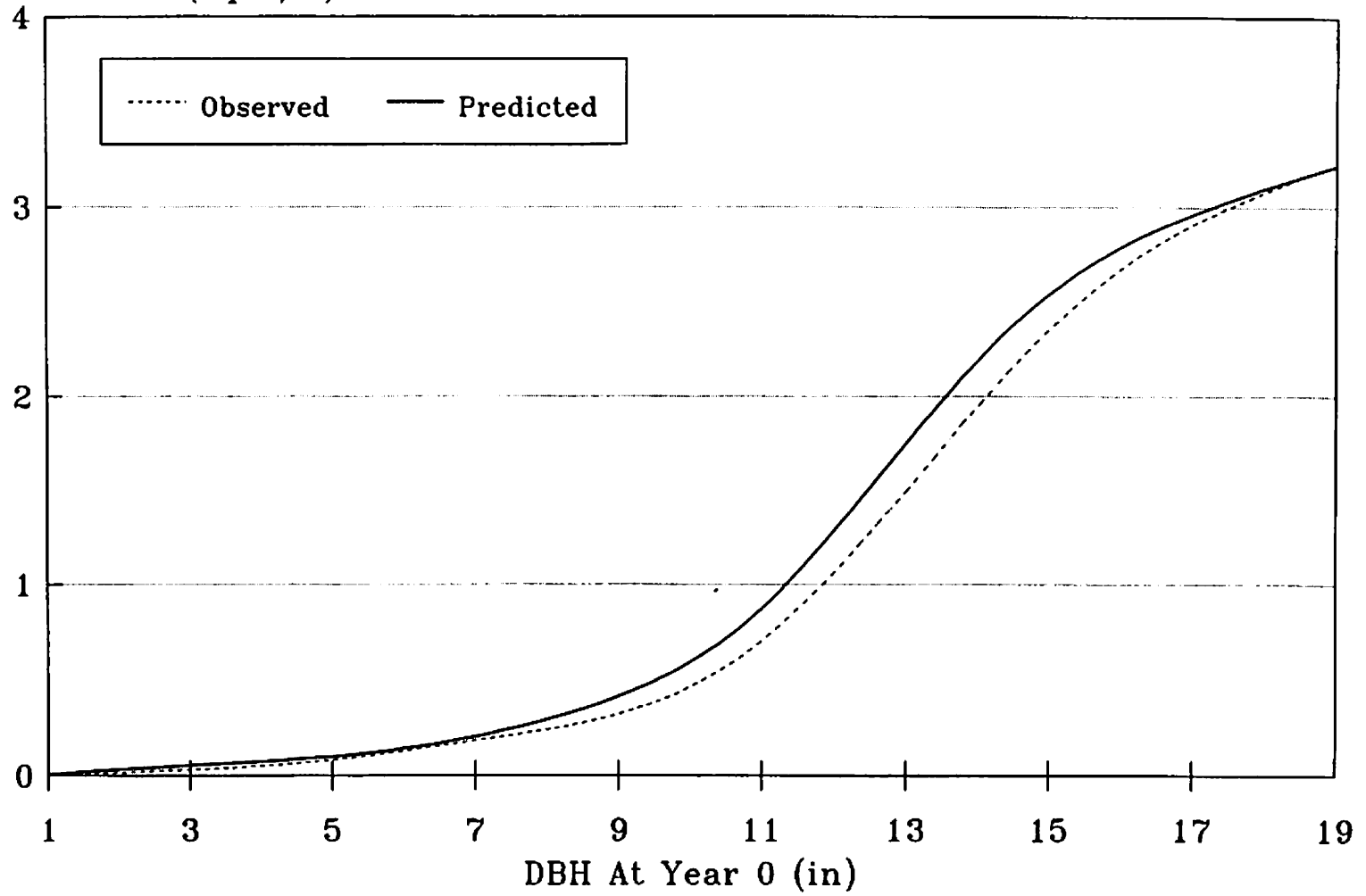


(b) 400 lb N/acre

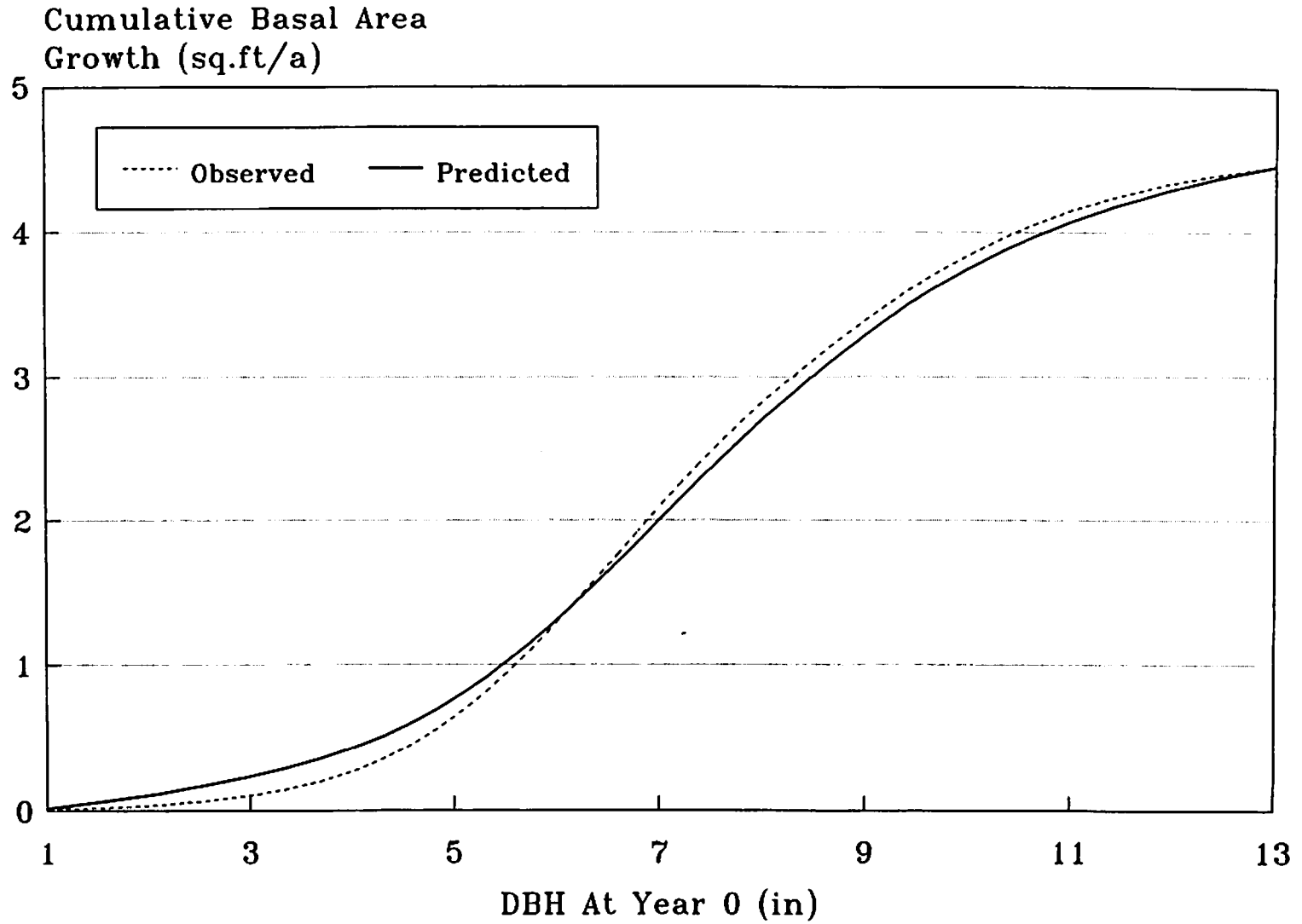


# (a) Control

Cumulative Basal Area  
Growth (sq.ft/a)



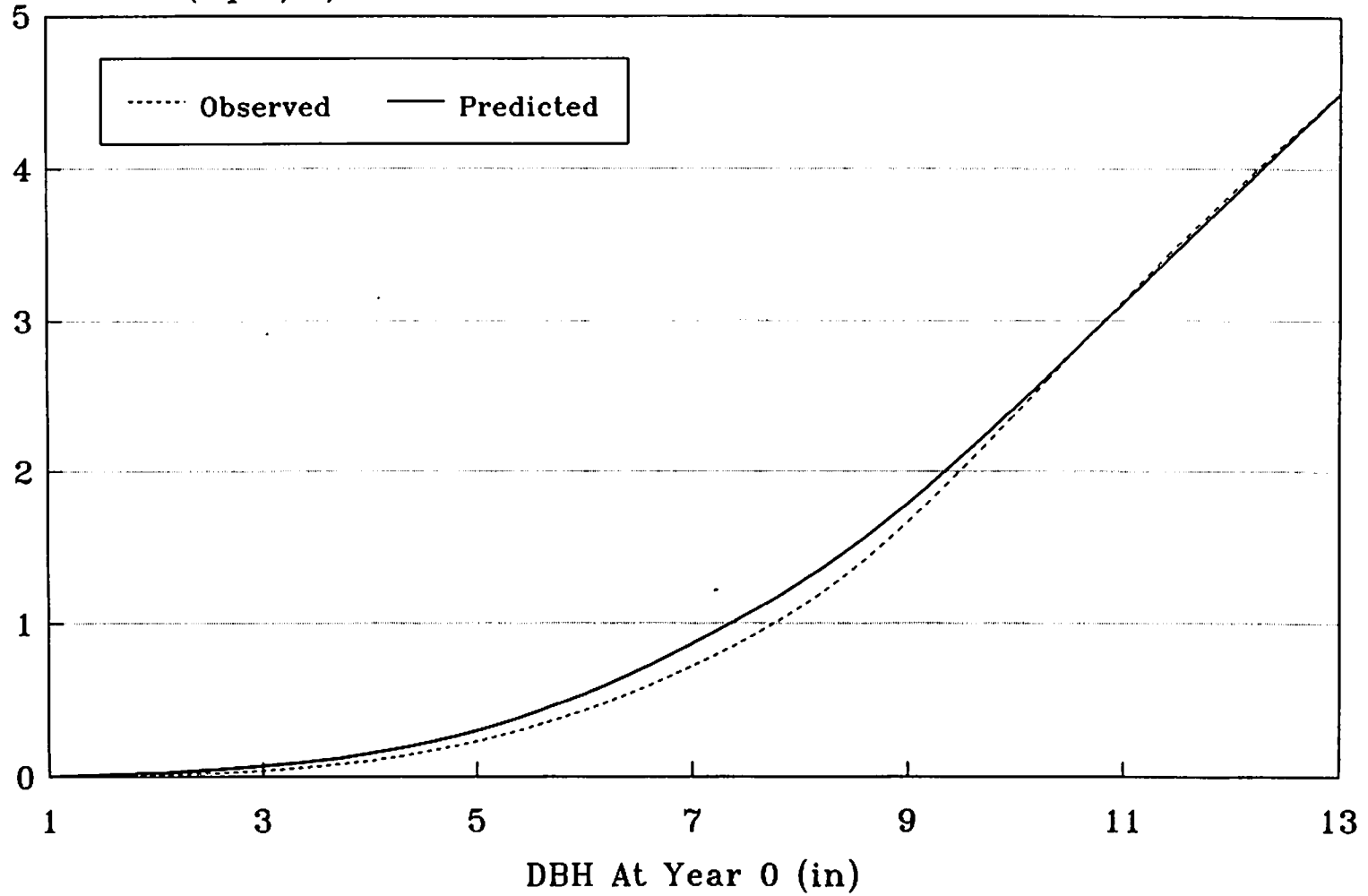
(b) Fertilized With 100 lb N/acre



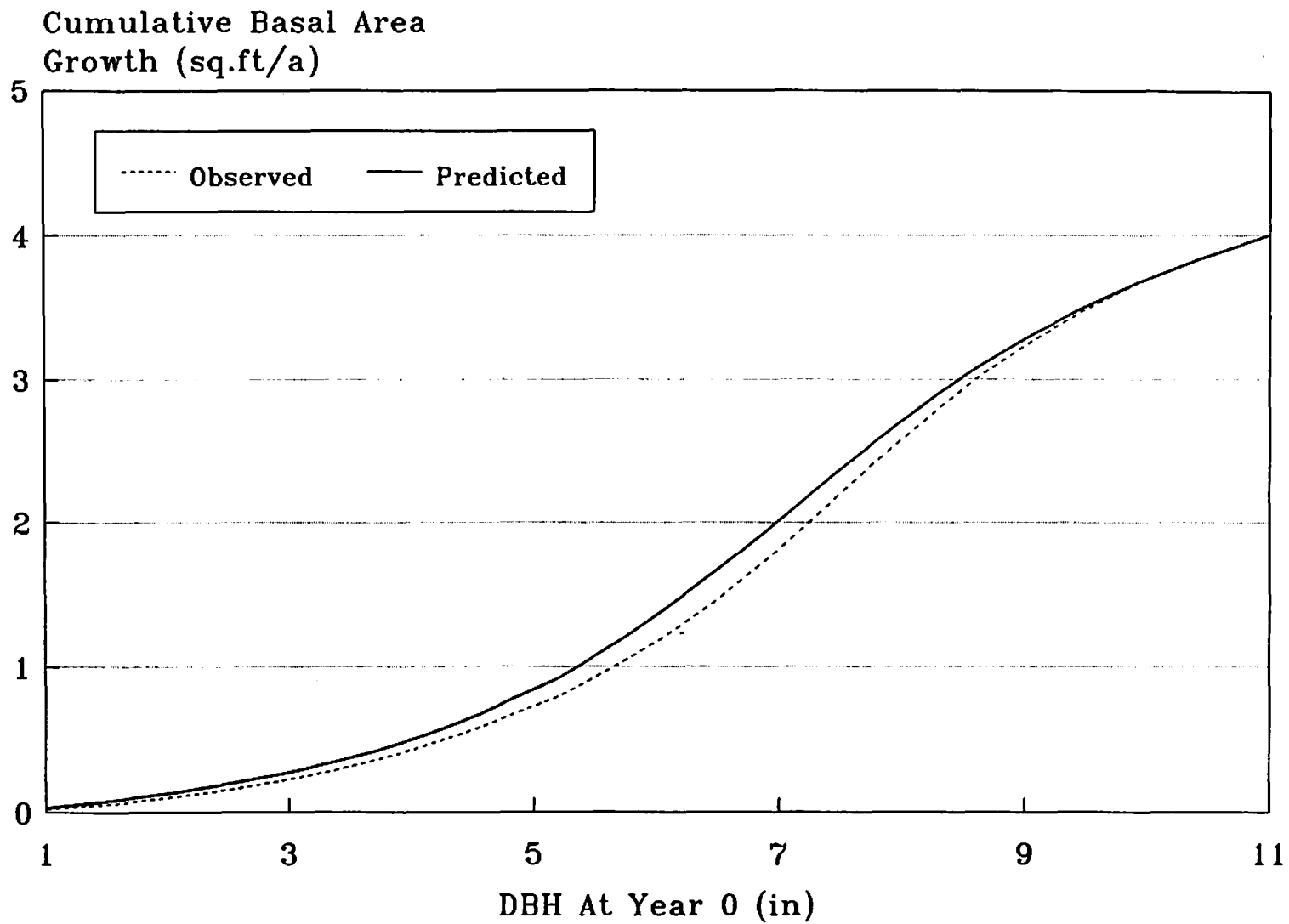


(c) Fertilized With 200 lb N/acre

Cumulative Basal Area  
Growth (sq.ft/a)

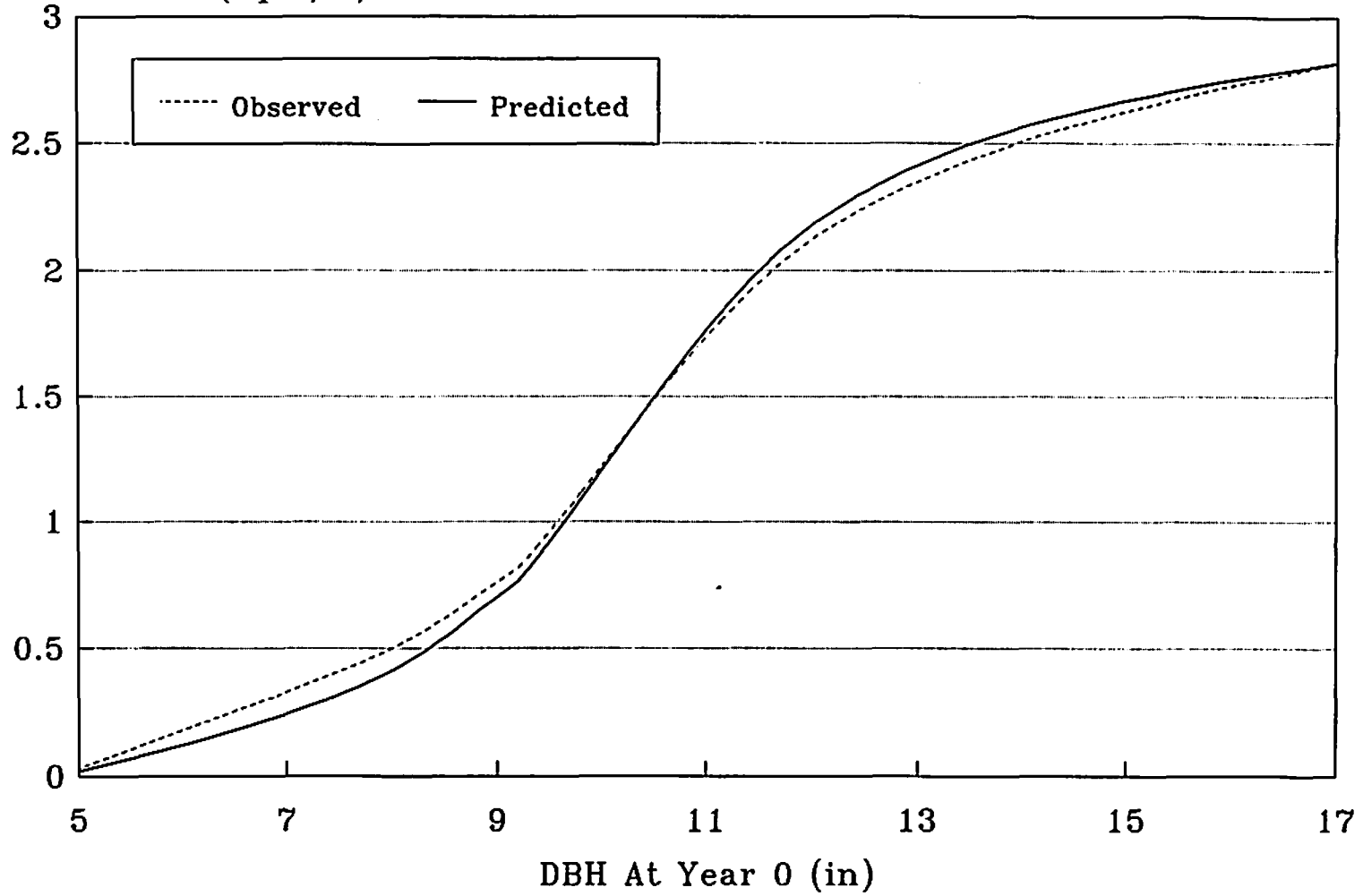


(d) Fertilized With 400 lb N/acre



(e) Thinned Only

Cumulative Basal Area  
Growth (sq.ft/a)



(f) Thinned and Fertilized 200 lb N/acre

