Research Project Technical Completion Report
Project A-025-Ida

WATER for IDAHO

Effect of Physical Properties of Porous Media on Water Movement

Principal Investigators G. L. Bloomsburg
Professor, Agricultural Engineering
University of Idaho

W. M. Carson
Assistant Professor, Bio-Resources Engineering
Nova Scotia Technical College

Water Resources Research Institute
University of Idaho
Moscow, Idaho
February 1971

RESEARCH TECHNICAL COMPLETION REPORT
PROJECT A-025-IDA

EFFECT OF PHYSICAL PROPERTIES OF POROUS MEDIA ON WATER MOVEMENT

PRINCIPAL INVESTIGATORS - G. L. Bloomsburg
Professor, Agricultural Engineering
University of Idaho

W. M. Carson
Assistant Professor
Bio-Resources Engineering
Nova Scotia Technical College

formerly

Research Technologist,
Agricultural Engineering
University of Idaho

PERIOD OF INVESTIGATION - July 1968 to June 1970

The work upon which this report is based was supported in part by funds provided by the United States Department of Interior, Office of Water Resources Research as authorized under the Water Resources Act of 1964.

Water Resources Research Institute
University of Idaho
February, 1971

ABSTRACT

Theoretical and experimental work was conducted to determine the effect of various physical properties of porous media on permeability. The theoretical work involved obtaining a solution for a linearized but complete form of the Navier Stokes equation for flow around a sphere in a rectangular array of spheres. The linearization involved making Oseen's approximation for the convective acceleration terms. The Galerkin method was used and the solution was valid up to a Reynolds number of about 10.

The experimental and analytical work showed that the Darcy equation could be used over the entire range of Reynolds numbers as long as it is realized that in the so-called non-Darcy range the coefficient of permeability is dependent on Reynolds number. Experimental data are given for the permeability of various porous materials as a function of Reynolds number with porosity as a parameter.

Bloomsburg, G.L. and Carson, W.M.

"Effect of Physical Properties of Porous Media on Water Movement." Technical Completion Report Project A-025-IDA September, 1970.

Keywords: *Permeability/Hydraulic conductivity/Porous media/*Darcy's law/Saturated flow/Groundwater.

INTRODUCTION

This project was initiated to attempt to define relationships between the permeability of unconsolidated porous materials and physical properties of the porous material such as porosity, orientation of particles and the particle size. The work was divided into three specific objectives--each of which was the subject of a graduate thesis. The experimental work was conducted on the campus while the extensive computer runs were done at the University of Idaho, Washington State University and Nova Scotia Technical College, Halifax, Nova Scotia.

OBJECTIVES

The specific objectives were:

1. To investigate theoretically and experimentally the flow of fluid around various arrays of cylindrical and spherical particles.
2. To relate, theoretically and experimentally, the coefficient of permeability for unconsolidated porous media to the particle size and porosity of the media.
3. To determine, theoretically and experimentally, a capillary pressure saturation relationship for an arrangement of spherical particles.

PROCEDURES

Objective 1.

This objective was pursued by Mr. Carson as the subject of his Ph.D. dissertation. Initially it was planned to obtain solutions for the Navier

Stokes equations (including inertia terms) around a cylinder in an array of cylinders. This is a two dimensional problem and is simpler mathematically than flow around a sphere which is a three dimensional problem. However, due to the following reasons it was decided to go directly to the problem of flow around a sphere in a rectangular array of spheres:

1. Flow through porous media is more closely approximated by flow around spheres than around cylinders.
2. The solution to the sphere problem, while being only moderately more complex, would be of far greater value for studying the microscopic phenomena of flow through porous media.
3. The sphere problem solution could be verified using energy relationships for which there is a great deal of existing data, and it would not be necessary to conduct experimental work.

The numerical method selected for the solution of the Navier Stokes equations was the Galerkin method. This method was used by Snyder et. al., (1964) to obtain a solution for flow through an array of spheres at low Reynolds number, that is, with the inertia terms neglected. It was originally hoped that a solution to the complete equations could be obtained up to a Reynolds number of 100 by the Galerkin method, but after a year of work and many hours of computer time it became apparent that this was not feasible.

The problem which was encountered is that when using the Galerkin method on a set of non-linear differential equations, it is necessary to solve a set of non-linear algebraic equations after trial solutions have been selected. This was not feasible so it was necessary to linearize the Navier Stokes equations. When this was done by substituting the superficial velocity for the velocity components in the inertia terms the algebraic equations were also linearized and a solution was obtained. The solution was only valid up to a Reynolds number of about 10 and above this failed to converge. This is the first time that a solution has been obtained for flow through an array of spheres to this high a Reynolds number.

The solution is compared with an experimental curve from Ergun (1952) by the plot of friction factor versus Reynolds number shown in Figure 1. Although the solution obtained is for a rectangular packing of spheres, it could be extended to other regular packings quite easily.

In obtaining a solution, velocity profiles around a sphere were computed. Although these appear to be reasonable they were not verified experimentally.

An abstract of Mr. Carson's dissertation is included in the appendix.

Objective 2.

The work on this objective was done by Mr. Hou for his M.S. thesis. His thesis was concerned with experimentally investigating a drag resistence theory of permeability introduced by Rumer and Drinker (1966) for flow in the Darcy range. Basically this theory relates permeability to porosity and particle size for spherical particles of uniform size by looking at the resistance to fluid flow caused by an elemental group of particles. The only unknown coefficient in this expression is a coefficient λ expressing the effect of adjacent particles on the drag on one particle.

A plot of the coefficient λ versus porosity ϕ for several different porous media is shown in Figure 2, as given by Hou (1969). Also shown are lines representing the Kozeny-Carmen equation for permeability as developed from the hydraulic radius theory. The several lines are for values of the

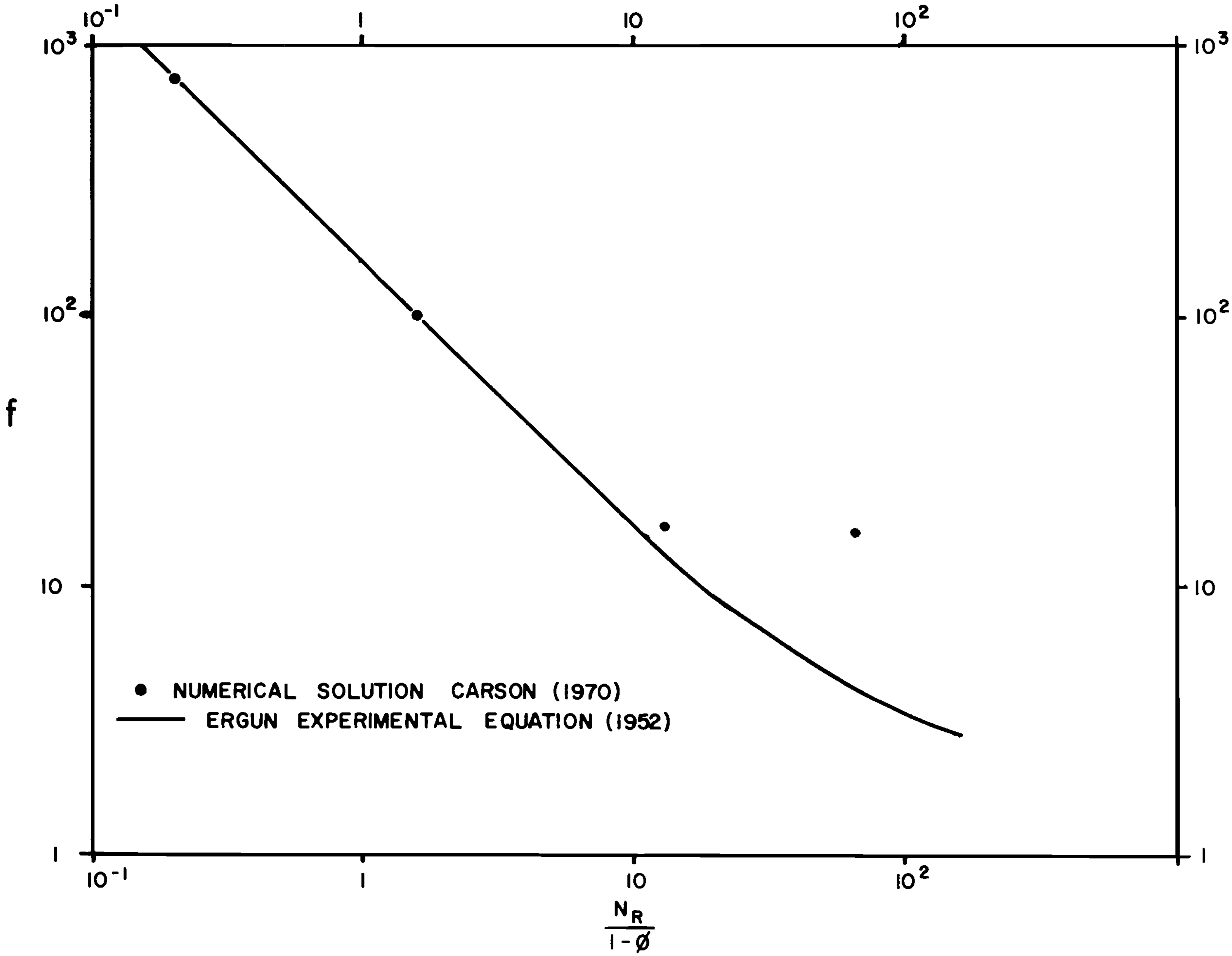

FIGURE - 1 FRICTION FACTOR VERSUS REYNOLDS NUMBER

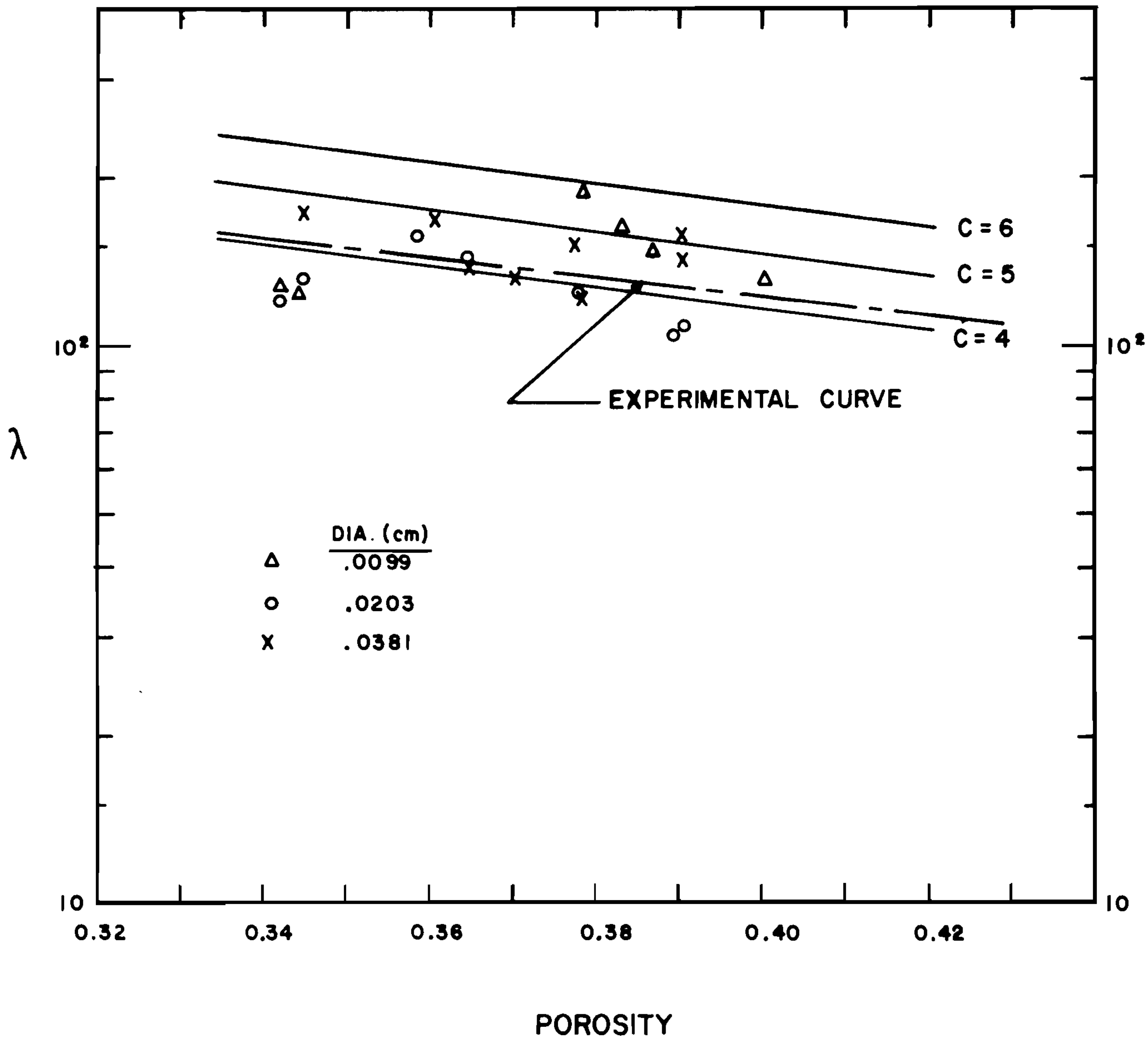

FIGURE - 2 RELATIONSHIP BETWEEN λ AND POROSITY

Kozeny-Carmen constant of 4, 5, and 6 which is the range of values ordinarily measured. In spite of the large experimental scatter of data points there appears to be agreement between the drag theory and the hydraulic radius theory.

The theoretical minimum value of λ is 3π at high porosity ($\phi \rightarrow 1.0$). From the data this value appears to be reasonable.

A great advantage of the drag theory over other theories for the development of the Darcy equation is that the upper range of the Darcy equation can be predicted. That is, from the solution of the viscous flow equations around a sphere it can be determined when the non-linear affects of the convective acceleration terms become significant. Thus, the only change in the flow equation necessary when non-linear terms are significant is the substitution of a different expression for the drag coefficient.

A disadvantage of the drag theory as compared to the hydraulic radius theory is that it does not logically lead into expressions for the effect of unsaturated conditions on the permeability coefficient as the hydraulic radius theory does. The hydraulic radius theory has been criticized because of the fact that it uses "fudge factors" such as tortuosity and a shape factor to bring about agreement with experimental data. The drag theory is not subject to this criticism which it is felt is a big improvement.

An abstract of Mr. Hou's thesis is given in the appendix.

Objective 3.

This work is being done by Mr. Roger Chen for an M.S. thesis but unfortunately is not completed at the present time. He has conducted an extensive literature review on regular packings of spherical particles and the effect of various packings on porosity. Theoretical curves for the desaturation of porous media initially filled with water will be calculated for several different porosities. These curves will be dependent on the porosities and therefore on the assumed packing of the spheres.

The experimental work consists of obtaining actual capillary pressure-saturation curves for packings at various porosities. These samples will include both random and regular packings.

The data from the random packings will be compared to the theoretical curve for the same porosity. This comparison will hopefully show whether the porosity is a satisfactory index to the average orientation or whether it is necessary to use some other orientation factor. There is some indication from the work in the following section that the porosity is a satisfactory index to the affect of orientation of particles on permeability.

Other Work

There have been numerous attempts to develop the Darcy equation in a rational manner by drawing on analogies with flow in pipes or around immersed objects. These analogies have led to various friction factors, drag coefficients and other experimental coefficients which have then been determined experimentally. Although some of the theories, particularly the drag theory of Rumer and Drinker previously mentioned, give some insight into when the permeability is no longer constant, they all introduce additional empirical coefficients.

Another unnecessary factor that is introduced is in the use of the Forchheimer equation

$$\frac{dh}{ds} = aq + bq^2$$

for the region where the Darcy law is no longer valid. In this equation $\frac{dh}{ds}$ is the hydraulic gradient, q is the superficial velocity and a and b are empirical coefficients. This equation was developed theoretically by Irmcey (1958) but has been in the literature for many years. It would be preferable to use the Darcy equation over the entire range of flow in porous media and recognize that the permeability coefficient is constant only over a certain range. It has often been shown that this upper limit is at approximately a Reynolds number of 1.0. Above this value of Reynolds number the value of the permeability coefficient could be selected from a curve of permeability versus Reynolds number.

To determine what variables to plot on such a diagram we can use dimensional analysis and assume that the following variables are of significance.

Let, $$f\ (\phi,\ q,\ d,\ \nu,\ k) = 0$$

where ϕ is porosity, q is the superficial velocity, d is some length parameter of the particle, ν is the kinematic viscosity of the fluid, and k is the permeability coefficient. According to the Buckingham π theorem the problem will be fully expressed by the relationship

$$\frac{k}{d^2} = f\left(\phi, \frac{\nu}{qd}\right)$$

$$= f\left(\phi, N_R\right)$$

where N_R is the Reynolds number. From this we can conclude that a plot of $\frac{k}{d^2}$ versus Reynolds number with porosity as a parameter should describe the relationship. It is realized that no consideration has been given to different particle shape or a gradation in particle size. It is felt, however, that if the relationship is suitable for uniform spherical particles then it can be modified by additional parameters such as particle shape or a gradation curve in the case of non-spherical and non-uniform particles.

Data from several investigators are shown in Figure 3 for uniform spherical particles. There is considerable experimental scatter in these data, but it is obvious that data from different sizes of particles with essentially the same porosity plot nearly on the same line, while data from the same sized material but with different porosity plot on a distinctly different line. It is felt that this plot gives enough evidence of the validity of the above analysis that an effort should be made to develop complete curves of $\frac{k}{d^2}$ versus Reynolds number for spherical particles packed at various porosities. If complete curves were developed it would be a simple matter to solve flow problems in spherical particles by using the Darcy equation and the permeability as selected from a curve. The procedure would be the same over the entire range of flow velocities, only the permeability coefficient would change.

To investigate whether the above analysis would work for non-spherical non-uniform particles, data on river gravel and sand given by Dudgeon (1964) were plotted in Figure 4. It is assumed that all these particles have, on the average, the same shape. There are two types of gradation curves for these materials, a quite uniform gradation for the sand, 3/4", 1 1/2" , 3" and 6" gravels and a similar curve except with a significant portion of fine material for the 1/4" and 3/8" gravel. The data for 3/4", 1 1/2", and 3" gravel are grouped together since these porosities are 0.367, 0.372 and 0.373 respectively. The sand and 6" gravel have higher values of $\frac{k}{d^2}$ since the porosities are 0.387 and 0.406 respectively. The data for 1/4" and 3/8" gravel are below the comparable porosity for the other materials but this is very likely due to the difference in the gradation curves.

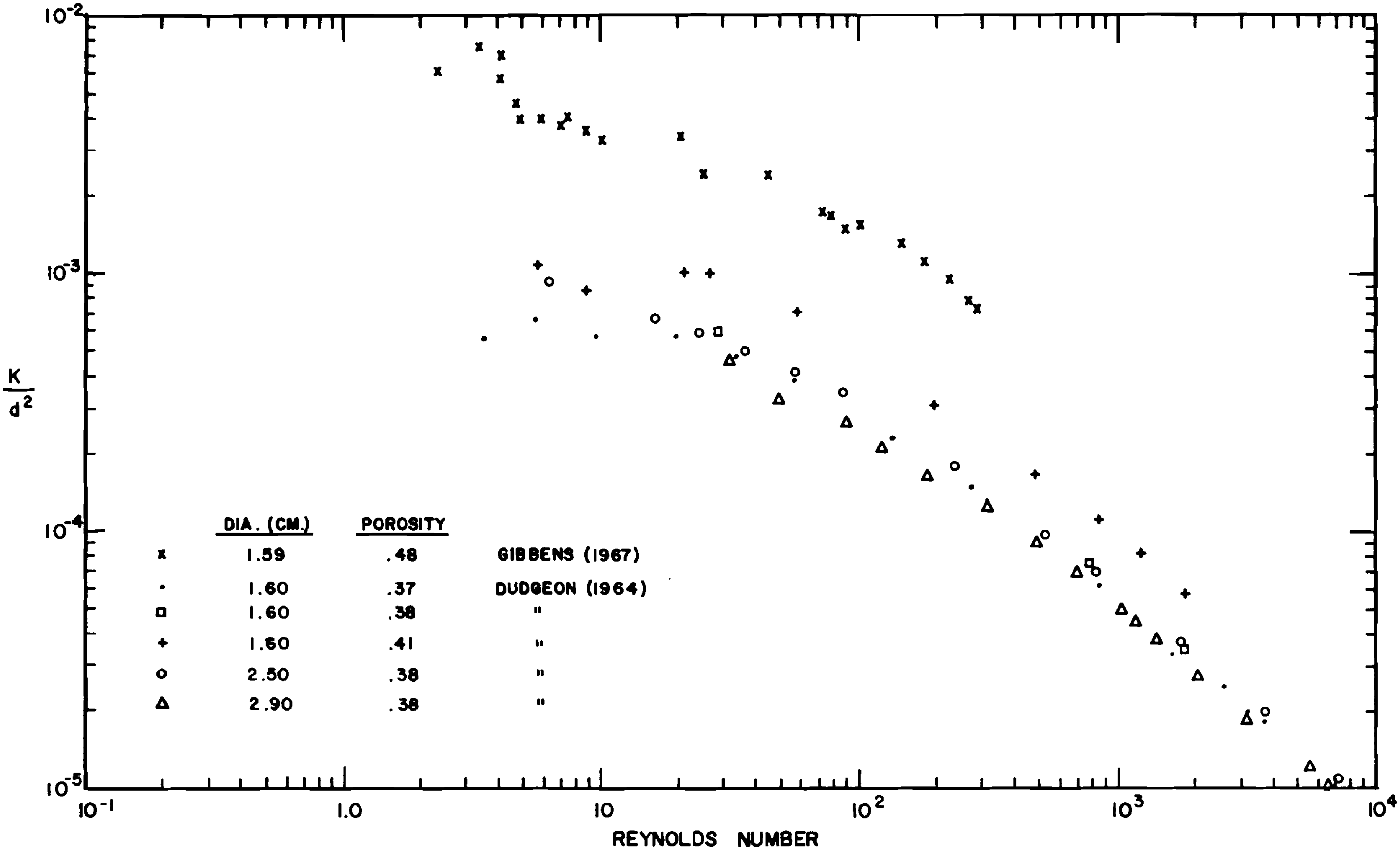

FIGURE - 3 PERMEABILITY VERSUS REYNOLDS NUMBER FOR UNIFORM SPHERES

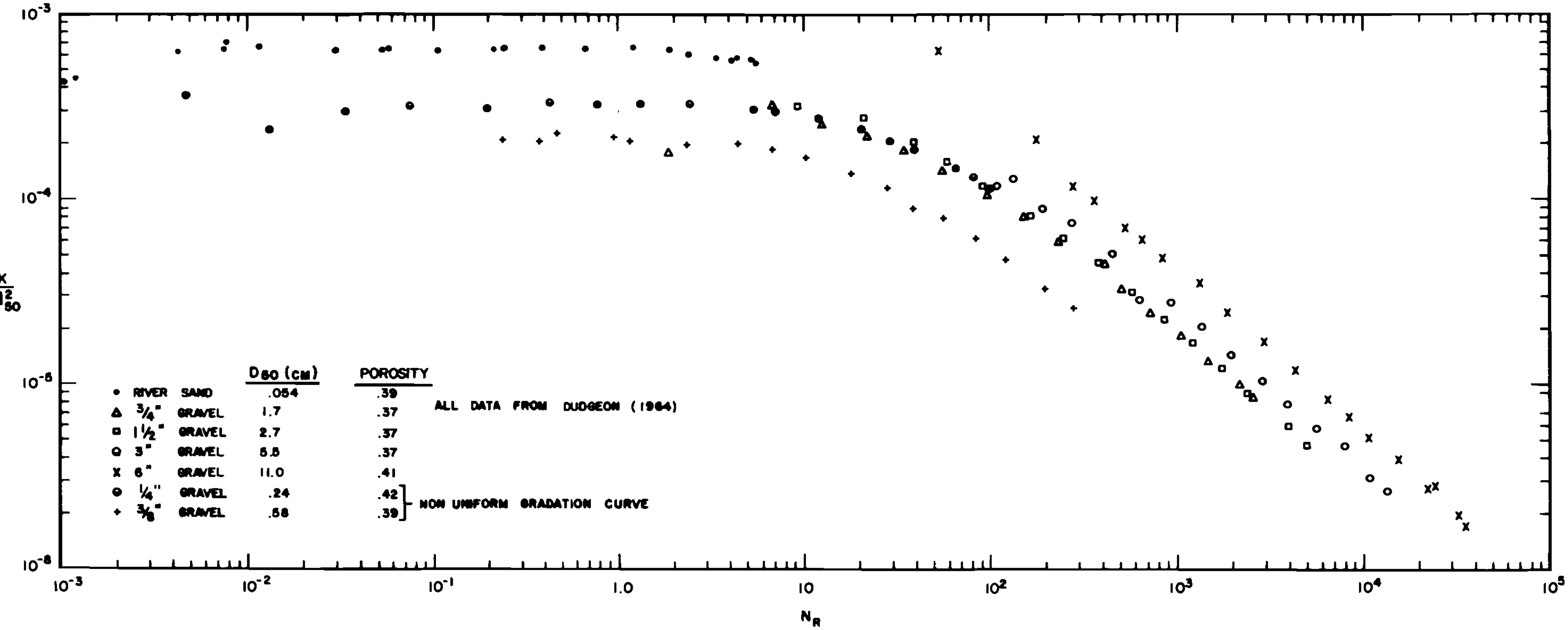

FIGURE - 4 PERMEABILITY VERSUS REYNOLDS NUMBER FOR SAND AND GRAVEL

If the data for non-spherical particles (Figure 4) are compared to that for spherical particles (Figure 3) it is noted that the permeability is less for non-spherical particles. This would be expected since the non-spherical particles do not have as uniform gradation and the individual particle is likely to have more drag than a sphere at low velocity flows.

The length parameter used in the Reynolds number and in $\frac{k}{d^2}$ is the d_{50} size picked off the gradation curve. The additional parameter that would be used to express non-uniform gradation would probably be the ratio $\frac{d^{85}}{d^{15}}$ although this was not tried. This ratio is the size of which 85% is smaller divided by the size by which 15% is smaller. This ratio would be 1.0 for a completely uniform gradation and greater than one for other gradation curves. For natural granular materials it would usually be less than 6.

Data from nearly spherical particles such as lead shot and ceramic balls have also been plotted but are not shown here. These data show essentially the same thing as Figures 3 and 4, that is, plotting the value of $\frac{k}{d^2}$ brings the data for different sizes together if the porosity is the same. If the porosity is different however, the permeability will be different.

Data for permeability as affected by porosity is shown in Figure 5 for 3 sizes of glass beads for flow in the Darcy range. According to the hypotheses in the preceding pages these data should all plot on one line. It is apparent that there is wide scatter from a single line, however, there is just as much scatter in the data from a particular size as there is in the data of different sizes. It is also apparent that the permeability increases with increasing porosity.

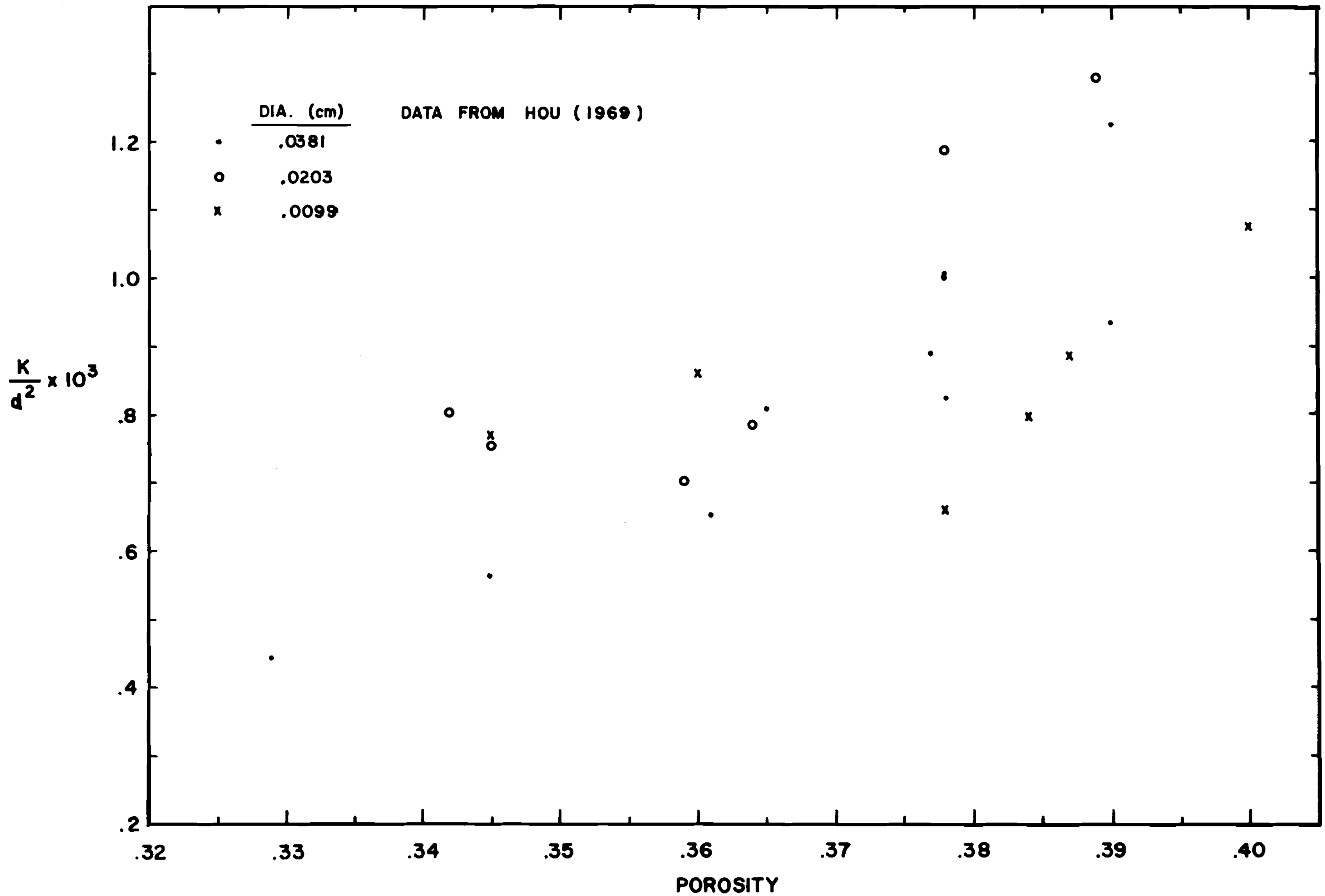

FIGURE - 5 THE EFFECT OF POROSITY ON PERMEABILITY FOR GLASS BEADS AT R < 1.0

Results and Conclusions

The various segments of this project have explored many aspects of the effect of physical properties of porous media on the permeability. It is felt that the following conclusions may be made.

1. It is impractical to solve the full Navier Stokes equations for flow around a sphere in an array of spheres. A solution was obtained up to a Reynolds number of 10 by using Oseens approximations but the computer time involved makes it impractical.
2. It is not necessary from the standpoint of application, to use any equation relating velocity and energy gradient other than the Darcy equation. It must be recognized, however, that there is a range where the permeability coefficient is no longer constant, but is a function of the Reynolds number.
3. The best variables to use in describing the effect of physical properties of porous media on the permeability are $\frac{k}{d^2}$ and Reynolds number with porosity and some function of the gradation curve as parameters.

Recommendations for Further Work

1. Data should be obtained to develop a complete set of curves of $\frac{k}{d^2}$ versus Reynolds number for porous media made of spherical uniform particles at porosities ranging from 0.26 to 0.47. Both regular and random packings should be used.
2. Data should be obtained to develop similar curves for sand and gravel with both uniform and non-uniform gradation curves. These

data should be accurate enough that reproducibility can be assured. The curves developed from this data could then be used in the determination of permeability values for various materials.

REFERENCES

Carson, W. M., (1970) Approximate solution of the Navier Stokes equations for flow through a rectangularly packed bed of spheres. University of Idaho, Ph.D. dissertation.

Dudgeon, C. R., (1964) Flow of Water through coarse granular materials, Report No. 76, Water Research Laboratory, University of New South Wales, Australia.

Ergun, S., (1952) Fluid flow through packed columns, Chemical Engineering Progress, Vol. 48, No. 2.

Gibbens, M. R., (1967) Laminar and turbulent flow through spherical particles, University of Idaho, M.S. thesis.

Hou, J. P. L., (1969) Relation of permeability to particle size and porosity, University of Idaho, M.S. thesis.

Irmay, S. (1958) On the theoretical derivation of the Darcy and Forschheimer formulas, Transactions of the American Geophysical Union, Vol. 39, No. 4.

Rumer, R. R., and Drinker, P., (1966) Resistance to laminar flow through porous media, Journal Hydraulics Division, ASCE, Vol. 92, No. 5.

Snyder, L. J., Spriggs, T., and Stewart, W. E., (1964) Solution of the equations of change by Galerkins method, American Institute of Chemical Engineers Journal, Vol. 10, No. 4.

PUBLICATIONS ARISING FROM THIS WORK

Carson, W. M, (1970) Approximate solution of the Navier Stokes equations for flow through a rectangularly packed bed of spheres. University of Idaho, Ph.D. dissertation, August, 1970.

Hou, J. P. L., (1969) Relation of permeability to particle size and porosity, University of Idaho, M.S. thesis, June, 1969.

Carson, W. M. and Bloomsburg, G. L., Oseen approximation for packed bed flow - Velocity profiles and friction factors to be submitted for publication in Transactions American Society of Agricultural Engineers.

APPENDIX

ABSTRACT

W. M. Carson

Approximate solutions for the Navier-Stokes equations describing fluid flow through a rectangular packing of spheres were obtained for Reynolds numbers of 0.1, 1, 7 and 35.

Initial attempts to solve the Navier-Stokes equations with the inertia terms intact were unsuccessful. However, the methods used in these solution attempts are given in detail.

The results reported are based on an Oseen linearization of the full Navier-Stokes equations. The solutions were approximated by triple trigonometric series and the unknown coefficients evaluated using the Galerkin method for error distribution.

Velocity components and pressure in the void space of the bed are given as explicit functions of the spacial coordinates. Friction factors for the packed bed and superficial velocity were evaluated from the velocity functions and are shown to agree with the experimental observations of previous investigators.

The viscous and kinetic contribution to the energy dissipation are partitioned using first principles of the mechanical energy balance and evidence is given that the viscous and kinetic effects determined by semi-empirical methods do not show the actual relationship between viscous and kinetic losses in the intermediate Reynolds number range.

Based on friction factor and superficial velocity, the Oseen linearization is shown to be valid for packed flow at Reynolds numbers less than seven, and invalid for a Reynolds number of 35.

Suggestions for future research are included.

ABSTRACT

J. P. L. Hou

Different theories dealing with the relationship between porosity and permeability in porous media are presented. The laboratory work conducted in this project to test the validity of Rumer and Drinker's drag theory is described in the thesis.

The theory shows that a drag coefficient should depend only on porosity for flow in the Darcy range. Data obtained in the experimental work show that λ does depend on porosity but there is considerable variation in λ which is not explained by the variation of porosity. The data also show that λ varies with Reynolds number beyond the Darcy range.