

# Development of a Mathematical Groundwater Model 

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A Thesis for the Degree of Master of Science in Civil Engineering

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# DEVELOPMENT OF A MATHEMATICAL GROUNDWATER MODEL 

A Thesis<br>Presented in Partial Fullfillment of the Requirements for the DEGREE OF MASTER OF SCIENCE<br>Major in Civil Engineering<br>in the<br>UNIVERSITY OF IDAHO GRADUATE SCHOOL<br>by<br>JOSEPH L. J. DE SONNEVILLE

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#### Abstract

The development of a mathematical model for study of water management and budget of the irrigated area of the Snake River Fan in eastern Idaho was initiated in May, 1970. The study was initiated to develop alternate management solutions for correcting the problem of increasing high water table problems which has been causing inconvenience and financial hardships to local residents. A more fundamental reason for the study was to develop methods of solving regional groundwater problems. Water: levels in wells raise as much as 40 feet during the irrigation season to within one - three feet of the surface in some areas.

The study area has a dense network of canals with seepage rates averaging 3.5 cubic feet per square foot per day, irrigated soils with high infiltration rates and in some cases inefficient irrigation practices have contributed to the fact that diversions for irrigation are significant in excess of the state average. In 1970 the net diversion from the Snake River was 16.5 acre feet per irrigated acre and the net irrigation application was 9.6 acre feet per acre for the entire irrigation season.

The mathematical model is a digital model and utilizes alternating direction implicit procedures for finite difference solutions to the basic flow equations. A special treatment of a flow boundary is incorporated in the model. Data on water table fluctuation, soils, crop distribution, irrigation diversions, distribution system losses and wastes, evapotranspiration, and irrigation practices were obtained for input to the model. The input data to the model is evaluated in two separate input programs in order to keep the actual program of the model as general as possible.

An attempt is made to apply the model to the study area.


## BIOGRAPHICAL SKETCH OF THE AUTHOR

The author was born in Beverwijk, The Netherlands, on February 3, 1947. He attended the Pius X College for five years. In September, 1964, he entered the University of Technology of Delft majoring in Civil Engineering from where he received his candidate's degree in February, 1970. During these years, the author was increasingly interested to study in the United States to broaden his knowledge as well as to personally experience the social environment of another country.

In the summer of 1970, he applied for one of the fellowships and assistantships offered by the Universities Council on Water Resources to the United States, a program for the International Hydrological Decade (UCOWR/IHD, 1965-1974). This was encouraged by Professor C. C. Warnick, Director, Water Resources Research Institute, University of Idaho who at that time was in The Netherlands. In August, 1970, the Author came to the United States and entered the University of Idaho to continue his study.

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## CHAPTER I

## INTRODUCTION

The research study on water use was initiated in the Rigby-Ririe area of the upper Snake River Basin about May 1, 1970. Concern for the high water table problems being experienced in this area resulted in the formation within Jefferson County of a sub-water committee to investigate possible causes and solutions to the problem. This committee has been very cooperative with the University and the Agricultural Research Service in formulating procedures for data collection and in securing some field data. The County Commissioners of Jefferson County also have been cooperative and have supplied at the county court house an office for the full-time field man presently working in the area. Meetings have been held with the Jefferson County Commisioners and with the members of the Great Feeder Canal Board under whose jurisdiction a majority of the irrigation water is delivered to this area. Good cooperation has been received from the local farmers in the area and from the government agencies involved.

In August 1970 a questionnaire was sent to residents of Jefferson County requesting information on any damage or problems associated with the sub-water. A total of 700 questionnaires were mailed with the cooperation of the University of Idaho Extension Service. Seventy-eight affirmative returns were received which indicated problems such as water in basements, potato cellars, flooded fields or corrals. The locations of the affirmative returns closely correlated with the high water table areas as indicated in Figure 1. Estimates of damages for 1970 amounted to nearly $\$ 24,000$.


## Objectives

Recognizing that the problem of high groundwater tables and drainage that exists in the Rigby - Ririe area is typical also of other areas in the State of Idaho and because of the known high rates of irrigation diversions to this area, this project area affords an excellent opportunity to study various aspects of water management in irrigation distribution systems.

Therefore, the objectives of this study are:

1. To develop a suitable mathematical model such that theoretical solutions can be obtained in the form of flow patterns for a general three dimensional nonhomogeneous groundwater basin with any water table configuration.
2. To investigate, using the mathematical model, the qualitative and quantitative changes in inputs to the aquifer and the underlying geology, the configuration of the water table and the ground water flow system in the Snake River Fan in eastern Idaho.

## Study Area

The water management study area as shown in Figure 2 is located in Jefferson and Bonneville counties in Townships 3, 4, and 5N and Ranges 37 through 40E of the Boise Meridian. The area selected for study comprises approximately 100,500 acres. The city of Rigby, Idaho, is located approximately in the center of the study area and is the county seat of Jefferson County. Other communities in the area are Lewisville, in the western part of the area; Menan and Lorenzo in the


Figure 2. Location of the Study Area.
northern part of the area; and the city of Ririe in the eastern part. This area which is an old alluvial fan of the Snake River is served by an irrigation system which was developed in the late 1800's by private and cooperative groups. A former channel of the Snake River runs east and west through the area and is used as a canal for the greater part of its length. This canal, referred to as the Great Feeder Canal, delivers water to some 17 smaller canals each one operated by a separate and independent canal company or irrigation district. The lands served by canals from the Great Feeder system and other canals diverting directly out of the Snake River have some of the earliest water rights in the upper Snake River Basin. As a result of these early water rights, water for irrigation has generally not been in scarce supply. With the construction of the Palisades Dam and reservoir in 1954 by the U. S. Bureau of Reclamation, new storage rights purchased by the individual canal companies have firmed up the irrigation water supply so that a shortage of water is not likely to occur.

The soils of the area are generally quite coarse, varying in texture from east to west along the alluvial fan. The material from which the irrigation canals are excavated is generally rocky and quite permeable, especially in the upper areas of the fan.

## CHAPTER II

## LITERATURE REVIEW

In 1940, M. King Hubbert published his classic paper, "The Theory of Ground-Water Motion". In this paper the physical laws governing the steady state flow of groundwater were presented for the first time in their exact mathematical framework. Later studies concerning groundwater hydrology are that from Jacob (1950), Todd (1959) and De Wiest (1965), which are applied in the determination of local aquifer conditions (well and well fields). This resulted in the mathematical solution of almost all the practical problems in well hydraulics (19351960).

In the 1960's attention has turned to the regional picture and the groundwater basin has been established as an acceptable unit of hydrological study. The publications of J. Toth $(1962,1963)$ are in this respect very important. As in Toth's work the solutions therefore were limited to homogeneous media and apply only to the specific cases treated. There is a need to extend the solutions to more general cases. The mathematical expression governing the flow of water through porous mediums can be determined from the equations of motion and continuity, and the laws of thermodynamics. This leads to a nonlinear multidimensional equation which is most amenable to computer solutions.

Computer solutions have been obtained by analog (Tyson and Weber, 1964), digital (Weber, Peters, and Frankel, 1968), and hybrid (Vemuri and Dracup, 1967; Femuri and Karplus, 1964) computers. Versatility in generality of the models is dependent on the extent of the simplifying assumptions used to solve the groundwater equation.

A short summary is given about further publications in this field:

- 'Selected Analytical Methods for Well and Aquifer Evaluation' by William C. Walton, 1962. The practical application of selected analytical methods to well and aquifer problems in Illinois is described in this report. Aquifer tests are useful tools in this respect.
- 'Analyzing Ground-Water Problems with Mathematical Models and a Digital Computer,' by W. C. Walton and J. C. Neill, 1961. In this paper complex aquifer conditions are simulated with a mathematical model based on the image-well theory and the non-equilibrium formula. Problems associated with geohydrologic boundaries are simplified to consideration of an infinite aquifer in which real and image wells operate simultaneously. The image well theory is applied to multiple boundary systems by taking into consideration successive reflections on boundaries.
- 'A Mathematical Model of an Existing Municipal Well Field' by Richard N. De Vries, 1968. In this case a mathematical model using the Theis non-equilibrium equation was formulated. Recharging image wells were used to represent the physical boundary of the aquifer.
- 'Analysis of Nonlinearities in Ground Water Hydrology: A Hybrid Computer Approach,' by Vemuri and Dracup, 1967. A hybrid computer method to solve nonlinear partial differential equations describing the flow of fluids in underground formations is discussed. Using finite difference methods, the
problems are first programmed by flow chart for a pure digital computer solution, however, major matrix inversion subroutines in the digital flow chart are replaced by an analog resistance network hardware resulting in a reduction of computation time.
- 'Digital Model of Alluvial Aquifer,' by Peter C. Trescott, George F. Pinder, and John F. Jones, 1970. In this paper a mathematical model using finite difference was developed to simulate the response of a confined aquifer to pumping from one or more wells. The groundwater reservoir may be irregular in shape and nonhomogeneous with infiltration from one or more lakes and streams.
- 'A Digital Model for Aquifer Evaluation,' by George F. Pinder, 1970. This program simulates the response of a confined aquifer to pumping at a constant rate from one or more wells. The groundwater reservoir may be irregular in shape and nonhomogeneous with infiltration from one or more lakes and streams.
- 'Digital Computer Simulation of an Áquifer: A Case Study,' by Hassan Dabiri, Don Green, and John Winslow, 1970. A digital computer model, based on the alternating - direction implicit method as is used by Pinder, is applied in a study of the Equus-beds aquifer of the Wichita, Kansas, water-well field. This model was developed for a confined aquifer. It may be used in an unconfined aquifer if the draw-
down does not change significantly in relation to the saturated thickness.


## CHAPTER III

## SYSTEM SIMULATION

In order to efficiently determine and evaluate the alternative solutions for relieving the high groundwater table, a method of predicting the response of the aquifer to varying degrees of change in the input and output is necessary. Analytical solutions to the basic flow equations are not applicable because of the complexity of the hydrogeologic system, varying boundary conditions and high degree of simplification needed to secure a solution. Analog models of the resistance - capacitance type would suffice. However, despite the reduction of computation time, construction of an analog is costly and the size and flexibility required for this study would be difficult to achieve.

The availability of large digital computers and new finite difference techniques for solving the flow equations make a digital model most feasible. The General Electric time-share unit in Los Angeles is available through the terminal at the Snake River Conservation Research Center and access is available to the large IBM 360-75 system of the Atomic Energy Commission in Idaho Falls.

The model being designed describes the response of the aquifer due to a wide range of conditions. It is general enough to be applied to other aquifers and areas and will accommodate non-homogeneous, anisotropic, confined and unconfined aquifers. All boundary conditions normally encountered can be handled, while experimental treatment is given to the flow boundary problems. Inputs to the modeled area include precipitation, canal seepage, irrigation application, river losses, or drainage well inputs. Output includes crop evapotranspiration, well
discharges, natural spring discharges and aquifer leakage. Inputs and outputs for this model may be different for each node and variable in time.

## Mathematical Theory

Forces

There are two forces acting on a fluid element. The forces may be classified as driving or resisting forces according to whether they tend to produce the motion or are a consequence of the motion.

Pressure gradient and gravity represent the two driving forces acting on fluid elements.

If $\rho$ is constant in space, it is possible to combine the two potentials. The combined potential can be expressed in three different ways:

$$
\begin{array}{ll}
p^{*}=p+\rho g h & \text { (energy/volume) } \\
H=\frac{p}{\rho g}+h & \text { (energy/weight) } \\
\phi=\frac{p}{\rho}+g h & \text { (energy/mass) }
\end{array}
$$

The driving force acting on a fluid element can be expressed as the negative gradient of the potential which existence is related to the space coordinates.

The resisting force is present in a nonstatic fluid system. For flow in porous media, the resisting force is a surface force acting tangentially on the surface of fluid elements called shear.

Shear is related to the rate of angular deformation of the fluid elements. For Newtonian fluids the relationship is linear over a wide range. (The coefficient of proportionality is called viscosity). The force per unit-volume can be expressed as:

$$
\mathrm{f}_{\tau}=\mu \nabla^{2} \underline{\mathrm{~V}} \quad \text { in which } \underline{\mathrm{v}} \text { is the total velocity vector. }
$$

## Fluid acceleration



The three components of $\underline{v}(u, v, w)$ can be written out as follows:

$$
\begin{aligned}
& \frac{d u}{d t}=u \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+w-\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t} \\
& \frac{d v}{d t}=u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+w \frac{\partial v}{\partial z}+\frac{\partial v}{\partial t} \\
& \frac{d w}{d t}=u \frac{\partial w}{\partial x}+v-\frac{\partial w}{\partial y}+w \frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}
\end{aligned}
$$

The partial derivatives with respect to space coordinates are known collectively as the convective acceleration. The terms containing the partial derivative with respect to time are known as the local acceleration.

Because the component velocities are very small in porous media the assumption is made that the convective terms can be neglected. For many cases of unsteady flow in porous material, the local acceleration is small and can be neglected.

By assuming that the acceleration term is negligible, it is implied that the inertial effects on the fluid elements are negligible. With this assumption, it can be said that for these cases Darcy's Law is valid.

## Equation of motion

According to Newton the rate of change of momentum with respect to time of an element of mass is equal to the resultant force acting on the element. It follows (Navier-Stokes) that

$$
\begin{equation*}
\rho \frac{\mathrm{D} \underline{v}}{\mathrm{Dt}}=-\nabla \mathrm{p}^{*}+\mu \nabla^{2} \underline{v} \tag{1}
\end{equation*}
$$

acceleration $=$ driving force + resisting force

The acceleration term for laminar flow in porous media is assumed to be zero so that equation (1) reduces to

$$
\begin{equation*}
\nabla \mathrm{p}^{*}=\mu \nabla^{2} \underline{\mathrm{v}} \tag{2}
\end{equation*}
$$

$\underline{v}$ represents the actual velocity of the fluid elements within the pore space. For transformation of this equation (2) to one in which the velocity is an average velocity that refers to an element of space including the porous matrix as well as the enclosed fluid, an empirical correlation has been found between the average velocity or seepage velocity, $q$, and $\nabla^{2} \underline{\mathrm{~V}}$. This is the Darcy ${ }^{5}$ equation and is written as follows:

$$
\begin{equation*}
\nabla \mathrm{p}^{*}=-\frac{\mu}{\mathrm{k}} \mathrm{q} \quad \text { or } \quad \mathrm{q}=-\frac{\mathrm{k}}{\mu} \quad \nabla \mathrm{p}^{*} \tag{3}
\end{equation*}
$$

in which $k$ is proportionality constant that is dependent on the shape and orientation of the pore spaces. Substituting

$$
H=\frac{p^{*}}{\rho g} \quad \text { and } \quad K=\frac{k \rho g}{\mu}
$$

equation (3) is written as:

$$
\begin{array}{ll}
\mathrm{q}=-\mathrm{K} \quad \nabla \mathrm{H} \quad \mathrm{H} & =\text { hydraulic head }  \tag{4}\\
\mathrm{K} & =\text { hydraulic conductivity }
\end{array}
$$

## Equation of continuity

The principle of conservation of mass for a volume element requires:

Mass inflow rate - mass outflow rate $=$ change of mass storage in time. This relationship can be expressed as:

$$
\begin{equation*}
-\operatorname{Div}(\rho q)=\rho(\alpha+n \beta) \frac{\partial p}{\partial t} \tag{5}
\end{equation*}
$$

where $\alpha=$ the reciprocal of Es, the bulk modulus of elasticity of the intergranular skeleton.
where $\beta=$ the reciprocal of Ew, the bulk modulus of elasticity of water.
or

$$
\begin{equation*}
-\left[\frac{\partial\left(\rho v_{x}\right)}{\partial x}+\frac{\partial\left(\rho v_{y}\right)}{\partial y}+\frac{\partial\left(\rho v_{z}\right)}{\partial z}\right]=\rho(\alpha+n \beta) \frac{\partial p}{\partial t} \quad \text { or, } \tag{6}
\end{equation*}
$$

$$
-\rho\left[\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{y}}{\partial y}+\frac{\partial v_{z}}{\partial z}\right]-\left[v_{x} \frac{\partial \rho}{\partial x}+v_{y} \frac{\partial \rho}{\partial y}+v_{z} \frac{\partial \rho}{\partial z}\right]
$$

$$
\begin{equation*}
=\rho(\alpha+n \beta) \frac{\partial p}{\partial t} \tag{7}
\end{equation*}
$$

Writing

$$
\rho \beta \mathrm{d} p=\mathrm{d} \rho, \quad \rho=\rho(x, y, z, t)
$$

or

$$
\begin{equation*}
\frac{\partial \rho}{\partial x}=\rho \beta \frac{\partial p}{\partial x}, \frac{\partial \rho}{\partial y}=\rho \beta \frac{\partial p}{\partial y} \text { and } \frac{\partial \rho}{\partial z}=\rho \beta \frac{\partial p}{\partial z} \tag{8}
\end{equation*}
$$

With substitution of these terms, equation (7) contains partial derivatives of $p$ with respect to the space coordinates and time and the terms $v_{X}, v_{Y}$, and $v_{z}$. Now,

$$
\begin{equation*}
v_{x}=-K x \frac{\partial H}{\partial x}, \quad v_{y}=-K y \frac{\partial H}{\partial y}, \quad v_{z}=-K z \frac{\partial H}{\partial z} \tag{9}
\end{equation*}
$$

If $\rho$ is a constant*, it follows that:

$$
\begin{align*}
& \frac{\partial p}{\partial x}=\rho g \frac{\partial H}{\partial x}, \frac{\partial p}{\partial y}=\rho g \frac{\partial H}{\partial y}, \frac{\partial p}{\partial z}=\rho g \frac{\partial H}{\partial z} \text { and } \\
& \frac{\partial p}{\partial t}=\rho g \frac{\partial H}{\partial t} \tag{10}
\end{align*}
$$

Substituting these relationships into equation (7) and for convenience substituting $h$ for $H$ :

$$
\begin{align*}
& \frac{\partial}{\partial x}\left(K x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K z \frac{\partial h}{\partial z}\right)+\rho \beta g\left(K x\left(\frac{\partial h}{\partial x}\right)^{2}+K y\left(\frac{\partial h}{\partial y}\right)^{2}\right. \\
&\left.+K\left(\frac{\partial h}{\partial z}\right)^{2}-K z \frac{\partial h}{\partial z}\right)=\rho g(\alpha+n \beta) \frac{\partial h}{\partial t} \tag{11}
\end{align*}
$$

Since the terms $\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}$ and $\frac{\partial h}{\partial z}$ are small, the quadratic terms are small and can be neglected. This leads to:

$$
\begin{equation*}
\frac{\partial}{\partial x}\left(K x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K z \frac{\partial h}{\partial z}\right)-\rho \beta g K z \frac{\partial h}{\partial z}=\rho g(\alpha+n \beta) \frac{\partial h}{\partial t} \tag{12}
\end{equation*}
$$

In order to solve this equation another simplification is made by considering water as an ideal fluid. This implies that $\beta$ (already very small) becomes 0 , which leads to:

$$
\begin{aligned}
& \frac{\partial}{\partial x}\left(K x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K z \frac{\partial h}{\partial z}\right)=\rho g(\alpha) \frac{\partial h}{\partial t} \\
& \rho g \alpha=S s
\end{aligned}
$$

As can be seen this equation is a confined aquifer equation with no terms for leakage or dewatering of the aquifer present. The only *In order to convert partial derivatives of $p$ into partial derivatives with
respect to $H, a$ constant $\rho$ is assumed.
conflict in the use of $\beta$ which is deads to a The impact of this upon the validity of equation (11) is nullified by further simplifying assumptions.
way that water can be released is by the compression of the intergranular skeleton since water released due to the expansion of water is not considered $[\beta \approx 0$ ]. In order to be able to deal with vertical leakage and dewatering of the aquifer the equation can be modified without difficulty by adding a flux term $\mathrm{W}(\mathrm{x}, \mathrm{y}, \mathrm{t})$. The storage coefficient S , which represents the storage in the saturated depth of aquifer is equal to Ss.b where $b$ is the saturated thickness of the aquifer. If $b$ is variable in time then equation (13) applies to water table conditions.

$$
\begin{equation*}
\frac{\partial}{\partial x}\left(K x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K z \frac{\partial h}{\partial z}\right)=\frac{S}{b} \frac{\partial h}{\partial t}+W^{\prime}(x, y, t) \tag{14}
\end{equation*}
$$

where $W^{\prime}(x, y, t)=\frac{W(x, y, t)}{b}$
This general equation can be reduced to a more simple approximate equation which is easier to solve mathematically if it is assumed that vertical velocities in the aquifer are small compared to horizontal velocities.

This means that $\frac{\partial h}{\partial z}$ can be disregarded with respect to $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ and implies that the hydraulic head $h$ can be considered to be constant over the height of the aquifer $b$. This is actually the Dupuit-Forchheimer ${ }^{10}$ assumption which is a good approximation if the gradient of the water table is small. Observation of the properties of the Snake River Fan Area indicatesthat this assumption can be made. With this assumption equation (14) reduces to:

$$
\begin{gather*}
\frac{\partial}{\partial x}\left(K x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K y \frac{\partial h}{\partial y}\right)=\frac{S}{b} \frac{\partial h}{\partial t}+W^{\prime}(x, y, t)  \tag{15}\\
\text { or } \\
K x \frac{\partial^{2} h}{\partial x^{2}}+\frac{\partial K x}{\partial x} \frac{\partial h}{\partial x}+K y \frac{\partial^{2} h}{\partial y^{2}}+\frac{\partial K y}{\partial y} \frac{\partial h}{\partial y}=\frac{S}{b} \frac{\partial h}{\partial t}+W^{\prime}(x, y, t) \tag{16}
\end{gather*}
$$

## Finite differences

Equation (16) can be solved by using a finite difference scheme. A rectangular grid is superimposed on a plan view of the aquifer (groundwater reservoir). See Figure 3.


Figure 3. Grid System for Finite Difference Equations Intersection of Any Two Lines Is a Node Location. (i Denotes the x-Direction; j Denotes the $y$-Direction).

The first and second derivatives of the differential equation are approximated by finite differences, using Taylor's expansion where $k$ designates the time dimension.

$$
\begin{align*}
& \frac{\partial h}{\partial x}=\frac{h_{i+1, j}-h_{i-1, j}}{2 \Delta x}+O\left[(\Delta x)^{2} 1\right.  \tag{17}\\
& \left.\frac{\partial h}{\partial y}=\frac{h_{i, j+1}-h_{i, j-1}}{2 \Delta y}+O(\Delta y)^{2}\right]  \tag{18}\\
& \frac{\partial h}{\partial t}=\frac{h_{i, j, k+1}-h_{i, j, k}}{\Delta t}+O(\Delta t)  \tag{19}\\
& \frac{\partial^{2} h}{\partial x^{2}}=\frac{h_{i+1, j}-2 h_{i, j}+h_{i-1, j}}{(\Delta x)^{2}}+O\left[(\Delta x)^{2}\right]  \tag{20}\\
& \frac{\partial^{2} h}{\partial y^{2}}=\frac{h_{i, j+1}-2 h_{i, j}+h_{i, j-1}}{(\Delta y)^{2}}+O\left[(\Delta y)^{2}\right] \tag{21}
\end{align*}
$$

$S$ is a constant but may be different for each nodepoint (i,j). To eliminate terms introduced by the terms $\frac{\partial K x}{\partial x}$ and $\frac{\partial K y}{\partial y}$ of equation (16). Taylor series is written for the permeability values at the midpoint between two nodes:

$$
\begin{aligned}
& K x_{i-1 / 2}=K x_{i, j}-\frac{\Delta x}{2} \frac{\partial K x_{i, j}}{x} \cdot \cdot \cdot \cdot \\
& K x_{i+1 / 2}=K x_{i, j}+\frac{\Delta x}{2} \frac{\partial K x_{i, j}}{x} \cdot \cdot \cdot \cdot \cdot \\
& K y_{i, j-1 / 2}=K y_{i, j}-\frac{\Delta y}{2} \frac{\partial K y_{i, j}}{\partial Y} \cdot \cdots \cdot \cdot \\
& K y_{i, j+1 / 2}=K Y_{i, j}+\frac{\Delta Y}{2} \frac{\partial K y_{i, j}}{\partial Y} \cdot \cdots \cdot \cdot
\end{aligned}
$$

Rearranging gives:

$$
\begin{align*}
& K x_{i, j}=K x_{i-l / 2, j}+\frac{\Delta x}{2} \frac{\partial K x_{i, j}}{\partial x}  \tag{22}\\
& K x_{i, j}=K x_{i+l / 2, j}-\frac{\Delta x}{2} \frac{\partial K x_{i, j}}{\partial x}  \tag{23}\\
& K y_{i, j}=K y_{i, j-1 / 2}+\frac{\Delta y}{2} \frac{\partial K y_{i, j}}{\partial y}  \tag{24}\\
& K y_{i, j}=K y_{i, j+l / 2}-\frac{\Delta y}{2} \frac{\partial K y_{i, j}}{\partial y} \tag{25}
\end{align*}
$$

Substituting the finite difference expressions (17) and (20) for the first two terms of equation (16) leads to:

$$
\frac{K x_{i, j}}{(\Delta x)^{2}}\left(h_{i+i, j}-2 h_{i, j}+h_{i-1, j}\right)+\frac{1}{2 \Delta x} \frac{\partial K x_{i, j}}{\partial x}\left(h_{i+1, j}-h_{i-1, j}\right)
$$

or:

$$
\begin{align*}
\left(\frac{K x_{i, j}}{(\Delta x)^{2}}+\frac{1}{2 \Delta x} \frac{\partial K x_{i, j}}{\partial x}\right)\left(h_{i+1, j}\right) & +\left(\frac{K x_{i, j}}{(\Delta x)^{2}}-\frac{1}{2 \Delta x} \frac{\partial K x_{i, j}}{\partial x}\right)\left(h_{i-1, j}\right) \\
& -\frac{2 K x_{i, j}}{(\Delta x)^{2}}\left(h_{i, j}\right) \tag{26}
\end{align*}
$$

Expression (26) rearranged gives:

$$
\left(\frac{K x_{i, j}}{(\Delta x)^{2}}+\frac{1}{2 \Delta x} \frac{\partial K x_{i, j}}{\partial x}\right)\left(h_{i+1, j}-h_{i, j}\right)+\left(\frac{K x_{i, j}}{(\Delta x)^{2}}\right.
$$

(Equation 27 continued on next page)

$$
\begin{equation*}
\left.-\frac{1}{2 \Delta x} \frac{\partial K x_{i, j}}{\partial x}\right)\left(h_{i-1, j}-h_{i, j}\right) \tag{27}
\end{equation*}
$$

Substituting expressions (23) and (24) for $K x_{i, j}$ in expression (27), the derivatives of $K x_{i, j}$ cancel out and (27) reduces to

$$
\begin{equation*}
\frac{K x_{i+1 / 2, i}}{(\Delta x)^{2}}\left(h_{i+1, j}-h_{i, j}\right)+\frac{K x_{i-1 / 2, i}}{(\Delta x)^{2}}\left(h_{i-1, j}-h_{i, j}\right) \tag{28}
\end{equation*}
$$

The same procedure is used for the third and fourth term of equation (16) which then becomes:

$$
\begin{equation*}
\frac{K y_{i, j+1 / 2}}{(\Delta y)^{2}}\left(h_{i, j+1}-h_{i, j}\right)+\frac{K y_{i, i-1 / 2}}{(\Delta y)^{2}}\left(h_{i, j-1}-h_{i, j}\right) \tag{29}
\end{equation*}
$$

Substituting $\quad K l=K x_{i-1 / 2, ~}$

$$
\begin{aligned}
\mathrm{K} 2 & =K x_{i+1 / 2, j} \\
\mathrm{~K} 3 & =K y_{i, j-1 / 2} \\
K 4 & =K y_{i, j+1 / 2}
\end{aligned}
$$

Equation (16) becomes

$$
\begin{align*}
& \frac{K 2}{(\Delta x)^{2}}\left(h_{i+1, j}-h_{i, j}\right)+\frac{K 1}{(\Delta x)^{2}}\left(h_{i-1, j}-h_{i, j}\right)+\frac{K 4}{(\Delta x)^{2}}\left(h_{i, j+1}-h_{i, j}\right) \\
& \quad+\frac{K 3}{(\Delta y)^{2}}\left(h_{i, j-1}-h_{i, j}\right)=\frac{S}{b}\left(\frac{h_{i, j, k+1}-h_{i, j, k}}{\Delta t}\right)+W^{\prime}(i, j, k+1) \tag{30}
\end{align*}
$$

## The explicit method of solution

Equation (30) has more than one possible method of solution. The explicit method of solution, in which the space derivatives (lefthand side of equation (16)) are approximated at the time level $t=k$, yields to a simple solution, except that the following restriction between the time and space increments must be observed to ensure stability:

$$
\Delta t<\frac{1}{2\left[(\Delta x)^{-2}+(\Delta y)^{-2}\right]}
$$

An explanation of stability is given by Carnahan, Luther, Wilkes ${ }^{1}$, pp. 452.

## The implicit method of solution

The implicit method in which the space derivatives are approximated at the time level $t=k+l$ leads to the difference equation.

$$
\begin{align*}
& \frac{K 2}{(\Delta x)^{2}}\left(h_{i+1, j, k+1}-h_{i, j, k+1}\right)+\frac{K 1}{(\Delta x)^{2}}\left(h_{i-1, j, k+1}-h_{i, j, k+1}\right) \\
&+\frac{K 4}{(\Delta y)^{2}}\left(h_{i, j+1, k+1}-h_{i, j, k+1}\right)+\frac{K 3}{(\Delta y)^{2}} \\
&\left(h_{i, j-1, k+1}-h_{i, j, k+1}\right)=\frac{S}{b} \frac{\left(h_{i, j, k+1}-h_{i, j, k}\right)}{\Delta t} \\
&+W^{\prime}(i, j, k+1) \tag{31}
\end{align*}
$$

The only known term in equation (31) is $h_{i, j, k}$, the hydraulic head at the previous timestep, and the term $W^{\prime}(i, j, k+1)$ is computed as a known value for each timestep.

It may be shown that this scheme is stable (Carnahan, Luther, Wilkes ${ }^{l}$ ) independent of the relative values of $\Delta x, \Delta y$, and $\Delta t$. There are now five unknowns per equation, and the simple version of the Gaussian elimination method given by the algorithm for the tridiagonal system is no longer applicable.

Gaussian elimination may still be used, however, but only at the expense of a considerable amount of computation. An alternative is to use the Gauss-Seidel iterative method (discussed in Carnahan, Luther, Wilkes ${ }^{l}$, Section 7.24h although this method may need a fair number of iterations for adequate convergence.

## The Implicit Alternating Direction Method

The implicit alternating direction method discussed by Peaceman and Rachford ${ }^{19}$, Douglas ${ }^{8}$ and Douglas and Rachford ${ }^{9}$ avoids these disadvantages and yet still manages to use a system of equations with a tridiagonal coefficient matrix for which a simple algorithm affords a straight forward solution.

Essentially, the principle is to employ two difference equations which are used in turn over successive timesteps, each of duration $\Delta t / 2$.

The first equation is implicit only in the x -direction and the second only in the $y$-direction. The first equation is solved for the intermediate values of $h_{i, j}$ at $t=k+1 / 2$ which are then used in the second equation, leading to the solution $h_{i, j, k+1}$ at the end of the whole time interval $\Delta t$. The process is repeated for the next time step. The advantage of this scheme over an entirely explicit technique is that it is unconditionally stable (investigated by the Von Neumann method) ${ }^{1}$, and there is no stability restriction on the size of $\Delta t$ as in the explicit technique. Convergence occurs with a discretization error $O\left[(\Delta t)^{2}+(\Delta x)^{2}\right]$ so that the choice of certain values for $\Delta t$ and $\Delta x$ (assured that ratio $\Delta x / \Delta y$ is constant) has an impact on the rate of this convergence. A more elaborate explanation about stability, consistency and convergence goes beyond the scope of this report but can be found in texts by Richtmeyer ${ }^{23}$, Lax ${ }^{16}$, O'Brien, Hymannand Kaplan ${ }^{18}$. In order to use the alternatingdirection method, equation (3I) is replaced by two similarly developed equations.

The first equation is implicit only in $x$-direction.

$$
\begin{align*}
& \frac{K l}{(\Delta x)^{2}}\left(h_{i-1, j, k+1 / 2}-h_{i, j, k+1 / 2}\right)+\frac{K 2}{(\Delta x)^{2}}\left(h_{i+1, j, k+1 / 2}\right. \\
& \\
& -h_{i, j, k+1 / 2)}+\frac{K 3}{(\Delta y)^{2}}\left(h_{i, j-1, k}-h_{i, j, k}\right) \\
&  \tag{32}\\
& +\frac{K 4}{(\Delta y)^{2}}\left(h_{i, j+1, k}-h_{i, j, k}\right)=\frac{S}{b} \frac{\left(h_{i, j, k+1 / 2}-h_{i, j, k}\right)}{\Delta t / 2} \\
& \quad+W^{\prime}\left(i_{i, j}, k+1 / 2\right)
\end{align*}
$$

As can be seen in Figure 4 a four other nodepoints are used to calculate the intermediate value $h$ at nodepoint $(i, j)$ of which $h_{i, j+1}$ and $h_{i, j-1}$ are known (from previous timestep). This leaves a maximum of three unknowns for equation (32), applied to nodepoint (i,j).

The equation must be written for each node of the row to completely define the physics of the system. This procedure generates for each row a system of equations with a tridiagonal coefficient-matrix. For the entire matrix, intermediate values are calculated (for $t=k+1 / 2$ ).

The intermediate values for the hydraulic head are used in the second equation, which is implicit only in $y$-direction.

$$
\begin{gather*}
\frac{K 1}{(\Delta x)^{2}}\left(h_{i-1, j, k+1 / 2}-h_{i, j, k+1 / 2}\right)+\frac{K 2}{(\Delta x)^{2}}\left(h_{i+1, j, k+1 / 2}\right. \\
\left.-h_{i, j, k+1 / 2}\right)+\frac{K 3}{(\Delta y)^{2}}\left(h_{i, j-1, k+1}-h_{i, j, k+1}\right) \\
+\frac{K 4}{(\Delta y)^{2}}\left(h_{i, j+1, k+1}-h_{i, j, k+1}\right)=\frac{S}{b} \\
\frac{\left(h_{i, j, k+1}-h_{i, j, k+1 / 2}\right.}{\Delta t / 2}+W^{\prime}(i, j, k+1) \tag{33}
\end{gather*}
$$



Figure 4a. Calculation of Node ( $\mathrm{i}, \mathrm{j}$ ) in Equation (32) x-implicit. Four Nodepoints Are Used to calculate Node (i,j). ©Are Nodepoints Which Values Are Unknown in Equation 1 .

Figure 4b. Calculation of Node ( $\mathrm{i}, \mathrm{j}$ ) in Equation (33) y-implicit. Four Nodepoints Are Used to calculate Node (i, j). © Are Nodepoints Which Values Are Unknown in Equation 2.


Figure 5. Leaky Aquifer for Nodepoint (i,j).

Similarly equation (33) must be written for each node but now in the column. For each column a tridiagonal system of equations is solved. (See Figure 4b.) This second half timestep calculates the hydraulic head at the end of the total time interval $\Delta t$. The transmissibility $T$ can be written as:

$$
T=K \cdot b \quad \text { where } \quad K=f(i, j) \quad \text { and } \quad b=b(i, j, k)
$$

Since $K$ is variable in place and $b$ variable in time and place, it follows for convenience:

$$
\begin{aligned}
& T x_{i-1 / 2, j, k}=b_{i, j, k} \cdot K 1=T 1 \\
& T y_{i+l / 2, j, k}=b_{i, j, k} \cdot K 2=T 2 \\
& T Y_{i, j-1 / 2, k}=b_{i, j, k} \cdot K 3=T 3 \\
& T Y_{i, j+1 / 2, k}=b_{i, j, k} \cdot K 4=T 4
\end{aligned}
$$

Written out in full and rearranged, equation (32) becomes:

$$
\begin{align*}
& \frac{T 1}{(\Delta x)^{2}}\left(h_{i-1, j, k}+1 / 2\right)-\frac{T 1+T 2}{(\Delta x)^{2}}+\frac{2 S}{\Delta t}\left(h_{i, j, k+1 / 2}\right) \\
& \\
& +\frac{T 2}{(\Delta x)^{2}}\left(h_{i+1, j, k+1 / 2}\right)=-\frac{T 3}{(\Delta y)^{2}}\left(h_{i, j-1, k}\right) \\
&  \tag{34}\\
& +\frac{T 3+T 4}{(\Delta y)^{2}}-\frac{2 S}{\Delta t}\left(h_{i, j, k}\right)-\frac{T 4}{(\Delta y)^{2}}\left(h_{i, j+1, k}\right) \\
& \\
& \quad+W i, j, k+1 / 2
\end{align*}
$$

with all known values on the right-hand side of the equation.

Wi,j,k+l/2 can be written as:

$$
\begin{equation*}
W i, j, k+1 / 2=W W_{i, j, k+1 / 2}-\frac{K v_{i, j}}{2 b c_{i, j}}\left(2 h c_{i, j}-h_{i, j, k+1 / 2}-h_{i, j, k}\right) \tag{35}
\end{equation*}
$$

in which $W_{i, j}{ }_{i}, k+1 / 2$ includes all inputs and outputs of each node which are not a function of the varying head of the aquifer. This term either can be read in as an array or generated in a separate input program.

The second term on the right hand side of equation (35) is the leakage term which is variable and dependent on the difference of head in the confining layer and aquifer, (see Figure 5.)
$K v_{i, j}=$ vertical hydraulic conductivity of the confining layer.
$b c_{i, j}=$ thickness of the confining layer.
$h c_{i, j}=$ hydraulic head in the confining layer and assumed to be constant for each node point.

Because of the large area, it is impossible to measure the head of the confining stratum at each nodepoint (this practically means one piezometer or well at each node point of the model). The assumption is made that the head of the confining layer is constant in time for all nodes and the change in vertical leakage is proportional to the change in aquifer head.

Equation (35) can then be substituted into equation (34). The leakage function contains a term with a yet unknown value ( $h_{i, j, k+1 / 2}$ ) which can be transferred to the left hand side of equation (34). Both sides of equation (34) are then multiplied by $(\Delta x)^{2}$.

Let $\mathrm{Fac}_{i, j}=\frac{K v_{i, j}}{2 b c_{i, j}}$

$$
\begin{aligned}
& \gamma_{1}=\left(\frac{2 S_{i, j}}{\Delta t}+F a c_{i, j}\right)(\Delta x)^{2} \\
& \gamma_{2}=\left(\frac{-2 S_{i, j}}{\Delta t}+F a c_{i, j}\right)(\Delta x)^{2} \\
& \delta l=\left(\frac{\Delta x}{\Delta y}\right)^{2}
\end{aligned}
$$

Equation (34), which is implicit in the $x$-direction and is written for every nodepoint in the row, becomes with these substitutions:

$$
\begin{align*}
T 1\left(h_{i-1, j, k+1 / 2}\right) & -(T 1+T 2+\gamma 1)\left(h_{i, j, k+1 / 2}\right)+T 2\left(h_{i+1, j, k+1 / 2}\right) \\
= & -\delta 1 T 3\left(h_{i, j-1, k}\right)+(\delta 1(T 3+T 4)+\delta 2)\left(h_{i, j, k}\right) \\
- & \delta 1 T 4\left(h_{i, j+1, k}\right)+(\Delta x)^{2} W^{O_{i, j}, k+1 / 2} \\
& -2(\Delta x)^{2} \mathrm{FaC}_{i, j}\left(h_{i, j}\right) \tag{36}
\end{align*}
$$

Let

$$
\begin{aligned}
\gamma 3 & =\left(\frac{2 S_{i, j}}{\Delta t}+F a c_{i, j}\right)(\Delta y)^{2} \\
\gamma 4 & =\left(\frac{-2 S_{i, j}}{\Delta t}+F a c_{i, j}\right)(\Delta y)^{2} \\
\delta 2 & =1 / \delta 1
\end{aligned}
$$

and multiplying by $(\Delta y)^{2}$, equation (33) which is implicit in $y$-direction and is written for every nodepoint in the column, becomes with these substitutions:

$$
\begin{aligned}
T 3\left(h_{i, j-1, k+1}\right) & -(T 3+T 4+\gamma 3)\left(h_{i, j, k+1}\right)+T 4\left(h_{i, j+1, k+1}\right) \\
= & -\delta 2 T 1\left(h_{i-1, j, k+1 / 2}\right)+(\delta 2(T 1+T 2)+\gamma 4)\left(h_{i, j, k+1 / 2}\right) \\
& -\delta 2 T 2\left(h_{i+1, j, k+1 / 2}\right)+(\Delta y)^{2} W_{i, j, k+1} O_{i,}-2(\Delta y)^{2}
\end{aligned}
$$

$$
\begin{equation*}
\mathrm{Fac}_{i, j}\left(\mathrm{hc}_{i, j}\right) \tag{37}
\end{equation*}
$$

Equation (36), which calculates the rows contains the term

$$
\begin{equation*}
(\Delta x)^{2} W_{i, j, k+1 / 2}^{O_{i}} \tag{38}
\end{equation*}
$$

or, written differently:

$$
\left(\frac{\Delta x}{\Delta y}\right)\left(\Delta x \Delta y W^{o_{i}}, j, k+l / 2\right)
$$

$\Delta x \Delta y W^{\circ}{ }_{i}, j, k+1 / 2$ is the total volume of water applied to or removed in one half timestep from one node that represents an area with dimensions of $\Delta x$ and $\Delta y$. For convenience let

$$
\begin{aligned}
\Delta \mathrm{x} \Delta y \mathrm{~W}^{\mathrm{O}}{ }_{i, j, k+1 / 2}=\frac{(\text { volume })}{\text { (half timestep) }} & =\frac{\mathrm{Qi}, j, k+l / 2}{(\Delta t / 2)} \\
& =\operatorname{mu}(\mathrm{Qi}, j, k+1 / 2)
\end{aligned}
$$

in which $m u=(2 / \Delta t)$
Qi,j,k+1/2 = volume of water, applied to node ( $\mathrm{i}, \mathrm{j}$ ) during onehalf timestep. ( $\mathrm{t}=\mathrm{k}$ to $\mathrm{t}=\mathrm{k}+\mathrm{l} / 2$ ).

Equation (37) contains the term $(\Delta y)^{2} W_{i, j, k+1}$
Similarly $(\Delta y)^{2} W_{i, j, k+1}=\left(\frac{\Delta y}{\Delta x}\right)\left(\Delta y \Delta x W^{0}{ }_{i, j, k+1}\right)$

$$
\Delta y \Delta x W^{O_{i}, j, k+1}=m u(Q i, j, k+1)
$$

Let $\frac{\Delta x}{\Delta y}=r l$ and $\frac{\Delta y}{\Delta x}=r 2$
Equation (38) becomes $\mathrm{rl}(\mathrm{mu}(\mathrm{Qi}, \mathrm{j}, \mathrm{k}+\mathrm{l} / 2)) \quad(\mathrm{t}=\mathrm{k}$ to $\mathrm{t}=\mathrm{k}+\mathrm{l} / 2)$
Equation (39) becomes $r 2(m u(Q i, j, k+1)) \quad(t=k+1 / 2$ to $t=k+1)$
For simplicity, let:

$$
Q C_{i, j}=2 F_{i c}, j(\Delta x \Delta y)
$$

Equation (36), utilized for calculation in the $x$-direction ( $x$ implicit),
is simplified further:
Let

$$
\begin{aligned}
& A=T 1 \\
& B=-(T 1+T 2+Y 1)
\end{aligned}
$$

$\mathrm{C}=\mathrm{T} 2$

$$
\begin{aligned}
D=-\delta l T 3 h_{i, j-1, k}+(\delta 1(T 3+ & T 4)+\gamma 2) h_{i, j, k}-\delta 1 T 4\left(h_{i, j+1, k}\right) \\
& +r l\left(m u Q i, j, k+1 / 2-Q C_{i, j}\right)
\end{aligned}
$$

$A, B$, and $C$ are coefficients of the unknown headvalues. $D$ is the sum of all known parameters. (The parameters on the right hand side of equation 36) 。

For each nodepoint in the row, equation (36) can be written as follows:

$$
\begin{equation*}
A\left(h_{i-1, j, k+1 / 2}\right)+B\left(h_{i, j, k+1 / 2}\right)+C\left(h_{i+1, j, k+1 / 2}\right)=D \tag{40}
\end{equation*}
$$

For calculation in the $y$-direction ( $y$ implicit) similar substitutions are made.
$A=T 3$
$B=-(T 3+T 4+\gamma 3)$
$\mathrm{C}=\mathrm{T} 4$
$D=-\delta 2 T 1\left(h_{i-1, j, k+1 / 2}\right)+(\delta 2(T l+T 2)+\gamma 4)\left(h_{i, j, k+1 / 2}\right)$

$$
\begin{aligned}
& -\delta 2 T 2\left(h_{i+1, j, k+1 / 2}\right)+r 2 \\
& \left(m u Q_{i, j, k+1}-Q C_{i, j}\right)
\end{aligned}
$$

For each nodepoint in the column, equation (37) becomes
$A\left(h_{i, j-1, k+1}\right)+B\left(h_{i, j, k+1}\right)+C\left(h_{i, j+1, k+1}\right)=D$
Looking at the equations (40) and (41) it is obvious that the solution of the system of equations in the $x$-direction is similar to that in $y$-direction so further examination is confined to the $x$-direction.

## Solution of the equations

For each row the following system of linear equations is developed for each time step:

$$
\begin{align*}
& \begin{aligned}
b_{1} h_{1}+c_{1} h_{2} & =d_{1} \\
a_{2} h_{1}+b_{2} h_{2}+c_{2} h_{3} & =d_{2} \\
a_{3} h_{2}+b_{3} h_{3}+c_{3} h_{4} & =d_{3}
\end{aligned} \\
& \begin{aligned}
b_{1} h_{1}+c_{1} h_{2} & =d_{1} \\
a_{2} h_{1}+b_{2} h_{2}+c_{2} h_{3} & =d_{2} \\
a_{3} h_{2}+b_{3} h_{3}+c_{3} h_{4} & =d_{3}
\end{aligned} \\
& \begin{aligned}
b_{1} h_{1}+c_{1} h_{2} & =d_{1} \\
a_{2} h_{1}+b_{2} h_{2}+c_{2} h_{3} & =d_{2} \\
a_{3} h_{2}+b_{3} h_{3}+c_{3} h_{4} & =d_{3}
\end{aligned} \\
& a_{i} h_{i-1}+b_{i} h_{i}+c_{i} h_{i+1} \quad=d_{i}  \tag{42}\\
& a_{N-1} h_{N-2}+b_{N-1} h_{N-1}+{ }^{c_{N-1}} h_{n}=d_{N-1} \\
& a_{N} h_{N-1}+b_{N} h_{N}=d_{N}
\end{align*}
$$

(43)

The first and the last equation are modified forms of equation (40). The first equation is written for the first node which is a boundary node. The boundary node expresses a property of the system which reduces the number of unknowns in the equation to two. (Either the value of the hydraulic head is known and the first term ( $a_{0} h_{o}$ ) is transferred to the right hand side of the equation, or the value of the head at the boundary is some function of the succeeding head value, which changes the coefficient to a new value $b_{1}$ and the equation is also reduced to two unknowns.) Another possibility is an impermeable boundary. In that case K (boundary) is zero which makes the first term zero. The same can be said of the last equation.

The matrix of coefficients $a, b, a n d c$ is called a tridiagonal matrix. With a maximum of three variables per equation the system can easily be solved by means of a recursion solution (Carnahan, Luther, Wilkes ${ }^{1}$ ).

$$
h_{i}=\gamma_{i}-\frac{c_{i}}{\beta_{i}} h_{i+1} \text { in which the constants } \beta_{i} \text { and } \gamma_{i} \text { are to be }
$$ determined. Substitution in the $i^{\text {th }}$ equation of set (43) gives

$$
h_{i}=\frac{d_{i-} a_{i} \gamma_{i-1}}{b_{i}-\frac{a_{i} c_{i-1}}{\beta_{i-1}}}-\frac{c_{i} h_{i+1}}{b_{i}-\frac{a_{i} c_{i-1}}{\beta_{i-1}}}
$$

subject to the following recursion relations:

$$
\beta_{i}=b_{i}-\frac{a_{i} c_{i-1}}{\beta_{i-1}} \quad, \quad \gamma_{i}=\frac{d_{i}-a_{i} i-1}{\beta_{i}}
$$

From the first equation of the set it follows that

$$
h_{1}=\frac{d_{1}}{b_{1}}-\frac{c_{1}}{b_{1}} h_{2} \quad \text { whence } \beta_{1}=b_{1} \quad, \quad \gamma_{1}=d_{1} / \beta_{1}
$$

Finally, substitution of the recursion solution into the last equation of the system yields:

$$
h_{N}=d_{n}-\frac{a_{N} h_{N-1}}{b_{N}}=\frac{d_{N}-a_{N}{ }_{r_{N-1}}-\left(\frac{c_{N-1}}{b_{N-1}} h_{N}\right)}{b_{N}}
$$

whence

$$
h_{N}=\frac{d_{N}-a_{N}{ }^{\gamma}{ }_{N-1}}{b_{N}-\frac{a_{N}{ }^{c}{ }_{N-1}}{{ }^{\beta}{ }_{N-1}}}=\gamma_{N}
$$

To summarize, the complete algorithm for the solution of the tridiagonal system is:

$$
\begin{aligned}
& h_{N}=\gamma_{N} \\
& h_{i}=\gamma_{i}-\frac{c_{i} h_{i+1}}{\beta_{i}}, \quad i=N-1, N-2, \ldots ., 1 \text { where the }
\end{aligned}
$$

$\beta^{\prime} s$ and $\gamma$ 's are determined from the recursion formulas:

$$
\begin{aligned}
& \beta_{1}=b_{1} \quad, \quad \gamma_{1}=d_{1} / \beta_{1} \\
& \beta_{i}=b_{i}-\frac{a_{i} c_{i-1}}{\beta_{i-1}}, \quad i=2,3, \ldots, N
\end{aligned}
$$

$$
\gamma_{i}=\frac{d_{i}-a_{i}^{\gamma}{ }_{i-1}}{\beta_{i}}, \quad i=2,3, \ldots N
$$

The values of ${\underset{1}{1}}$ and $\gamma_{i}$ for any row or column are computed in order of increasing $i$. When the last node in the row or column is reached, the head is calculated in order of decreasing $i$.

## Boundary Conditions

The hydraulic head is calculated in a system of tridiagonal equations in which the boundary equations have only two unknowns.

The general equation (row) for nodepoint ( $i, j$ ) is: ( $k$ denotes the $k^{\text {th }}$ half timestep).

$$
\begin{equation*}
a h_{i-1, j, k}+b h_{i, j, k}+c h_{i, j, k}=d \tag{44}
\end{equation*}
$$

where a, b, c, and d are known values.
The two most common boundary conditions are the impermeable boundary, where $\frac{\partial h}{\partial x}=0$ and the constant head condition where $h_{\text {(boundary) }}=C$. The flow boundary for which the head at the boundary is changing is more difficult to solve.

## Impermeable boundary

The impermeable boundary condition (Figure 6) can be described by letting $T(i-1, j)$ equal zero at nodes just outside the boundary. This causes the impermeable boundary to be at midpoint between the boundary node point and the node with zero $T$. This zero $T$ causes the coefficient ' $a$ ' in equation (44) to be zero so that the equation is reduced to a boundary equation with two unknowns.

$$
b h_{i, j, k}+c h_{i, j, k}=d
$$



Figure 6. Impermeable Boundary. $\mathrm{T}_{\mathrm{i}-1}$, Is Node Outside the Boundary. $T_{i-1, j}^{i-1}=0$ Causes the Impermeable Boundary ${ }^{i-1, j}$ to Be at Midpoint Between the Boundary Nodepoint and the Node with Zero T.


Figure 7. The Original Boundary Is (A-B), Bordering the Confined Aquifer, At Some Distance in the Confined Aquifer the Influence of the Initial Head, $H_{\text {, Is }}$ Is Negligible. If the Boundary Is Moved to or Beyond that Specific Point (C-D), This Boundary Can Be Described as a Constant Head Boundary.

## 'Constant' head boundary

Boundaries for which the head is known for every timestep could include either lakes, reservoirs or regulated river boundaries.

The river stage may be different in any timestep. Then $h_{i-1, j, k}$ is a known value and is transferred to the right hand side of the equation leaving a boundary equation with two unknowns.

$$
b h_{i, j, k}+c h_{i+1, j, k}=d-a h_{i-1, j, k}
$$

## Experiment in flowboundary

No flow and constant head boundaries can be handled quite easily; however, when flow occurs across boundaries for which the head at the boundary is changing then some function for defining the head becomes necessary. Several possible alternatives to the boundary problem are considered.

One solution is to write $h_{\text {(boundary) }}$ as a function of time. Historical records of the change in $h$ for a particular aquifer throughout the year can be approximated with a harmonic function. The program can then be run for the preceding years in which the computer must reproduce the historic values of the groundwater levels in the remainder of the aquifer. If computed and measured historical levels do not agree, the K values within the aquifer may be adjusted until a satisfactory fit of computed water table elevations with measured values is obtained. This method allows the adjustment and refinement of variables using historical data. However, if major changes in hydraulic activities are made (such as setting up a domestic well field or change in irrigation practices) the function which defines the hydraulic head of the boundary becomes invalid.

Another solution can be applied if across the flow boundary the unconfined character of the flow changes into a confined character. This problem can be solved easily in the mathematical model. The changes in hydraulic head of the unconfined aquifer migrate through the confined aquifer and can be described by:

$$
h / H_{o}=\operatorname{erfc}[x / 2 \cdot \sqrt{T t / S}]
$$

A problem similar to this is encountered in bank storage calculations as shown in Figure 7. This equation indicates that at some distance in the confined aquifer the influence of the initial head, $H_{o}$, is negligible. If the boundary is moved to or beyond that specific point this boundary can be described as a constant head boundary.*

A third possibility is as follows, (See Figure 8). The new hydraulic head at ( $t=k$ ) is calculated in a system of tridiagonal equations, in which the boundary equations have only two unknowns. In this case, by cutting off the aquifer at point (ib,j), three unknowns remain because nothing is known about the nodepoint with $h_{i b-1, j}$. ((ib,j) is the boundary node。)

To eliminate one of the unknowns a means must be found to express $h_{i b-1, j, k}$ in terms of $h_{i b, j, k}$ and $h_{i b+1, j, k}$ using hydraulic head values which are calculated in the previous half timestep.

The solution is considered to be valid if the following conditions are present:

1. That part of the aquifer that is cut off by the flow boundary extends sufficiently far so that the boundary condition (called condition end) which terminates this part will not influence the aquifer head in the vicinity of the flow

[^0]

Figure 8. Hydraulic Head in the Vicinity of the Flow Boundary. Flow Boundary is on Beginning of Row. al and $\alpha 2$ Are the Average Slopes of the Water Table at Time $(k-1)$. Bl and $\beta 2$ Are the Average Slopes of the Water Table at Time (k).
boundary. Only changes inside the aquifer, contemplated in the model, may influence the boundary nodes.
2. No major hydraulic activities take place close to those boundary: nodes.
3. If these major hydraulic activities do occur then they must occur for every nodepoint in the vicinity of the boundary nodes. For instance, if leakage to an underlying aquifer occurs in this model then this must be an activity that is shared by all nodes close to this special boundary.

Satisfying these three conditions dictates that the change in hydraulic head per timestep is not abrupt in the vicinity of the boundary, or the change is a gradual one.

With this in mind then: (See Figure 8)

$$
\begin{aligned}
\frac{\alpha 1}{\alpha 2}=\frac{\beta 1}{\beta 2} & \text { Ratios of average hydraulic slope close } \\
& \text { to boundary points. } \alpha 1 \text { and } \alpha 2 \text { are the } \\
& \text { average slopes of the watertable in the } \\
& \text { previous half timestep }(k-1) .
\end{aligned}
$$

Then

$$
\beta 2=\frac{\alpha^{2}}{\alpha 1} \beta 1
$$

and

$$
\begin{aligned}
& \beta 1=\frac{\left(h_{i b+1, j k}-h_{i b, j, k}\right)}{\Delta x}, \\
& \frac{\alpha 2}{\alpha 1}=\frac{\left(h_{i b, j, k-1}-h_{i b-1, j, k-1}\right)}{\left(h_{i b+1, j, k-1}-h_{i b, j, k-1}\right)} .
\end{aligned}
$$

and

$$
\beta 2=\frac{\alpha 2}{\alpha 1} \frac{\left(h_{i b+1, i, k}-h_{i b, j, k}\right)}{\Delta x}
$$

$h_{i b-1, j, k}$ can now be expressed in terms of $h_{i b, j, k}$ and $h_{i b+1, j, k}$ according to:

$$
h_{i b-1, j, k}=h_{i b, j, k}-\beta 2(\Delta x)
$$

Substituting for $\beta 2$ in this expression yields to:

$$
\begin{equation*}
h_{i b-l, j, k}=h_{i b, j, k}-\frac{\alpha 2}{\alpha l}\left(h_{i b+l, j, k}-h_{i b, j, k}\right) \tag{45}
\end{equation*}
$$

With this procedure $h_{i b-l, j, k}$ is defined for the initial timestep and the system of equations can be solved by substituting expression (45) for $h_{i b-1, j, k}$ in, the following equation:

$$
a h_{i b-1, j, k}+b h_{i b, j, k}+c h_{i b+1, j, k}=d
$$

This yields a boundary equation with two unknowns:

$$
b^{\prime} h_{i b, j, k}+c^{\prime} h_{i b+l, j, k}=d
$$

The routine of the program solves this system of row equations starting from nodepoint ( $\mathrm{ib}, \mathrm{j}$ ) and calculates successively for $\mathrm{t}=\mathrm{k}$ the values for nodes ( $i b+1, j),(i b+2, j)$. . .

$h_{i b-l, j, k}$ was calculated with equation (45) which is an approximation because the ratio $\alpha 2 / \alpha l$ uses the values for hydraulic head of the $k-1$ (previous) timestep.

Because now the hydraulic head values for nodepoints (ib,j), $(i b+1, j),(i b+2, j)$ etc. are known for $t=k$ a new value for $h_{i b-1, j, k}$ can be calculated using these nodepoints. Since the character of the
hydraulic slope has more resemblance to a quadratic function than to equation (45), the new value for $h_{i b-1, j, k}$ is calculated with a second degree polonomial which uses the boundary head $h_{i b, j, k}$ and two inside values $h_{i b+1, j, k}$ and $h_{i b+2, j, k}$ (backward differences). This refined approximation forces the outside boundary point (ib-1,j) to "behave" in the same way as points inside the boundary. The refined approximation of the hydraulic head for nodepoint (ib-1,j) will be used in the next half timestep $t=k+1$ to calculate new values for the ratio $\alpha 2 / \alpha 1$.

In checking out this procedure, it appeared necessary that the physical properties of the nodes close to this boundary be known fairly accurately. The relationship between the depth of aquifer, the hydraulic conductivity, the storage coefficient, the impedance of the leaky aquifer and the initial water table condition should be fairly well matched.

If the estimates of the hydraulic parameters, conductivity, storage coefficient or other are not accurate near the boundary, then a highly unsteady state condition exists in which the water table values will change rapidly in a few timesteps to match the mentioned input parameters. This is in contradiction with the conditions assumed for this boundary solution.

This highly unsteady state condition is prevented if this matching happens more gradually. Therefore, a check of the $\frac{\alpha 2}{\alpha 1}$ ratio is introduced in the main program to control the rate of change in hydraulic slope. In the program an upper limit for the head difference between the succeeding nodepoints is incorporated and an upper and lower limit for the hydraulic slope ratios ( $\alpha 2 / \alpha 1$ ). These checks are applicable to nodepoints ( $\mathrm{i} b, \mathrm{j}$ ), ( $\mathrm{ib}+\mathrm{l}, \mathrm{j}$ ) and ( $(\mathrm{ib}+2, \mathrm{j})$.

These limiting values can be chosen according to the character of the aquifer and will, after chosen, remain as constants in the program. Reference to these values will occur under the name CONTROL CONSTANTS. Solution of the limiting values depends on the aquifer properties at the location of the flow boundary. A discussion on the treatment and solution of the flow boundary controls is shown in Appendix A。

Provided conditions (1), (2), and (3) as described on pages 34 and 36 are satisfied, this solution for a flow boundary is applicable.

## Computer Program of the Model

## Computation of the hydraulic head

A computer program in FORTRAN-IV language is written to solve the finite difference equations. It calculates the new hydraulic head in a system of tridiagonal equations for both rows and columns in succeeding half timesteps, using the recursion formula as described on pages 29 through 31. Let N be the last node in a row and NN the first node. If one considers the calculation of a row at timestep $k$ the routine first generates the tridiagonal matrix coefficients $a_{i}, b_{i}, c_{i}$ and $d_{i}$ (i denotes $i^{\text {th }}$ node in row) for $i=N N, N N+1$, . . . . . . N.

It then calculates intermediate values:

for $i=N N, N N+1$, . . . . . N.
The new hydraulic head $h_{i}$ is calculated according to:

$$
\begin{aligned}
& h_{N}=\gamma_{N} \\
& h_{i}=\gamma_{i}-\frac{c_{i}}{\beta_{i}} h_{i+1} \quad ; \quad i=N-1, N-2, \ldots . . . N N .
\end{aligned}
$$

In FORTRAN IV language:
$a_{i}, b_{i}, c_{i}, d_{i}$

A, B, C, D,
$\frac{c_{i}}{\beta_{i}}$

$B E(I)=C / W$
$\beta_{i}=b_{i}-\frac{a_{i} c_{i-1}}{\beta_{i-1}}$
$\longrightarrow$
$W=B-A * B E(I-1)$
$\gamma_{i}=\frac{d_{i-} a_{i} \gamma_{i-1}}{\beta_{i}}$
$\longrightarrow$
$G(I)=(D-A * G(I-1)) / W$

Similar routine is used for calculation of the columns.

Input
As explained on page 29, the main program calculates the new hydraulic head in a system of tridiagonal equations for both rows and columns in succeeding half timesteps.

The general equation for a row or column for the nodepoint ( $\mathrm{i}, \mathrm{j}$ ) is: $a h_{i-1, j, k}+b h_{i, j, k}+c h_{i+1, j, k}=d$ while the boundary nodes have only two unknown head values on the left hand side of the equation.

For each nodepoint or boundary point the values of the constants a, b, c, and d are calculated. However, dependent on the "character" of the nodepoint, the computation of these constants is different. In order to distinguish between the characters of the different nodepoints and hence to allow for the right computation of the constants, both for computation in the $x$-direction (rows) and in the $y$-direction (columns), arrays are introduced, called

1. $\operatorname{NCX}(i, j)$ and
2. $\operatorname{NCY}(i, j)$, which.
give to each nodepoint a number according to the character of the nodepoint. In this way for either the x or y direction, the computer routine checks the character number of the nodepoint being calculated and carries out the right computation via an assigned "GO TO" statement. The following character numbers are introduced: ( $\mathrm{NC}=$ character number). For calculation of rows (x-direction) for nodepoint (i,j):
description nodepoint description boundary
NCX = 1: nodepoint is outside study area ..... :NCX $=2$ : nodepoint is inside study area :$\mathrm{NCX}=3$ : nodepoint left of this node is boundarynode at start row : constant headNCX = 4: nodepoint right of this node is boundarynode at end row : constant headNCX = 5: nodepoint is boundary node at start row : impermeable boundary toleft of this node

NCX = 6: nodepoint is boundary node at end row : impermeable boundary to right of this node

NCX = 7: nodepoint is boundary node at start row : flow boundary
NCX $=8$ : nodepoint is boundary node at end row : flow boundary

For calculation of columns ( $y$-direction) for nodepoint ( $\mathrm{i}, \mathrm{j}$ ): description nodepoint description boundary

NCY = l: nodepoint is outside study area :
NCY = 2: nodepoint is inside study area :
NCY $=3$ : nodepoint below this node is boundary node at start column : constant head

NCY = 4: nodepoint above this node is boundary node at end column : constant head

NCY = 5: nodepoint is boundary node at start column
: impermeable boundary below this node

NCY＝6：nodepoint is boundary node at end column

NCY $=7$ ：nodepoint is boundary node at start column ：flow boundary

NCY＝8：nodepoint is boundary node at end column
：impermeable boundary above this node
：flow boundary

Figures 9a，9b and 9c show the arrays $\operatorname{NCX}(i, j)$ and $N C Y(i, j)$ for the hypothetical case of a square island surrounded by a＇constant＇head water body on three sides and an impermeable boundary on the remaining side．

The remaining nine arrays that are entered in the program are：
3．$N N(i, j):$ an array that denotes whether the nodepoint（ $i, j$ ）is located in an unconfined or confined aquifer．

$$
\begin{aligned}
& \mathrm{NN}(\mathrm{i}, \mathrm{j})=0 \longrightarrow \text { unconfined } \\
& \mathrm{NN}(\mathrm{i}, \mathrm{j})=1 \longrightarrow \text { confined }
\end{aligned}
$$

4．$K X(i, j) \quad: \quad$ an array of the hydraulic conductivity in the $x$－direction ［ft／day］／100

5．$K Y(i, j):$ hydraulic conductivity in the $y$－direction 「ft／dayl／100．
6．$Z(i, j):$ the elevation of the aquifer bottom 「ft］．
7． $\operatorname{PHI}(i, j, 1)$ ：the initial hydraulic head in the aquifer as initial conditions for the finite difference equations［ft］．

8． $\operatorname{PSI}(\mathrm{i}, \mathrm{j})$ ：the hydraulic head of a possible leaky aquifer 「ft1．
9． $\operatorname{SURF}(\mathrm{i}, \mathrm{j}):$ the landsurface elevation［ft］．
10．$S(i, j):$ storage coefficients，dimensionless．
11． $\operatorname{FAC}(\mathrm{i}, \mathrm{j}):$ impedance of the leaky aquifer $\Gamma \mathrm{ft} / \mathrm{day}^{2} 1 \times 10$ ．
The total source term $Q(i, j, k)$ is read in as a three dimensional array from the input program in which $Q(i, j, k)$ is calculated $\left[f t^{3} / 1 / 2 \Delta t\right]$ ．

Other input data are：
MICX ：If MICX $=2$ the calculation of the rows（x－direction） will appear as a contour plot on microfilm．If MICX $=$ 0 no output on microfilm will be given．


Figure 9a. Plain View of Hypothetical Island Surrounded by Water and With One Impermeable Side.

Figure 9b. Array $\operatorname{NCX}(i, j)$ for Island.

1. Outside Boundary Node
2. Inside Boundary Node
3. Node to the Right of This Node Is a Constant Head (end of row)
4. Node has Impermeable Boundary to the Left (start row).

Figure 9c. Array $\mathrm{NCY}(\mathrm{i}, \mathrm{j})$

1. Outside Boundary Node
2. Inside Boundary Node
3. Node Below This Node Is a Constant Head (start column)
4. Node A bove This Node is a Constant Head (end column)

MICY : similar rules for MICX apply but for calculation of the columns ( $y$-direction).

NIM : variable which dictates the desired number of contour levels in the contour plot.

NSER : variable that is directly related to MICX $=2$ and MICY $=2$ such that if NSER $=1$ a contour map will be plotted for the half timesteps with $k=3,5,7,9,11$, etc. If NSER $=2$ a contour map will be plotted for the half timesteps with $\mathrm{k}=3,7,11,15$, etc.
$X A$ and YA : variables which relate to the size ( $x$ - and $y$-direction) of the contour plot.

NPX : if NPX $=2$ the calculation of the rows ( $x$-direction) will be printed out as an array for every timestep. If $N P X=0$ no printed output.

NPY : similar rules apply for NPX, but for calculation of the columns ( $y$-direction).

NQ : variable that relates to the total source term $Q(i, j, k)$. If $N Q=1$, the total source term $Q(i, j, k)$ will be read in from file as a three dimensional array calculated in a separate input program. If $N Q=2$, the total source term will be the same for each nodepoint i.e: $Q(i, j, k)=$ FLUX for $k=1,2$, . . NFLUX. If $N Q=3$, the total source term will be zero for each nodepoint for each $k$.

FLUX : variable which value represents the total input for one node per half timestep when $N Q=2\left[\mathrm{ft}^{3} / 1 / 2 \Delta \mathrm{t}\right]$ 。

NFLUX : denotes number of half timesteps (k) for which $Q(i, j, k)=$ FLUX. Used when $N Q=2$.

| FFF |  | ```amplification factor of total source term Q(i,j,k). Used always, independent of the value of NQ.``` |
| :---: | :---: | :---: |
| AMPL | : | amplification factor by which the KX array and KY array is |
|  |  | multiplied. |
| MORE |  | denotes the time dimension of the source term $Q(i, j, k)$. |
|  |  | It is the maximum value of $k$. |
| NROW |  | total number of nodes in row. |
| NCOL |  | total number of nodes in column. |
| DELT |  | length of one timestep [days]. |
| DELX |  | length of mesh in $x$-direction in grid system [ft]. |
| DELY |  | length of meshin $y$-direction in grid system [ft]. |
| LTS | : | total numbers of half timesteps for which length of timestep |
|  |  | is DELT. If $k>L T S$ length of timestep $=2$ DELT. |
| NTS |  | total number of half timesteps of the complete simulation. |
| NSU1 |  |  |
| NSU2 |  | variables which value indicates the number of half timesteps |
| NSU3 |  | for which an array of the depth to water table is printed out. |

Other variables are input variables which are related to the flow boundary:

CONTROL CONSTANTS: These are the variables which control the behavior of the flow boundary as described in Appendix A. The CONTROL CONSTANTS consist of:

DIF : maximum head difference between succeeding nodepoi nts at flow boundary 「ft1.
: suggested values of head differences between suceeding nodepoints at flow boundary. Utilized when flow boundary is on end of row or column.

RMAX : maximum value for ratio $(\alpha 2 / \alpha 1)$ for either row or column.
RMIN : minimum value for ratio $(\alpha 2 / \alpha 1)$ for either row or column.
AAN $\quad: \quad$ suggested value for ratio $(\alpha 2 / \alpha 1)$ if actual ratio exceeds the limits. Utilized for either row or column if flow boundary is on beginning of row or column.

AAN2 : suggested value for ratio $(\alpha 2 / \alpha 1)$ if the actual ratio exceeds the limits. Utilized for either row or column if flow boundary is on end of row or column.

Remaining for consideration are the input variables which are related to the location of the flow boundary in the grid system. These are the variables: NX, NXX, NYS. NYF, NYSS, NYFF, NY, NYY, NXS, NXF, NXSS, and NXFF. The need for these variables is discussed in Appendix B.

In discussion of the computer program, a flow chart and listing of the main program are included in Appendix C.

## Output

Depending on the values of input variables, output of the calculated hydraulic head for each half timestep will appear either as a contour plot on microfilm* and/or as a printout of hydraulic head values in an array. The input arrays and single variables are also listed with the output.

## Testing of the Model

To check the program several test problems were developed to show the validity of the model. The test problems were kept simple so that the computer solution could be checked analytically.
*A subroutine is included in the main program. If this subroutine is called the output will be a contour plot on microfilm.

Filterwell in a circular island, Test 1
Figure 10a and $10 b$ give the arrays $\operatorname{NCX}(i, j)$ and $\operatorname{NCY}(i, j)$ for this simulation. In this case a single well was placed in the center of a circular island. The radius of the island was 500 feet. The initial conditions show a static horizontal water table and a zero discharge of the pump. For $t>0$ Qpump $=24,960 \mathrm{ft}^{3} /$ day .

The storage coefficient was chosen at 0.15. Hydraulic conductivity, $K X=K Y=600.0 \mathrm{ft} /$ day, and $T X=T Y=6000 \mathrm{ft}^{2} / \mathrm{day}$.

The water table around the island was kept constant at the initial value of 10 feet. For the computer solution a timestep of 8 hours was chosen. The computer solution shows that the steady state is reached after 10 timesteps.

The analytical solution for the steady state condition is straight forward and can be described as:

$$
\mathrm{h}^{2}=\mathrm{H}_{\mathrm{o}}^{2}-\frac{\mathrm{Q}}{\pi \mathrm{~K}} \ln \frac{\mathrm{R}}{\mathrm{r}}
$$

utilizing the Dupuit assumption of horizontal flow.
Figure 11 shows the analytical solution of the steady state condition compared with the computer solution.

At the location of the well the two solutions differ substantially, however at this point both the computer and analytic solution differ from the exact solution. For every other grid point in the island, both methods yield the same results.

The next question is whether the rate of drawdown for both the computer and analytical solutions is the same or whether the steady state solution for both methods is reached in the same time span.

For the analytical solution the non-equilibrium equation of Theis ${ }^{25}$ can be applied. The Theis equation, however, assumes the perimeter of


Figure 10a. Array for $\operatorname{NCX}(1, j)$ of Island. Test 1.

1. Node Cutside Aquifer
2. Node Inside A guifer
3. Node to the Teft of This N.ode Is Constant Head Node Row)
4. Node to the Right of This Node Is Constant Head Node ROW)


Figure 10b. Array for $\operatorname{NCY}(i, j)$ of Island.

## Test 1.

1. Node Outside Aquifer
2. Node Inside Aquifer
3. Node Below This Node Is Constant Head Node (Column)
4. Node A bove This Node Is Constant H.ead Node (Column)


Figure 11. Comparison of the Analytically Determined Drawdown and the Results Obtained with the Computer Model.
the island at an infinite distance, so that a difference between the Theis solution and the computer solution (perimeter at 600 ft .) is expected. For small values of $r$ and/or large $t, J^{\prime} \operatorname{cobs}^{13}$ method for solution of nonequilibrium equation can be applied, which is a simplification of the Theis method.

For nodepoint $(7,6) r=100 \mathrm{ft}$; the drawdown at different times can be computed with Jacob's equation

$$
s=\frac{2.3 Q}{4 \pi T} \log \frac{2.25 T}{r^{2} S}+\frac{2.30 Q}{4 \pi T} \log t
$$

On semilogarithmic paper, this is the equation of a straight line of drawdown $s$ versus t. The slope of the line is equal to $\frac{2.30 Q}{4 \pi T}$. For this case the slope $=\frac{(2.3)(24,960)}{4 \pi 6,000}=0.76 \mathrm{ft}$. The drawdown obtained from the computer simulation is plotted on semilog paper for nodepoint $(7,6)$ (Figure 12). The slope can be found as the vertical projection of the intercept of the straight line between two numbers on the time scale that have logorithms one unit apart. From this graph, a slope of 0.76 ft . is obtained, which is in full agreement with the slope obtained with Jacob's solution.

## Two wells in rectangular island, Test 2

In this case a square island is considered over which an $18 \times 18$ grid is superimposed. (See Figure 13).

Location $(7,7)$ contains one discharge well.
Location (12, 12) contains one recharge well.
Both wells pump at the rate of $16,960 \mathrm{ft}^{3} /$ day.
The initial condition shows a static (horizontal water table with no discharge or recharge at the well sites. Outside the island the water table was held constant at 15.0 feet, and a storage coefficient of 0.15 was selected. In the computer solution, hydraulic head values are computed


Figure 12. Jacobs Method for Solution of Nonequilibrium Equation. One Observation Well.


Figure 13a. Three-Dimensional Representation of Square Island with a Recharge Well at Node $(12,12)$ and a Discharge Well at Node $(7,7)$. The Water Surrounding the Island致s a Constant Level.


Figure 13b. Grid System of Square Island. Diagonal I-I Is the Same Diagonal I-I as Shown in Figure 13a.
for every timestep for grid points which are 200 feet apart and for $\Delta t=0.333$ days.

## Expected results

1. Because of the symmetric location of the two wells, a diagonal line between the two wells will appear at which the water table does not change and will maintain the initial head value.
2. The drawdown at the discharge well location will be greater than the build up at the reonege well. This is causea ly the different transmissibilities at the two locations. (At the discharge well the depth of aquifer is less than at the recharge well site ( $T$ discharge $\langle T$ recharge).

Because no easy analytical solution is available for this case, no absolute value check can be made. However, a check of the expected results is possible.

At the computer site where this program was checked, a microfilm plotter-printer was available. With the main program, a subroutine was linked which plotted a contour map of the solution for each timestep. Figure 14 represents a computer plot of water table contours for the $18^{\text {th }}$ timestep.

As shown in Figure 14, the diagonal of constant head is easy to detect.

The com puter solution indicates that this system reaches the steady state after 18 timesteps or 6 days. For the steady state situation, the head at the location of the discharge well is 12.86 ft which is 2.14 feet below the initial value, and the head at the location of the recharge


Figure 14. A Computer Plot of Water Table Contours of Test 2 for the 18 th Timestep. Discharge Well at Location $(7,7)$. Recharge Well at Location $(12,12)$.
well is 16.87 which is 1.86 feet higher than the initial value.
The foregoing problems dealt with the validity of the program as a simulation tool itself and showed that the constant head boundary treatment was valid.

## Flowboundary Solution - Setup of experiment

With the flow boundary solution it is possible to terminate an aquifer by a boundary across which flow occurs for conditions as described on pages 34 and 36. To test this boundary two cases were considered which were subject to the same input conditions. Case l considers an aquifer which is terminated both by impermeable boundaries and constant head boundaries as shown in Figures 15 a and $15 b$. Case 1 shows a case of flow (steady or unsteady) through a dam. The boundaries on each side of the dam are bodies of water which are kept at a constant level.

Case 2 is actually the same as Case 1 except that it is cut in half with a flow boundary at the location of the cut, Figures 16 a and 16 b . Figures 17 and 18 show the conceptional idea of respectively Case 1 and Case 2.

AREA INTEREST is the study area for which a model is proposed.
Under certain conditions which are described on page 34 and 36,
it is not necessary to expand the model to a larger model, which covers also the NO VALUE AREA (which is not of interest), in order to terminate the model with a conservative boundary like a constant head boundary. Instead, the model can be confined to the AREA OF INTEREST and a flow boundary will terminate the model without introducing a significant difference in results.

It must be proven then that for both Case 1 and Case 2, the model shows the same results for the AREA INTEREST if subjected to the same


Figure 15a. Three-Dimensional Representation of Case I. Case I Is an Aquifer Terminated by Constant Head Boundaries and Impermeable Boundaries.


Figure 16a. Three-Dimensional Representation of Case II. Case II Is Actually the Same as Case I Exaept That It Is Cut in Half with a Flow Boundary at the Location of the Cut.


Figure 16b. A Schematic Representation of Figure 16a, Case II.


Figure 17, Under Certain Conditions, It Is Not Necessary to Expand the Model to a Larger Model (Case I, Shown Above). This Covers Also the NO VALUE AREA (Which Is Not of Interest) in Order to Terminate the Model with a Conservative Boundary Like a Constant Head Boundary. Instead, the Model Can Be Confined to the AREA OF INTEREST (Case II, Sh own Below) and a Flow Boundary Will Terminate the Model Without Introducing a Significant Difference in Results.


Figure 18. Case II: The Model Is Confined to the AREA OF INTEREST Using a Flow Boundary.
hydraulic activities.

## Flow boundary, Test 3

For a situation as shown in Figure 19, the hydraulic head values for the steady state condition are determined analytically. The continuity equation for unsteady flow in an isotropic homogeneous medium is:

$$
-K \frac{\partial}{\partial x} h\left(\frac{\partial h}{\partial x}\right)=-\phi_{d} \frac{\partial h}{\partial t}
$$

in which $\phi_{d}$ is the drainable porosity and $K$ is the hydraulic conductivity. For steady flow and under the Dupuit-Forchheimer ${ }^{10}$ assumptions, the equation yields:

$$
\mathrm{h}^{2}=\frac{\mathrm{H}_{\mathrm{O}}^{2}-\mathrm{H}_{1}^{2}}{\mathrm{~L}} \mathrm{x}+\mathrm{H}_{\mathrm{l}}^{2}
$$

In the computer solution, Figure 19 is identical to Case 1 (Figure 20a).
Case 2 is the flow boundary solution for this problem which is actually Figure 19 terminated in the middle of the aquifer. (Figure 20b). The initial conditions for both cases were the steady state values for hydraulic head. For both cases, the model was run for 21 timesteps. Since the initial condition was a steady state, the computed head values for Case 1 were the same for all timesteps and equal to the initial conditions. In Case 2 with the flow boundary and the same initial conditions, the calculated head values were the same as calculated for Case 1, Figure 20a, i。e. for steady state the flow boundary maintains a steady state.

## Flow boundary, Test 4

In this case both cases were subjected to an input term which was the same for each node, as shown in Figure 21.

Each nodepoint was given recharge of $3,500,000 \mathrm{ft}^{3} / \mathrm{sq}$. mile. day during the total simulation time. $\Delta t=7.0$ days. $K X=K Y=2000 \mathrm{ft} / \mathrm{day}$, $S=0.15$.


Figure 21. Schematic Representation of Test 4. Case I and Case II Are Subjected to an Input Term, the Same for Each Node. Each Node Has a Recharge of $3.10^{6} \mathrm{ft}^{3} / \mathrm{sq}$ mile/day. $\Delta t=7.0$ days, $S=0.15, \quad K X=K Y=2000 \mathrm{ft} /$ day. Total Simulation Time Is 21 Timesteps.

The initial condition for both cases was the steady state condition as described in Test 3. Both were run for 21 timesteps. Figures 22, 23, and 24 compare the rise of head as a function of time at nodepoints 1,2 , and 3 for the two cases.

Figure 22:
Point 1 represents the nodepoint at the flow boundary. For the first 11 timesteps, there is almost no difference between the results of Case 1 and Case 2. After the llth timestep, the hydraulic head in Case 2 takes on a greater value. The total rise for point 1 in Case 1 is 89 feet. The total rise for point 1 in Case 2 is 81 feet. The difference is 8 feet or $8.1 \%$ of the rise after 21 weeks of this input conditior, or $2.2 \%$ of the aquifer depth. There are several reasons why this is on the large side.

1. For this test run no experimentation was done with the CONTROL CONSTANTS for this flow boundary. A better adaption of the CONTROL CONSTANTS to this aquifer will yield a better resemblance between the two graphs.
2. The input term of 3.5 million feet ${ }^{3}$ per square mile per day is quite large, which results in a fast rise of the water table for which a smaller applied timestep would also yield more accurate results. As can be seen after 21 timesteps, no steady state condition is reached which is also caused by the rather large input term. An analytical comparison for this case with Cases 1 and 2 is not possible because the rise of water table compared to the depth of aquifer (350 feet) is 80 feet or $23 \%$. This is very large so that a comparison of the results will not be relevant because the Dupuit Forchheimer ${ }^{10}$ assumptions, on

Figure 23. Hydraulic Head as a Function of Time for Point 2, Calculated in
which both the analytical solution and the computer model are based, are violated. It would be useful to repeat this test with a smaller input term. With a smaller input term, both cases would reach a new steady state condition. Lack of time, however, made it impossible to carry out such a test.

Figure 23:
For point 2 the total rise in Case 1 is 84 feet and the total rise in Case 2 is 79 feet. A difference of 5 feet or $6.3 \%$ of the rise. This is a smaller difference (or better fit) than for point 1 which is to be expected because point 2 is farther from the flow boundary.

Figure 24:
For point 3, the total rise in Case 1 is 79 feet and total rise in Case 2 is 76 feet. This shows a difference of 3 feet or $3.8 \%$ in rise which is tolerable. The result is obtained on a distance of $2 \Delta x$ from the flow boundary. A graph which depicts the difference in rise between the two cases as a percentage of the total rise versus distance from the flow boundary in percentage of total length of aquifer is shown in Figure 25.

This graph shows that for $75 \%$ of the aquifer the difference between the two models expressed as a percentage of the rise is less than or equal to $3.0 \%$.

Experimentation with the CONTROL CONSTANTS will yield a better result so that Case 2 gives a good simulation of Case 1.

## Flow Boundary, Test 5

In both cases the model was subjected to an input term for each node in the respective aquifers. Figure 21 applies here also.


Figure 25. Test 4: Difference in Rise in Percentage of Total Rise Between Case I and Case II as a Function of Distance From Flow Boundary in Percentage of Total Length of Aquifer.
$\Delta t=$ one week. Initial conditions for both models is the steady state condition as described in Test 3. Each nodepoint was given a recharge of 1 million $\mathrm{ft}^{3} /$ day for the first $91 / 2$ weeks. After the recharge was terminated, the computer runs were continued until both cases had reached again their initial steady state condition.

Objective of this was whether comparable nodepoints in the two cases showed the same changes in hydraulic head as a function of time. Figures 26,27 and 28 respectively compare the rise of hydraulic head as a function of time at nodepoints 1, 2, and 3 for the two cases.

Figure 26:
Point 1 represents the nodepoint at the flow boundary. The maximum rise of the water table is 27 feet for Case 1 and 25 feet for Case 2.

During the 74 weeks it takes to regain the steady state condition, the difference is nowhere greater than 2 feet except for values of the $59^{\text {th }}$ until the $61^{\text {st }}$ week. Because the computer printout is given to the nearest feet, this could also be caused by a roundoff error.

As can be seen both Case 1 and Case 2 reach the maximum rise at the same timestep and the decrease in head after the maximum values takes place identically. Case 2 also gives here a good simulation of Case l. Another test for both cases, with as initial head values the maximum head values obtained in above simulation (the cases then have an unsteady state condition as start), would yield the same final results.

Figure 27:
The same can be said for this nodepoint on a distance of $\Delta x$ ( $=1$ mile) from the flow boundary with a slightly closer result for Case 1 and Case 2.

Figure 26. Hydraulic Head as a Function of Time for Point 1, Calculated in


Figure 27. Hydraulic Head as a Function of Time for Point 2, Calculate in Test 5.


Figure 28. Hydraulic Head as a Function of Time for Point 3, Calculated in Test 5.

Figure 28:
Again this graph, showing the hydraulic head as function of time for nodepoint 3 , shows an almost identical curve for Case 1 and Case 2. The maximum difference varies between 1 and 2 feet to the nearest foot.

## Conclusions

Test 1, Filterwell in island, shows the same results for both the analytical solution and the computer solution which also shows that the constant head boundary in the computer model works correctly.

Test 2 was more or less a qualitative test, but nevertheless shows that the computer model behaves according to the expectations.

Test 3, the steady state condition, shows the validity of the impermeable boundary in the computer model. Also the flow boundary solution proves to be valid for a steady state condition.

Test 4 and Test 5 show that in several different unsteady conditions, identical results are obtained with an aquifer that is terminated by a flow boundary, and comparable aquifer that is terminated by a conservative boundary (constant head boundary or impermeable boundary). Only close to the nodes of the flow boundaries are significant differences noticed, but they are mostly in the $5 \%$ range which is considered to be negligible. By a good choice of the CONTROL CONSTANTS, which are dependent on the shape of the aquifer inside and outside the study area, a termination of an aquifer by a flow boundary can be shown to be valid. This, of course, is as̀sumed as long as conditions on pages 34 and 36 are applicable. This computer model has shown to be a valid simulation tool.

## CHAPTER IV

# APPLICATION OF THE MODEL TO THE SNAKE RIVER FAN IN SOUTHEASTERN IDAHO 

## The Study Area

## Plan of the study area, grid system

Figure 29 shows a map of the study area. Over this area a square grid is superimposed in which $\Delta x=\Delta y=1$ mile. Every intersection of grid lines represents a nodepoint. The location of the grid lines is chosen in this manner so that the nodepoints are situated in the center of the public land survey sections. Input or output from the aquifer is possible at every nodepoint.

## Boundaries of the study area

Figure 29 also shows the choice and location of the boundaries. The Snake River serves as a constant head boundary for the northeast and northern part of the area. For these parts, the river has a fully saturated connection with the groundwater aquifer.

The southeast part of the aquifer is bounded by a mountainous area (called the South Hills) which serve as a natural no flow - or impermeable boundary.

In the southwest corner the groundwater aquifer connects with the deep groundwater table of the Snake River Plain via a sharp drop down in hydraulic head. Since the flow is generally in southwest direction, the boundary is composed of two types:

1. artificial no flow boundary. This boundary appears on the north and south side of the flow and is chosen perpendicular to the general direction of the existing equipotential lines.

2. the flow boundary, as described on pages $34,36,37$, and 38 closes the study area. This solution for a flow boundary is applied because the properties of this part of the study area satisfy the limiting conditions of this solution, as described on pages 34 and 36 .

The aquifer outside the study area connects with the (general) deep water table in the Snake River Plain which has a more or less confined character just west of the study area. The boundary of this Snake River Plain aquifer is some one hundred miles away.

All nodepoints close to this specific boundary have the same source term. All belong to the same irrigation district which distributes approximately an equal irrigation application per node. The stratum which underlies the aquifer acts as a leaky aquifer and does this for all nodes close to this boundary.

From this information the input arrays $\operatorname{NCX}(i, j)$ and $\operatorname{NCY}(i, j)$ which denote the character of the nodepoints in the study area are determined. They are included in Appendix $H$.

## Data Collection for the Study Area

The data which are entered in the main program include 11 arrays, as described on pages 40,41 , and 42.

1. $\operatorname{NCX}(\mathrm{i}, \mathrm{j})$
2. $\operatorname{NCY}(i, j)$
3. $N N(i, j)$
4. $K X(i, j)$
5. $K Y(i, j)$
6. $Z(i, j)$
7. $\operatorname{PHI}(i, j, 1)$
8. $\operatorname{PSI}(i, j)$
9. $\operatorname{SURF}(i, j)$
10. $S(i, j)$
11. $\operatorname{FAC}(\mathrm{i}, \mathrm{j})$

Other input data is data which specifies the output, relates to the total source term $Q(i, j, k)$, specifies the dimensions of the grid system, specifies the location of the flow boundary, and assigns values for the CONTROL CONSTANTS.

Remaining is the source term $Q(i, j, k)$ which is generated in separate input programs.

Only the above mentioned arrays are directly related to the geohydrologic properties of the study area.

In order to get reliable data for the arrays, a large amount of field data is necessary. A continuous data collection program was begun in May, 1970, in the area. Cooperating local residents have greatly assisted in the data collection program.

## Geology (Progress Report ${ }^{22}$ )

Available well logs from the Department of Water Administration and local residents indicate that the gravel aquifer is extensive over the Fan. However, very few of the domestic wells for which logs are available are over 100 feet deep so that the depth of the gravels is not discernible over the entire Fan. One exploratory well, one and one-half miles northwest of Rigby is 1008 feet deep and indicates a 300 foot depth of gravel underlain by 170 feet of clay above basalt. Basalt is encountered at shal low depths south of Ririe and fingers of basalt extend eastward from the Lewisville Knolls on the western edge of the area into the gravels.

Because of the lack of geologic information in the area, a cooperative geophysical study was undertaken in May 1971 with the U. S. Bureau of Reclamation. Electrical resistivity transects were run from east to west across the fan from the Lewisville Knolls to the Snake River. Results of these transects showed that electrical resistivity tests as they were run in this area were not effective in revealing the geological aspects of this aquifer, because of a highly resistant layer at the surface which obscured the deeper formations.

In general, an unconfined gravel aquifer is assumed, 100 feet deep in the southeast corner and along the Snake River, gradually increasing to $300-400 \mathrm{ft}$ at the location of the flow boundary. Clay layers of varying thickness and at varying depths are interspersed throughout the fan.

Ground water table elevations (Progress Report ${ }^{22}$ )
A network of some 40 wells in the area is being used to monitor changes in the groundwater table throughout the year. Figure 30 shows the locations of wells and well points being measured in the network. Water stage recorders were installed on seven of the wells in order to detect fluctuations of shorter period than the normal weekly measurements. In the vicinity of Rigby, water table rises of 30 feet or more have been recorded for the period from the beginning of the irrigation season until midJuly. Figure 31 shows the hydrograph of the Clark well northwest of Rigby for several seasons beginning with 1963.

Water table contours for the area indicate a general east-west water flow with a rapid increase in water table depth on the west and southwest boundaries. Figure 32 shows the water table contours interpreted from well measurements made on July 15,1970 . The flow is generally from east to west with the flow under the Snake River south of the city of Roberts,



being at greater depths where the gravels of the fan apparently interfinger into basalts of the Snake Plain aquifer.

Maximum groundwater table elevations usually occur in the month of August and associated problems are prevalent during August and September. Figure 1, page 2 shows the depths to the water table on August 30, 1970 as computed from the groundwater contours. The area north and west of Rigby as indicated in the figure had depths to water of five feet or less during July, August and September of 1970. The area around the city of Ririe is a local groundwater mound and some reports of damage have been received from this area.

## Evaluation of the input arrays

From above described data and maps of the area (Progress Report ${ }^{22}$ ) values can be attached to arrays like:

1. $\operatorname{NCX}(i, j)$ ?
2. $\operatorname{NCY}(\mathrm{i}, \mathrm{j})$ \}

They are simply derived from the boundary conditions as described on pages 72 and 73.

The arrays NCX and NCY are shown in
Appendix H .
3. NN(i,j), array denoting the confined or unconfined character of the aquifer. The Snake River Fan is an unconfined aquifer.
4. $Z(i, j)$, the elevation of the aquifer bottom.
5. $\operatorname{PHI}(i, j, 1)$, the initial hydraulic head, determined from the measurements in existing wells.
6. $\operatorname{SURF}(i, j)$, the land surface elevation.
7. Geological data reveal a gravelly aquifer with a coarser medium close to the river and a finer type at an interesting distance from the. Snake River. From these data average values
for the storage coefficient are determined, varying from $S=0.22$ to $S=0.15$. This gives: $S(i, j)$.
8. $\operatorname{PSI}(i, j)$ the hydraulic head in the confining layer and
9. $\operatorname{FAC}(i, j)$ the impedance of the confining layer. $\operatorname{PSI}(i, j)$ and FAC(i,j) define the leaky aquifer in this system. From water budget data it is certain that there is a leaky aquifer situated under a great part of the aquifer. Little, however, is known about this and only an approximation can be made. The discussion of the determination of these arrays is shown in Appendix D.
10. $K X(i, j)$, the hydraulic conductivity in $x$-direction and
11. $K Y(i, j)$, the hydraulic conductivity in $y$-direction remain to be determined. The discussion of determination of $K X(i, j)$ and $K Y(i, j)$ is also shown in Appendix D.

The total source term $Q(i, j, k)$ is calculated in two input programs respectively Input Program I and Input Program II, and entered in the main program as a three-dimensional array. The other input data are chosen according to the specific in and output requirements.

## The Total Source Term $Q(i, j, k)$

## General

The source term $\mathrm{rl}(\mathrm{muQi}, \mathrm{j}, \mathrm{k})$ (see page 27 ) for computation of the rows and the source term $\mathrm{r} 2(\mathrm{muQi}, \mathrm{j}, \mathrm{k})$ for computation of the columns both contain the term Qi,j,k which represents the flux in an area $\Delta x$ long, $\Delta y$ wide. That is the total flux during a half time step in cubic feet for each nodepoint.

As can be seen the term has two space dimensions and one time dimension which makes it possible to apply different inputs for each node
for each half timestep. This is an appropriate approach for aquifers with nodepoints that have inputs which differ substantially during the time increments. The Snake River Fan in eastern Idaho is composed of many different irrigation districts in which input or output due to irrigation practices varies substantially. Due to this and the physical properties of the aquifer, the rise of the water table varies from 5 to 50 feet. Maximum rise for all the nodepoints does not occur at the same time. To make a reasonable simulation of this hydrogeological system, it was considered necessary to approximate as accurately as possible input for each nodepoint at earh timestep. For the Snake River Fan the grid size is $25 \times 20$ nodes with a time cycle of a year, divided into 42 half timesteps. A ( $25 \times 20 \times 42$ ) matrix increases the dimension size of the model considerably, but grid systems of this size do not overload the computer core capacity. However, large sized systems with a three dimensional input term do need large computer facilities. In that case, it would be convenient to assume an input that is constant in time. For an aquifer in which the source term consists mainly of pumped wells, this is an entirely appropriate method.

## The source term $Q(i, j, k)$

The source term $Q(i, j, k)$ can be written as follows:

$$
\begin{gathered}
Q(i, j, k)=-Q I(i, j, k)-Q S(i, j, k)-P E(i, j, k)+E(i, j, k)+A M O(i, j, k) \\
+P U M(i, j, k)-Q C L(i, j, k)
\end{gathered}
$$

where
QI = input due to irrigation at each node during time increment $(k-1)$ to $k\left[L^{3} / T\right]$

QS = input due to canal seepage at each node for time increment $(k-1)$ to $k\left[L^{3} / T\right]$
$\mathrm{PE}=$ precipitation $\left[\mathrm{L}^{3} / \mathrm{T}\right]$
$\mathrm{E}=$ evapotranspiration $\left[\mathrm{L}^{3} / \mathrm{T}\right]$
$A M O=$ change of average water content in soil profile above the water table $\left[\mathrm{L}^{3} / \mathrm{T}\right]$

PUM $=$ out or input due to a pumped well at node location ( $\mathrm{i}, \mathrm{j}$ ) for time increment $(k-1)$ to $k\left[L^{3} / T\right\rceil$

QCL $=$ in or output due to a leaky aquifer which leakage per node is constant in time $\left[\mathrm{L}^{3} / \mathrm{T}\right]$

The total time cycle of the digital model is a full year. However. the major agricultural and hydraulic activities take place during a certain part of the year. In winter and early spring there is no irrigation, and because of the low temperatures, the evapotranspiration is negligible. The rest of the year irrigation is being practiced. A time table for this area is made in which the timesteps in the irrigation season are $\Delta t$ and are increased to $2 \Delta t$ for the winter season.

The irrigation season starts when all irrigation canals are filled for the first time. It is extended to the date after which no irrigation takes place and the evapotranspiration is negligible. For this part of the year the source term is calculated as a volume per half timestep (per mesh surface) so that $\Delta t$ (irrigation) $=\Delta t / 2$. The rest of the year is called the winter season, and for this part of the year the source term is also calculated as a volume per half timestep but now $\Delta t$ (winter) $=2 \Delta t / 2=\Delta t$.

For the study area the major irrigation canals are filled in the first half of May. Diversions take place until November $25^{\text {th }}$ or later.

For the model the irrigation season starts May $1^{\text {st }}$ and ends December $11^{\text {th }}$. $\Delta t$ is chosen as 14 days.

With this in mind the following scheme can be set up.
I. Irrigation Season, $\Delta t$ (irrigation) $=1$ week

$$
Q(i, j, k)=-Q I-Q S-P E+E+A M O+P U M-Q C L
$$

II. Winter Season, $\Delta t($ winter $)=2$ weeks

$$
Q(i, j, k)=Q S+P U M-Q C L \quad(Q I=P E=E=A M O=0)
$$

Table 1 shows the two seasons for a year simulation.

Table 1. Time Table of the Two Seasons for Computer Model

| K | K | K |
| :---: | :---: | :---: |
|  |  |  |

$Q(i, j, 2)=$ source term for the first half timestep $Q(i, j, 43)=$ source term for the last half timestep k represents the number of half timesteps +1
$Q(i, j, 2)$ is the first term calculated
With the arrow a full time step is visualized. The irrigation season is from $k=2,1 . . . .33$. Winter season is from $k=34, . . . . .43$.

For this area the source term $Q(i, j, k)$ is not calculated in one input program. The evapotranspiration term and precipitation term are calculated in a separate program, Input Program I. The results of this program is a part of the input of the second program, called Input Program II in which the total source term is calculated.

## Input Program I: Calculation of the Precipitation and Evapotranspiration Terms of $Q(i, j, k)$

## Climate (Progress Report ${ }^{22}$ )

The Rigby - Ririe are of Jefferson County lies in the climatological area known as the Upper Snake River Plains with moderately warm summers and rather severe winters. Temperatures average about $68^{\circ} \mathrm{F}$ in July and $17^{\circ} \mathrm{F}$ in January and $0^{\circ} \mathrm{F}$ temperature or lower generally occurs for at least 16 days a year.

The growing season averages 123 days in duration and the growing degree days above $40^{\circ} \mathrm{F}$ average 3710 degree days. Precipitation averages 8.7 inches with $25 \%$ occurring in May and June and sunshine averages from 80 to $85 \%$ of possible in July - September. Average windspeeds, generally from the southwest range from 10-15 miles per hour with high winds of 40 - 60 miles per hour occurring most often in April.

Crop distribution (Progress Report ${ }^{22}$ )
The crop evapotranspiration is calculated with data based on the crop distribution for the 22,000 acres of the Burgess Canal Company, an irrigation district in the study area.

Crop distribution throughout the study area is nearly uniform so that the assumption is made that the crop distribution for each nodepoint
( 159 mile $=640$ acres) in the study area is the same. The evapotranspiration term is then only a function of time and the subscripts with respect to the space coordinates are dropped. Evapotranspiration is $E(k)$.

## Potential evapotranspiration: PET

Potential evapotranspiration is the evapotranspiration for a well watered crop with an aerodynamically rough surface like alfalfa with about 12 to 18 inches of top growth.

Under these conditions the rate of PET is controlled mainly by heat energy available. For this area the evaporitive flux was calculated using the Penman ${ }^{20}$ Combination equation, an equation which is based on the energy balance concept and aerodynamic data.

$$
\begin{equation*}
\operatorname{PE}=\frac{\Delta}{\Delta+\gamma}(\operatorname{Rn}+G)+\frac{\gamma}{\Delta+\gamma}(15.36)(1.0+0.0092 W)\left(e_{s}-e_{d}\right) \tag{46}
\end{equation*}
$$

The coefficient 0.0092 of the wind factor is slightly different from the original value in the Penman Equation since it is adapted to the specific height at which the anemometer is installed for these measurements.

Input Program I first calculates PE in $\mathrm{cal} / \mathrm{cm}^{2}$ and then converts it to a depth equivalent in inches, using $585 \mathrm{cal} / \mathrm{gm}$ as latent heat of vaporization.

Potential evapotranspiration (PET) = PE/1485.9 [inches/day1

## Data oollection 1

In order to utilize equation (46), daily data of the following variables are necessary.

1. day and month of year, respectively $\mathrm{MO}(\mathrm{I})$ and $\mathrm{ND}(\mathrm{I})$
2. mean or maximum air temperature, $\mathrm{TX}(\mathrm{I})$ in ${ }^{\circ} \mathrm{F}$
3. minimum air temperature, $\mathrm{TN}(\mathrm{I})$ in ${ }^{\mathrm{O}_{\mathrm{F}}}$
4. dew point temperature, $\mathrm{TD}(\mathrm{I})$ in ${ }^{\mathrm{O}} \mathrm{F}$
5. wind velocity, $W(I)$ in miles/day
6. solar radiation, $\mathrm{RS}(\mathrm{I})$ in Langleys
7. Clear day radiation, $\mathrm{RSO}(\mathrm{I})$ in Langleys
$\operatorname{PET}(1)=$ potential evapotranspiration at first day of irrigation season (May $\left.1^{\text {st }}\right)$. In the computer program the total number of days (starting May $1^{\text {st }}$ ) in the irrigation season is called NDAYS.

For this area most of the data was available; however, for solar radiation no data of the study area was available. Solar radiation data was available from the irrigation districts near Twin Falls, and a correlation between RS Twinfalls and RS Study area was made. Via a dummy variable NOWN, the program converts the RS Twinfalls to RS Study area as follows:

If $(\mathrm{NOWN}=0)$ then $\operatorname{RS}(\mathrm{I})=37.015+0.86433 \mathrm{RS}(\mathrm{I})$. A zero value for NOWN indicates that no solar radiation data of the study area is available and therefore uses correlation with another area. If NOWN is nonzero, this correlation is neglected andown data is used.

The function ENl(A) converts the temperature to equivalent saturation vapor pressure value. After calculating the variables which are used in equation (46), the evapotranspiration is calculated as follows:

$$
\operatorname{PET}(\mathrm{I})=\operatorname{Tl}(\mathrm{I}) *(\operatorname{RN}(\mathrm{I})-\operatorname{GEST}(1))+\mathrm{T} 2(\mathrm{I}) *(15.36) *(1.0
$$

$$
\begin{equation*}
+0.0092 \mathrm{~W}(\mathrm{I})) * \operatorname{VDEF}(\mathrm{I})) / 1485.9 \tag{47}
\end{equation*}
$$

in which (in comparison with equation (46))
Tl (I) $=\frac{\Delta}{\Delta+\dot{\gamma}}$
$\mathrm{RN}(\mathrm{I}) \quad=\mathrm{Rn}$
$\operatorname{GEST}(\mathrm{I})=\mathrm{G}$
T2 (I) $=\frac{\gamma}{\Delta+\gamma}$
$\mathrm{W}(\mathrm{I})=\mathrm{W}$
$\operatorname{VDEF}(\mathrm{I})=\left(\mathrm{e}_{\mathrm{s}}-\mathrm{e}_{\mathrm{d}}\right)$
$1485.9=$ conversion factor (cal/day into inches/day)

## Crop coefficient

In order to obtain the actual evapotranspiration for a crop the potential evapotranspiration has to be multiplied by a crop coefficient varying usually between 0 and 1.0. The crop coefficient (Jensen ${ }^{14}$ ) represents "The combined relative effects on the resistance of water movement from the soil to the various evaporating surfaces and the resistance to the diffusion of water vapor from the surfaces to the atmosphere and the relative amount of radiant energy available as compared to the reference crop." A crop coefficient curve is shown in Figure 33.

For the different crops, crop coefficient curves have been developed (Jensen ${ }^{14}$ ) which are described by a polynomial like: $\mathrm{KC}=\mathrm{C} 1+\mathrm{C} 2 * \mathrm{~A}$ $+C 3 * A^{2}+C 4 * A^{3}$. Two such polynomials compute the crop coefficient in which A represents the total time since start, expressed as a percentage of the total time from planting to effective cover. The beginning and end for this curve is the day of planting and the day of effective (full) cover or cutting. The second polynomial computes the rest of the curve, starting at the effective cover or cutting date with $A$ expressed as time in days after effective (full) cover.

To obtain the daily values of the evapotranspiration for each crop, the crop coefficient is multiplied by the potential evapotranspiration.

## Data collection 2

In order to calculate a combined crop coefficient, if more crops are present, the following data is necessary:

1. number of crops (LAST)
2. the coefficients $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4$ for the crop coefficient polonomial of each crop.


Figure 33. Crop Coefficient Curve. Polonomial 1 Computes the Crop Coeficient from Planting to Effective Cover. Polonomial 2 Computes the Crop Coefficient for Days After Effective Cover.


Figure 34. Time Scheme with Depiction of Computer Variables to Calculate the Evapotranspiration Term if Day of Planting Is After Start Model.


Figuce 35. Time Scheme with Depiction of Computer Variables to Calculate the Evapotranspiration Term if Day of Planting Is Before Start of Model.
3. the day of planting of each crop
4. day of effective coverage
5. day of cutting
6. day of planting relative to start of the irrigation season in the model (May $1^{\text {st }}$ ). This is necessary in order to correlate the right value of potential evapotranspiration with crop coefficient value for specific days. This correlation is shown in Figure 34 and 35.

DAYS $=$ total number of days between planting and effective cover.
UNM = number of days between planting and start of model if planting is before start model.

NPL = number of days between planting and start model if planting is after start model ${ }_{\text {. }}$
$\mathrm{LF}=$ number of days between start model and last day crop is on the field.

LTS = number of weeks after which "winter" season starts.

## Actual evapotranspiration

The computer program first computes the daily evapotranspiration of each crop in inches. The half timestep in the irrigation season is 1 week. In order to get the value per $\Delta t$ (irrigation), daily values are summed up to weekly values.

For this study 4 crops were considered for which the crop distribution is as follows:

| Grain | $31 \%$ | Weighting factor $00(1)=0.31$ |
| :--- | :--- | :--- |
| Alfalfa Hay | $27 \%$ | $00(2)=0.27$ |
| Potatoes | $31 \%$ | $00(3)=0.31$ |
| Pasture | $11 \%$ | $00(4)=0.11$ |

For each crop these weekly values are multiplied by the appropriate weighting factor $(\mathrm{OO}(\mathrm{I}), \mathrm{I}=1,4$ ), divided by 12 to convert inches into feet, and multiplied by the area of the node $(\Delta x * \Delta y)$ to obtain the weighted value as a total volume per node per $\Delta t$ (irrigation) in cubic feet.

For each crop the weighted crop evapotranspiration in $\mathrm{ft}^{3} / \Delta t$ (irrigation is known; $E(j, i), j=$ crop number, $i=n u m b e r ~ o f ~ w e e k . ~$

In order to get the total crop evapotranspiration/node the weighted values are summed up. For half timestep (I):
$\operatorname{EVAP}(\mathrm{I})=E(1, I)+E(2, I)+E(3, I)+E(4, I)$

## Precipitation

Daily values of precipitation were obtained from the weather stations in the area. Here the assumption is made that the average rainfall in the area is the same for each node. Because of the size of the area, this is a satisfactory approximation.

Values for precipitation in $\mathrm{ft}^{3} / \Delta \mathrm{t}$ (irrigation) are calculated under the name $\operatorname{PRE}(\mathrm{K})$ in the last part of the program.

The weekly values for the crop evapotranspiration and precipitation are now used in the second input program, Input Program II, which calculates the total source term.

A flow chart and listing of Input Program I is included in Appendix E.

Input Program II: Calculation of the Total Source Term $Q(i, j, k)$

## General

The source term $Q(i, j, k)$ is composed of two parts:
I. Source term for irrigation season

$$
\begin{aligned}
& \Delta t \text { (irrigation }=1 \text { week } \\
& k=1,2, . . ., 33
\end{aligned}
$$

$$
\begin{gather*}
Q(i, j, k)=-Q I(i, j, k)-Q S(i, j, k)^{*}-\operatorname{PE}(k)+E(k)+A M O(i, j, k) \\
+\operatorname{PUM}(i, j, k)-Q C L(i, j) \tag{48}
\end{gather*}
$$

II. Source term for winter season

$$
\begin{align*}
& \Delta t(\text { winter })=2 \text { weeks } \\
& k=34,35, \cdot \cdots 43 \\
& Q(i, j, k)=\operatorname{PUM}(i, j, k)-\operatorname{QCL}(i, j)-\operatorname{QS}(i, j, k) \tag{49}
\end{align*}
$$

They will be discussed successively.

## Part I of input program II, source term for irrigation season

The irrigation term $Q I(i, j, k)$
The irrigation system for this area was developed in the latel800's.
A former channel of the Snake River runs east and west through the area and is used as a canal for the greater part of its length. This canal delivers water to some 17 smaller canals each one operated by a separate and independent canal company or irrigation district. The gross acres which are served by each canal company varies from 33,000 to 1,100 and the total water diversion per gross acre per year is also different for the various canal companies (Progress Report ${ }^{22}$ ).

For the study area a square grid is superimposed on the plan $\Delta x=\Delta y=1$ mile $=5,280 \mathrm{ft}$. Hence one nodepoint covers 1 square mile $=640$ acres.
*General note on the seepage term. The canal seepage term can be expressed as $\operatorname{QS}(i, j, k)=\operatorname{AS}(i, j, k) \cdot \operatorname{IS}(i, j, k)$. wheré:, $A S=$ total wetted area of canal at time $k$ per node surface ( $\Delta x \cdot \Delta y$ in square ft.) IS = seepage rate in cubic feet per square foot per day at time $k$.

Since the normal operating procedure for canals in the study area is to maintain water levels as near maximum as possible (Progress Report ${ }^{22}$ ) the change in wetted area with time is small and becomes a function of ( $\mathrm{i}, \mathrm{j}$ ) only. $\quad \mathrm{QS}(\mathrm{i}, \mathrm{j}, \mathrm{k}) \longrightarrow \mathrm{QS}(\mathrm{i}, \mathrm{j})$

Functional relationships between canal seepage rate and depth to water table require knowledge of the vertical hydraulic conductivity in the aquifer and would be difficult to determine. Since little data is available on $K$ values and several measurements of IS in canals throughout the area have been made, $\operatorname{IS}(i, j)$ is read in as an array and the values changed in different computer runs if necessary.

Each canal company is given a number, e.g. Burgess Canal Company has number 1 , and all nodepoints which belong to this Canal Company are given the same character number (which is 1 in this case). The character name for the nodes is called $\operatorname{NIR}(i, j)$.

The total number of nodes within one irrigation district is represented by the variable $D(I)$ in which $I$ is the number of the irrigation district. For the case of the Burgess Canal Company, (25 nodes) $I=1$, and $D(1)=25$. Figure 36 shows an arbitrary canal district with main irrigation canal. Over this map a square grid is superimposed. In this example the number of canal district is 3 ; so nodepoints $(i, j)$ within this district have $\operatorname{NIR}(i, j)=3$. There are 24 nodes within the district so $D(3)=24$. Figure 37 shows array $\operatorname{NIR}(i, j)$ for this district.

Total numbers of irrigation districts in study area is NI. The assumption is made that within the canal company or irrigation district, the irrigation application per node $(Q I(i, j, k))$, for an arbitrary timestep is the same. QI $(i, j, k)$ then becomes $Q I(m, k)$ in which $m$ denotes the irrigation district and $k$ the timestep.

Considering irrigation district 1 , the total net diversion per $\Delta t$ (irrigation) is: (Total inflow in district - Total outflow)/ $\Delta t$ (irrigation).

The total net diversion is a multiplication of a term called TA( $m, k$ ) times DIFA/7.0 in which subscript $m$ denotes the irrigation district. In this case $m=1$ 。
$T A(m, k)$ is a special form of the total net diversion, and is read in as a total volume per irrigation district per half timestep in cubic feet per second which flows for one day. TA(m,k) is in cfs-days because recorded diversions are published in this form.

The constant DIFA $=3,600.0(\mathrm{sec}) 24.0$ (hours) 7.0 (days); hence, if $T A(m, k)$ is multiplied by DIFA/7.0, the total net diversion for one half timestep in cubic feet is obtained.


Figure 36. Map, Showing Irrigation District No. 3. $D(3)=24$ or; There Are 24 Nodes in Irrigation District 3.


Figure 37. Array of NIR(i,j) for Irrigation District 3. All Nodepoints Within District No. 3 Have NIR(i,j) of 3. This Array Enables the Computer to Scan Through the Matrix and Calculate the Total District Seepage for Every Irrigation District.

This total net diversion is not used totally as irrigation application since part is lost by seepage out of the district canals called the district seepage. To bring clarity the following variables are defined:

The canal distribution for each node in a particular irrigation district $m$ is composed of
a. district canals: canals in node (i,j) belonging to irrigation district m.
b. non-district canals: canals of other irrigation districts which pass over node ( $i, j$ ) of irrigation district $m$.
c. non-irrigation canals: canals other than irrigation canals which of mot deliver to the area and which remain full during the entire year (e.g. Snake River or Great Feeder).

From this, it follows that the seepage for each node in a particular irrigation district m is divided into three parts:
district seepage: seepage from district canals
non-district seepage: seepage from non-district canals
non-irrigation seepage: seepage from non-irrigation canals
RATE( $\mathrm{i}, \mathrm{j}$ ) is the average seepage rate per node read in as an array in $\mathrm{ft} / \mathrm{day}$.

ARTOT(i,j) is the total wetted area per node except for the wetted area of the non-irrigation canals, read in as an array in thousands of $\mathrm{ft}^{2}$.

ARW ( $\mathrm{i}, \mathrm{j}$ ) is the total wetted area per node of the non-irrigation canals, read in as an array in thousands of $\mathrm{ft}^{2}$ 。
$\operatorname{ARFOR}(\mathrm{i}, \mathrm{j})$ is the total wetted area per node of the non-district canals, read in as an array in thousands of $\mathrm{ft}^{2}$.
$\operatorname{AROWN}(\mathrm{i}, \mathrm{j})$ is the total wetted area per node of the district canals and is calculated by subtracting $\operatorname{ARFOR}(i, j)$ from $\operatorname{ARTOT}(i, j)$; $\operatorname{AROWN}(\mathrm{i}, \mathrm{j})=(\operatorname{ARTOT}(\mathrm{i}, \mathrm{j})-\operatorname{ARFOR}(\mathrm{i}, \mathrm{j}))$ in thousands of $\mathrm{ft}^{2}$.

To obtain the value for the district seepage term as a volume for one half timestep, this amount has to be multiplied by ( $1,000 * \Delta t$ (irrigation)). Define the multiplication factor $\operatorname{MULT}(i, j)$ as
$\operatorname{MULT}(\mathrm{i}, \mathrm{j})=1,000 * \operatorname{RATE}(\mathrm{i}, \mathrm{j}) * \Delta \mathrm{t}$ (irrigation)
and the district seepage becomes
district seepage $(i, j)=\operatorname{MULT}(i, j) * \operatorname{AROWN}(i, j)$ in $\mathrm{ft}^{3} / \Delta t$ (irrigation)
To calculate the total district seepage term for irrigation district 1 , the following routine was developed.

The computer scans through the rows and columns and checks the value of $\operatorname{NIR}(i, j)$. If $N \operatorname{IR}(i, j)$ is $l$ (node in district 1), the district seepage term for this node is obtained by multiplying $\operatorname{MULT}(i, j)$ and $\operatorname{AROWN}(i, j)$. The next node ( $\mathrm{i}, \mathrm{j}$ ) for which $\operatorname{NIR}(\mathrm{i}, \mathrm{j})$ is also 1 , is treated the same, and this value for district seepage is added to the previous total.

The total district seepage for district 1 is called $\operatorname{SSEEP}(1)$.
$\operatorname{SSEEP}(1)=\sum_{\operatorname{NIR}(i, j)=1} \operatorname{MULT}(i, j) * \operatorname{AROWN}(i, j)$
This is done for all irrigation districts. With this in mind, it follows that the total irrigation application AT for timestep $k$ in irrigation district $m$ is:

$$
\operatorname{AT}(\mathrm{m})=\mathrm{TA}(\mathrm{~m}, \mathrm{k}) * \mathrm{DIFA} / 7.0-\operatorname{SSEEP}(\mathrm{m})
$$

To be able to study the effects of a change in total net diversion on the hydraulic head, the term $T A(m, k) * D I F A / 7.0$ is multiplied by $C H$, a factor which can be changed for different production runs. Within irrigation district the irrigation application is the same for each node.

Let $A(m)$ be irrigation application per node $/ \Delta t$ (irrigation.
Then

$$
A(m)=(T A(m, k) *(D I F A / 7.0) * C H-\operatorname{SSEEP}(m)) / D(m)
$$

Note that in the way the district seepage term $\operatorname{SSEEP}(\mathrm{m})$ is calculated, the seepage of the district canals sections which are situated in other irrigation districts (these sections are denoted as non-district canals) is not incorporated in $\operatorname{SSEEP}(\mathrm{m})$, as should be. The error introduced by this is negligible because the percentage of total district canal area denoted as non-district area, is negligible compared to the percentage of total district canal area which lies in the own irrigation district, so error made in the calculation of the total irrigation application $m$ for district $m$ is negligible.

The reason for the distinction between district andnon-district canals is that on a microscopic scale (i.e. per nodepoint), the ratio between non-district and district canal area may be quite large. For some nodepoints the nondistrict seepage term is large while no district canal passes over this node (district seepage $=0$ ) so that the nondistrict canal seepage has a large impact on the water balance for that particular point.

Summarized: The irrigation term per node in irrigation district $m$ is for timestep $k$

$$
\mathrm{QI}(\mathrm{~m}, \mathrm{k})=\mathrm{A}(\mathrm{~m})=\overbrace{(\mathrm{TA}(\mathrm{~m}, \mathrm{k}) *}^{\text {Total net diversion }}, \underbrace{\sum \text { district }}_{(\operatorname{DIFA} / 7.0)} * \overbrace{\operatorname{CH}-\operatorname{SSEEP}(\mathrm{m})}^{\text {seepage }} \text {, } \overbrace{\mathrm{D}(\mathrm{~m})}^{\begin{array}{c}
\text { total } \\
\text { of } \\
\text { of } \tag{50}
\end{array}}
$$

## The seepage term $Q S(i, j)$

As said in the general note on the seepage term on page 9l, QS is only a function of the space coordinates.

The total seepage for one nodepoint is a sum of district seepage/node non-district seepage/node non-irrigation seepage/node

To be able to study the effects of changes in these seepage terms on the hydraulic head in successive production runs, the following terms are added:
$\Delta$ district seepage (DAOWN $(i, j)$ ), and
$\Delta$ non-district seepage (DAFOR(i,j)).
The total seepage term/node is called $\operatorname{RESEEP}(\mathrm{i}, \mathrm{j})\left[\mathrm{ft}^{3} / \Delta \mathrm{t}\right.$ (irrigation].
$\operatorname{RESEEP}(\mathrm{i}, \mathrm{j})=\operatorname{MULT}(\mathrm{i}, \mathrm{j}) *(\operatorname{AROWN}(\mathrm{i}, \mathrm{j})+\operatorname{ARFOR}(\mathrm{i}, \mathrm{j})+\operatorname{ARW}(\mathrm{i}, \mathrm{j})$
$-\operatorname{DAOWN}(\mathrm{i}, \mathrm{j})-\operatorname{DAFOR}(\mathrm{i}, \mathrm{j}))$
in which
DAOWN = change in district wetted area
DAFOR = change in non-district wetted area
The total input from irrigation and seepage for nodepoint (i,j) and half timestep $k$ is $A M(m)+\operatorname{RESEEP}(i, j)$. This is for nodes in irrigation district m. Therefore,

1. Any change in irrigation application can be represented by a change in total net diversion (change CH )
2. Any change in district seepage or non-district seepage can be expressed in $\Delta$ district seepage and $\Delta$ non-district seepage. This can also be done in combination with the $\operatorname{Rate}(i, j)$.

In here the assumption is made that a decrease of seepage will result in an excess of canal waste and not as an increase of net diversion, which would increase the irrigation application.

## The precipitation term $\mathrm{PE}(\mathrm{k})$

This term is read in from Input Program I under the name $\operatorname{PRE}(k)$.

## The evapotranspiration term $E(k)$

This term has also been discussed in Input Program I and is read in under the name $\operatorname{EVAP}(k)$ from this program.

## Change of water content term $A M O(i, j, k)$ as volume per nodepoint per $\Delta \mathrm{t}$ (irrigation)

The term AMO which is a function of $x, y$ and $t$ can be written:
$\operatorname{AMO}(x, y, t)=\frac{\partial \bar{\theta}}{\partial t} Z T(x, y) *$ AREA
$\vec{\theta}=$ average moisture content over profile in feet per foot of soil moisture.
$\frac{\partial \bar{\theta}}{\partial t}=$ the change in time of this average moisture content. $Z T(x y)=$ the profile above the saturated zone in which $\frac{\partial \bar{\theta}}{\partial t}$ is significant AREA $=$ area of the nodepoint,

The term $\frac{\partial \bar{\theta}}{\partial \mathrm{t}} \mathrm{ZT}$ may or may not be significant depending on the value of $Z T$. Values for $\bar{\theta}$ at the beginning of the irrigation season ( $t=0$ ) can be estimated or measured. During the irrigation season $\bar{\theta}$ will increase to a certain value around which it alternates, depending on irrigation practices (Figure 38).

At sometime $t=t_{C}$, the term $\frac{\partial \bar{\theta}}{\partial \mathrm{t}} \mathrm{ZT}$ becomes insignificant if the fluctuation of $\bar{\theta}$ about the average moisture content called $\bar{\theta}_{e}$, is small.
$\Delta \mathrm{T}=\mathrm{t}_{\mathrm{C}} \mathrm{t}_{\mathrm{o}}$ is a function of $\bar{\theta}_{\mathrm{o}}$, porosity, ZT , and the rate of water supply. If $Z T$ is small, $\Delta T$ will be small and the change in storage may be neglected. However, if ZT is large this term could have a significant damping effect on the rise of the water table, and


Figure 38: Average Soil Moisture Content in Feet per Foot of Soil Profile as a Function of Time, Influenced by Irrigation Practices.
therefore influence the time response of the water table in the region.

In the finite difference model, the term
$\frac{\partial \bar{\theta}}{\partial \mathrm{t}} \mathrm{ZT}(x, y)$ AREA is replaced by:
$\left(\frac{\operatorname{DELM}(k) Z T(i, j) \operatorname{DELX} \operatorname{DELY}}{100.0}\right)$
in which $\operatorname{DELM}(k)=$ moisture content difference in percentage between the $k^{\text {th }}$ and $(k-1)^{\text {th }}$ half timestep.

DELM (k) is read in for every half timestep and is expressed as an average value for the study area. $\mathrm{ZT}(\mathrm{i}, \mathrm{j})$ is the height of the soil profile which is significant for $\operatorname{DELM}(k)$ in feet and is read in as an array.

## The pumping term $\operatorname{PUM}(i, j, k)$

The pumping term represents the recharge or discharge caused by a pumped well. Normally, these wells are pumped at a rate which will be constant during all timesteps. So PUM will be only dependent on the rate of pumping. The pumping rate $P U(i, j)$ is read in as an array in cfs. (Zero pump rate indicates no pump at that nodepoint). To obtain the pump term as a volume per $\Delta t$ (irrigation), $P U(i, j)$ is multiplied by DIFA(see page 92 ).

$$
\operatorname{PUM}(i, j)=\operatorname{PU}(i, j) * \text { DIFA }
$$

The constant leakage term QCL( $i, j)$
$\operatorname{QCL}(\mathrm{i}, \mathrm{j})=\operatorname{QL}(\mathrm{i}, \mathrm{j}) * \operatorname{DELX} \mathrm{DELY}^{*} \Delta \mathrm{t}$ (irrigation) in which
$Q L(i, j)=$ leakage in ft/day.
In this case input to or output from an unconfined aquifer from underlying strata is constant with time at each node so QL for each node can be read in as an array.
(Leakage which is variable and dependent on the difference of head in the confining layer and aquifer is already incorporated in the finite difference equation). In the Input Program II (DELX*(DELY)* $\Delta t$ (irrigation) is replaced by the term COS.

$$
\operatorname{QCL}(i, j)=\operatorname{QL}(i, j) * C O S
$$

With the discussion of the last term, the total source term $\mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ for part I of Input Program II which was written as:

$$
\begin{gathered}
Q(i, j, k)=-\operatorname{QI}(i, j, k)-\operatorname{QS}(i, j, k)-\operatorname{PE}(k)+E(k)+A M O(i, j, k) \\
+\operatorname{PUM}(i, j, k)-\operatorname{QCL}(i, j)
\end{gathered}
$$

is transformed in the computer program to:

$$
\begin{array}{r}
Q(i, j, k)=-A(m)-\operatorname{RESEEP}(i, j)-\operatorname{PE}(k)+E(k)+\operatorname{AMO}(i, j) \\
+\operatorname{DIFA*PU(i,j)-\operatorname {COS*QL}(i,j)} \tag{51}
\end{array}
$$

in $\mathrm{ft}^{3} / \Delta \mathrm{t}$ (irrigation).

## Input Program II, Source Term for Winter Season

For this part the input term $\mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ is reduced to:
$Q(i, j, k)=\operatorname{PUM}(i, j)-Q C L(i, j, k)-Q S(i, j, k)$

The pumping term $\operatorname{PUM}(i, j)$
The pumping term remains the same. However, the timestep is two times as long. $\Delta t($ winter $)=2 \Delta t$ (irrigation) so that the factor DIFA is replaced by 2*DIFA.

## The constant leakage term QCL(i,j)

The leakage term was expressed as: COS*QL(i,j). COS is replaced by WCOS in the computer program. $\mathrm{WCOS}=2 * \operatorname{COS}$
$\operatorname{QCL}(i, j)=W C O S^{*} Q L(i, j) \quad$ in $\mathrm{ft}^{3} / \Delta \mathrm{t}$ (winter)

## The seepage term $Q S(i, j, k)$

Since the irrigation canals are empty during this time of the year, the only seepage present is that from the non-irrigation canals with $\Delta t($ winter $)=2 \Delta t$ (irrigation).
$\operatorname{QS}(\mathrm{i}, \mathrm{j}, \mathrm{k})=2 * \operatorname{MULT}(\mathrm{i}, \mathrm{j}) * \operatorname{ARW}(\mathrm{i}, \mathrm{j})$
The total source term $Q(i, j, k)$ for part II of Input Program II can be written as follows:

$$
\begin{gather*}
Q(i, j, k)=2 * \operatorname{DIFA} * P U(i, j)-W C O S * Q L(i, j)-2 * \operatorname{MULT}(i, j) \\
* \operatorname{ARW}(i, j) \tag{52}
\end{gather*}
$$

in $\mathrm{ft}^{3} / \Delta \mathrm{t}$ (winter).

## Alternative Solution for the Source Term $Q(i, j, k)$

The above described program approximates as accurately as possible the source term for each nodepoint in the study area by simulation of the irrigation practices carried out in this area.

If a new irrigation practice were set up for this area, the irrigation requirements will be interpreted as follows: for timestep $k$,

1. plant requirement $=\frac{\operatorname{EVAP}(k)}{\operatorname{COE}} \quad \begin{aligned} & \text { where COEl } \\ & \text { coefficient }\end{aligned}$ is an efficiency
2. leaching requirement $=(\operatorname{COE} 2 * \operatorname{EVAP}(\mathrm{k}))$, the leaching requirement can be expressed as a percentage of the $\operatorname{EVAP}(\mathrm{k})$.
3. conveyance losses and operation losses $=\operatorname{COE} 3 * \operatorname{EVAP}(\mathrm{k})$.

The total irrigation requirement is then

$$
\operatorname{EVAP}(k) *\left(\frac{1}{\operatorname{COE} 1}+\mathrm{COE} 2+\mathrm{COE} 3\right)
$$

The total input caused by seepage and irrigation is

$$
\operatorname{EVAP}(k) *\left(\frac{1}{\operatorname{COE} 1}+\operatorname{COE} 2+\operatorname{COE} 3\right)+\operatorname{RESEEP}(i, j),
$$

while the total source term for nodepoint $(i, j)$ at time $k$ is:

$$
\begin{align*}
Q(i, j, k)=-\operatorname{EVAP}(k) & *\left(\frac{1}{\operatorname{COE} 1}+\operatorname{COE} 2+\operatorname{COE} 3\right)-\operatorname{RESEEP}(i, j) \\
& +\operatorname{AMO}(i, j)+\operatorname{EVAP}(k)-\operatorname{PRE}(k)-\operatorname{QL}(i, j) * \operatorname{COS} \\
& +\operatorname{DIFA*PU(i,j)} \tag{53}
\end{align*}
$$

To be able to investigate the effects of this approach on the aquifer, this solution is included in Input Program II and may be tried in the production runs of the study area.

## Data Collection Input Program II

General information of the study area (Progress Report ${ }^{22}$ )
Water supply
All of the irrigated lands in the study area are served from the Snake River with water rights dating as early as June 1880. The majority of the area is served by irrigation canals diverting from the Great Feeder Canal which serves as a by-pass for the Snake River main stream and generally runs continuously. Management of deliveries to the smaller independent canals is the responsibility of a cooperative group called the Great Feeder Board.

Historical diversions to the area are recorded in the reports of the watermaster for Water District 36 of the State of Idaho commencing in 1919 through the present. Some canals serve lands in the study area as well as land south of the area. A list of canals diverting from the Snake River and Great Feeder is contained in Appendix F. Figure 39 shows the annual diversions for all canals for the period 1919 through 1970 and a mass curve of accumulated discharges for the same period. The area irrigated under these canals, as indicated in the District 36 records, has not increased Significantly over the period so that the trend of increase in total diversions is indicative of the trend in total diversion per acre. Several increases in slope of the mass curve can be seen after dry periods such as 1931-1935 and also after the Palisades Reservoir became operational.

Total diversions per acre for the period 1 May to 30 September have increased over the period and were recorded as $10.2 \mathrm{af} / \mathrm{acre}$ in 1969. On some canals, diversions were made in October and November which are not recorded in District 36 records.


Figure 39. The Annual Diversions for All Canals for the Period 1919 through 1970 and A Mass Curve of Accumulated Discharges for the Same Period.

## Soils

Agricultural soils of the Snake River Fan are dominated by medium texture soils as found on fans and stream bottom lands. This soil group is identified as Group A in a recent soil survey by the U. S. Department of Agriculture Soil Conservation Service. Approximate locations of the soil association in the study area are shown in Figure 40.

The A2 association primarily in the lower elevations of the northwest section of the fan comprises some of the better crop land and consists of silt and clay loams of 40-60 inches depth underlain by sand and gravel. The available water holding capacity ranges from 3.5 to 13 inches and averages about 10 inches. These soils are moderately well drained and are presently utilized as cropland.

In the southern part of the study area and extending into Bonneville County, the soils are primarily of the A3 association consisting of the Bock and Bannock loam series. These soils are well drained with water holding capacities from 4 to 10 inches and depths of 20-60 inches over sand and gravel. In some areas top soils contain from 0 to 50 percent gravels.

The northeast part of the study area just south of the Snake River is classified as soil group A4 consisting primarily of the Blackfoot silt loam. These soils are well drained with depths generally from 20 to 60 inches above sand and gravel; however, one soil comprising about 15 percent of the area is very shallow with depths of 5 to 20 inches. Most of the soils are cropped with the gravelly soils being used for pastureproduction.

A soil group C4 overlies the Lewisville Knolls area and is presently used only for range and wildife. Water holding capacities are low, 1.0 to 6.5 inches, and depths are 10 to 40 inches above basalt bedrock.


## Irrigation practices

Border irrigation is used extensively on grains, alfalfa and pasture. Large stream sizes are prevalent with turnouts from the canals regulated in most cases by individual farmers. Large siphon tubes are used for border irrigation and overnight sets are not uncommon.

Irrigation practice on three potato fields and two bordered grain fields was evaluated in 1970. Test results show that intake rates are very high. Border irrigation of grains resulted in 12 to 15 inches of water being applied to irrigate a $l_{q} 300 \mathrm{ft}$. run. Many fields are longer than this and strams used were as much as oould be held in the border. Overnight irrigation with reduced stream size can result in almost twice this application while the soil profile will hold only 5 inches of water.

Furrow irrigation of potatoes was about $50 \%$ efficient for 2 inches irrigation on furrows of $1,000 \mathrm{ft}$. length early in the season where the wheel compacted furrow was used. Use of non-compacted rows early in the season gave about $20 \%$ efficiency.

Later in the season the vines fall into the rows and intake rates increase. Stream sizes used ranged from 10 to 80 gpm . Wheel compaction at the right soil moisture can be very effective in reducing the intake rate of these soils. Nearly all fields could benefit from more leveling and the high intake rates measured indicate that this area could benefit from sprinkler irrigation。

## Surface water diversions

Irrigation diversions for the major canals in the study area for the May l - September 30 period are recorded in the reports of Water District 36. These measurements performed by District 36 and U. S. Geological Survey personnel are obtained primarily by periodic current metering of the canals
at selected rating sections and reporting of daily staff gage readings by watermasters. No standard water measuring devices are in use on the major canals. During the 1970 season measurements were extended by University and ARS personnel past the normal September 30 cutoff date through November 30 or until all canals had been shut down for the winter. Return flows to the Snake River at the Burgess Canal spill, Long Island Slough, and Great Feeder were measured throughout the season. Water transported out of the study area to the south was measured in the Anderson, Farmers Friend, and Harrison canals.

Figure 41 shows the recorded May - September diversions per acre from the Snake River for all canals on the Snake River fan. The acre feet per acre values were computed from discharges and acreages for each irrigation district as recorded in District 36 records. An increasing trend can be observed with the 1967-69 diversions approaching 10 acre feet per acre. The seasonal distribution of net diversions for 1970 is shown in Figure 42. The net diversion is calculated as the measured diversion at the canal headgates minus all surface wastes. An irrigated acreage of 82,250 acres in the study area was determined from 1966 aerial photographs and does not include roads, canal right of ways, farmsteads and undeveloped lands within the study area. Total diversions for May - November 1970 season were $1,507,000$ acre feet of which 100,000 acre feet was measured as irrigation water transported out of the study area and 48,200 acre feet returned to the Snake River. The total net diversion for 1970 was 16.5 acre feet per irrigated acre of which $14 \%$ was diverted after September 30.

## Canal seepage losses

The main canals of the systems have a total water surface area of 820 acres of slightly less than $1 \%$ of the irrigated area. Seepage tests were


Figure 41. Graph of the Recorded May-September Diversions per Acre from the Snake River for All Canals of the Snake River Fan (Annual Irrigation Diversion).

made in 20 locations in the late summer and fall of 1970 and in 16 locations in early spring of 1971. These tests were made by inflow-outflow measurement methods in reaches with little or no diversion at the time of measurement. The accuracy of any one flow measurement is probably $\pm 5$ or $10 \%$ which can lead to much variability in seepage loss measurements, but the average of many loss measurements will give values that are good estimates. The average of the 1970 tests was $3.55 \mathrm{ft} /$ day with a range from $0.2 \mathrm{ft} / \mathrm{day}$ to $11.5 \mathrm{ft} /$ day. The average of the 16 tests made in early May, 1971 was 3.28 feet/day with a range of 0.67 to $12.07 \mathrm{ft} / \mathrm{day}$ 。

The spring tests had an average value that was $8 \%$ less than the average found in late summer, but the reaches tested were not the same and the channel flow depths were $11 / 2$ to 2 feet lower in the spring.

The average loss of $3.43 \mathrm{ft} / \mathrm{day}$ for all of the tests would amount to a total loss of 2810 acre ft . per day (or 1805 cfs ) if it were applicable to all the main canals in the system. Assuming a 150 day irrigation season, this seepage rate would involve a loss of 428,500 acre feet per season or about 28 percent of the yearly diversion. Assuming a 200 day season, which more nearly approximates the actual operating procedure, canal seepage adds 570,000 acre feet to the aquifer.

Further seepage loss determinations were made in 1971 to improve on the accuracy of the estimates and to determine if some area may have seepage losses that are consistently above or below the average.

From the aerial photographs, the total wetted area per nodepoint was determined. Also the total wetted area of non-district canals per nodepoint (sq. mile) and the total wetted, area of the non-irrigation canals was determined.

## Snake River losses

Recognizing that losses in the Snake River as it flows over the fan can contribute to the groundwater table rise, an attempt was made to evaluate these losses. Stearns ${ }^{24}$ in 1928 reported losses from 20 cfs to 830 cfs or an average loss of 288 cfs or 3.3 percent of the flow from the measuring station at Heise to Lorenzo, a distance of about 13.8 miles. In 1970 current meter measurements were made at three times during the year, during which the river was maintained at constant discharge at Palisades Dam. Discharge varied from $3,340 \mathrm{cfs}$ to $17,000 \mathrm{cfs}$ and computed losses varied from 808 cfs to 208 cfs or an average of 408 cfs. Based on a loss of 408 cfs for a 200 day season, losses from the Snake River account for 41,600 acre feet of water added to the aquifer.

## Input data

From above described information, the variables and arrays, necessary to compute the source term $\mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ in Input Program II, can be generated and read in the program. The input data is shown in Appendix G.

List of variables and arrays, read in:

1. $\operatorname{DELM}(\mathrm{I})=$ the change in moisture content of the soil during the different time steps. Assumed to be zero, considering the general information of the soils.
2. NROW: number of nodepoints in row

NCOL: number of nodepoints in column
DELX: size of mesh surface [ft].
NTS: number of timesteps in irrigation season
LTS: total number of timesteps
NI: number of irrigation districts in study area

NCHOIC: dummy variable relating to choice of computation.

> If $\mathrm{NCHOIC}=2, ~ \mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ is calculated according to the alternative solution (page l02
> If $\mathrm{NCHOIC}=1, \mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ is calculated according
> to the actual data of the study area.

CH : multiplication factor of net diversion per node
3. PRE(I),EVAP(I) read in from Input Program I $\mathrm{fft}^{3} /$ half $\left.\Delta t\right\rceil$
4. TA(I,N) net diversion per irrigation district per timestep 〔cfs〕.
5. Arrays of: NIR(i,j): denoting to which irrigation diotwi mode

$$
(i, j) \text { belongs }
$$

$\operatorname{ARFOR}(\mathrm{i}, \mathrm{j}):$ total wetted area per node $\left[\mathrm{ft}^{2} / \mathrm{l}_{\boldsymbol{p}} 0001\right.$
ARTOT(i,j): total non-district area per node [ft ${ }^{2} / 1,0001$
$\operatorname{ARW}(\mathrm{i}, \mathrm{j})$ : total non-irrigation area per node $\left[\mathrm{ft}^{2} / 1,0001\right.$
$\operatorname{RATE}(\mathrm{i}, \mathrm{j})$ : canal seepage rate [ft/day]
$\operatorname{DAOWN}(\mathrm{i}, \mathrm{j}):$ change in district wetted area per node $\left[\mathrm{ft}^{2} / 1,000\right\rceil$
$\operatorname{DAFOR}(\mathrm{i}, \mathrm{j})$ : change in non-district wetted area per node $\left[\mathrm{ft}^{2} / 1,0001\right.$
6. $P U(i, j)$ : the pumping term per node [cfs]

QL(i,j): constant leakage term per node [ft/day]
ZT(i,j): moisture holding profile above water table [ft]
7. COE1, COE2, COE3: irrigation coefficients
8. $D(I)$ : total number of nodes in irrigation district (I)

A flow chart and listing of Input Program II is also included in Appendix G.

## CHAPTER V

CALIBRATION OF THE MODEL

With the approximate values of the geohydrologic parameters and the source term $Q(i, j, k)$, a calibration of the model is feasible since the calculated hydraulic head values of the model can be compared with historical data, available from a network of 40 wells throughout the area.

The calibration is attempted in two steps.

1. Calibration without source term $(Q(i, j, k)=0)$
2. Calibration with the source term $(Q(i, j, k) \neq 0)$

## Calibration Without the Source Term

Figure. 43 shows the hydrograph of a typical well in the study area. The water level in the well rises quite sharply after the irrigation season starts. After reaching a peak in the last half of August the curve starts to decline. The level decreases slowly till it reaches an approximate steady state value at the end of winter.

To simulate this regime without the source term, the input term is set to zero. The values for the initial water table array are the steady state values recorded at the end of the winter season. The program is run for 42 half timesteps. Since this is a steady state starting condition, the computed head values in the succeeding timesteps should not differ from the initial conditions. If changes appear, they are due to incorrect values of input parameters. For each succeeding run, parameters are changed to obtain a better result. This trial and error method procedure is continued till no changes occur in the succeeding timesteps. This adjustment is time consuming because the influence of each hydrologic

parameter is different. In order to distinguish between influences of different parameters, only one parameter at a time is changed. It is also necessary to detect the interaction between the various parameters. A combination of adjusted parameters yields the final solution. It is important to note that no conclusions can be made about the uniqueness of the final solution obtained. The values of the arrays obtained by calibration of a steady state may be incorrect if checked in the unsteady state condition. For example, the value of the storage coefficient array has a large influence on the rise and fall of the groundwater table under unsteady state conditions, but much less influence in steady state conditions. Under unsteady state conditions the leakage term will vary by a change in hydraulic head. Then the effect of the values for the impedance and hydraulic head of the confining layer on the calculation of the hydraulic head in unsteady state conditions may be different than the effect of these parameters on the calculation in steady state conditions.

The calibration without the source term yielded a good result. The deviation of the hydraulic head after 42 half timesteps from the initial value stayed mostly within 3 feet with an aquifer depth of 300 feet.

In order to get a good fit the overall values for the hydraulic conductivities KX and KY , were a factor $10^{-1}$ less than initially assumed. The values for the impedance of the leaky aquifer showed more variation. The values for the variables and arrays which yielded the final steady state calibration are included in Appendix H. Figure 44 shows the watertable elevation as a computer contour plot.


Figure 44. Computer Plot of Water Table Contours for the Steady State Calibration. Multiply Numbers by 10 to Obtain Water Table Elevation Above Mean Sea Level.

## Calibration with the Source Term

In this calibration the computer program was run for the simulation of the year starting May 1, 1970 and ending April 31, 1971, utilizing the source term as is calculated in Input Programs I and II. The rise and fall of the water table as occurs in the course of the simulation is compared with the hydrographs of a number of wells in the study area.

This simulation showed clearly that the parameter values obtained in the calibration without source term did not apply to the calibration with the source term. Some parts of the aquifer showed a higher rise than the historical rise while other parts showed a smaller rise.

The conclusion can be drawn that the selection of parameters that correctly simulates historical unsteady state response must also cause the correct maintenance of the steady state conditions with no source term. This is conceivable since the steady state solution is not unique and therefore more solutions are possible. Actually, this "steady state" calibration is only carried out to obtain parameter values which are at least in the right order of magnitude. This is because the state at the end of the winter time is an approximate steady state, i.e. with no input succeeding the winter time, the water table would follow a depletion curve as is shown in Figure 54 which is approximately similar to the depletion curve of a runoff hydrograph. (Figure 45).

For this calibration the same trial and error method is used. It appeared very difficult to get a good simulation of the historic data. There are three necessary conditions:

1. model must be a valid simulation tool.
2. input (source term) $\mathrm{Q}(\mathrm{i}, \mathrm{j}, \mathrm{k})$ must be correct.
3. hydrogeological parameters of the aquifer must be correct.


Figure 45. A Runoff Hydrograph


Figure 47. Left Graph Shows the Historical and Computed Rises of the Water Table Before Changing Parameters. The Graph on the Right Shows the Respective Rises After Changing of the Parameter Values.

The first two conditions are reasonably satisfied. Determinating a reasonable value for the hydrogeological parameters is more difficult. By changing one parameter at the time the influence of that parameter can be detected. However, the interactions between parameters cannot be determined. Their relationship can only be obtained via a considerable number of trials. This procedure has to be done for every pair. With 11 arrays, of which 7 represent the geohydrologic properties of the aquifer, it is possible to construct numerous pairs, or even a combination of pairs, of which one combination renders the correct solution. Irivestigation of all possible (significant) combinations takes considerable time. In this case more or less at random, values of parameters were changed and the newly obtained head values compared with the historical data. This was considered to be a fair approach since from the beginning, the assumption was made that only a few parameters were incorrect.

In the course of these trials, a good choice was sometimes made. A next choice following the analysis of previous data showed sometimes a poorer fit to historical data. Often, the outcome of a trial showed that in one part of the aquifer the simulated rise of the water table was too small. After analyzing the data some parameters were changed. This yielded a correct rise, but the water table did not return to the historical value at the end of the simulation period. In general for these cases, the peak of the water table occurred at an earlier stage (Figure 46). Other parts of the aquifer showed a simulated rise which was too high. A change of parameters yielded a correct rise; but in this case a decline of the water table below the historical value at the end of the winter time (end simulation) occurred,accompanied by a later peaking of the water table (Figure 47).

Sometimes a correct simulation for one part of the aquifer was achieved; but in order to obtain this correct simulation, other areas which originally had a good fit were disturbed.

These experiences made clear that the assumption of only a few incorrect parameters was highly optimistic. Values for the hydraulic conductivities KX and KY , the impedance of the leaky aquifer, FAC ; the hydraulic head of the confining layer, PSI; the aquifer bottom elevation, Z; the storage coefficient, $S$; and the choice for confined or unconfined aquifer, NN, which were assumed or roughly calculated, appeared to be incorrect.

In this case, it is obvious that a random choice of parameter combinations does not yield the correct solution in the most effective way and a clearly defined systematic approach should be taken.

# CONCLUSIONS AND RECOMMENDATIONS 

## Conclusions

One part of the objectives was to develop a suitable mathematical model. The testing of the model showed that impermeable boundaries, constant head boundaries and in this model also flow boundaries can be handled quite easily. In testing the model with simmle cases a very good agreement was reached with the analytical solution.

The model is general enough to be applied to other aquifers and areas and will accomodate nonhomogeneous anisotropic, confined and unconfined, leaky and nonleaky aquifers.

Inputs and outputs for this model may be different for each node and variable in time and are generated in a separate input program.

The other part of the objectives was to apply the model to the Snake River Fan in southeast Idaho. In order to apply the model correctly, it appeared necessary to have accurate data of:

1. Input and output of every node in the study area.
2. Hydrogeological properties of the aquifer.

The first set of data is calculated in separate input programs. In these programs input and output for every nodepoint is calculated. Inputs include precipitation, canal seepage, irrigation application, river losses and drainage well recharges. Outputs include crop evapotranspiration, pumped well discharges, natural spring discharges and aquifer leakage. The study area can accomodate numerous different irrigation districts with different irrigation application rates. Via change in parameters, a range
of irrigation application rates can be simulated in this program as well as lining of canals and installing of drainage canals.

The input programs are general enough to be applied to different study areas. The second set of data was, because of lack of data, estimated as accurate as possible.

The calibration of the model for steady state conditions (no input or output that is variable in time) is easy to achieve. This also appears from the results of similar developed models that were applied to study areas in which the only output existed of a constantly pumped well field.

For this area, input as well as output varies with time. It appeared that with variable input, the water table configuration is much more sensitive to incorrect values of the hydrogeological parameters.

The correct combination of parameter values can only be obtained if a systematic approach is taken.

## Recommendations

The model can accomodate aquifers over which a square or rectangular grid is superimposed. It may be useful to include the possibility of having a grid system consisting of different grid spacings in the $x$-direction as well as the $y$-direction. Then boundaries, input, and output can be located more accurately.

More can be said about the flow boundary solution. For convenience of discussion, it is assumed that the groundwater flow can be described by

$$
\begin{equation*}
\frac{\partial^{2} h}{\partial x^{2}}+\frac{\partial^{2} h}{\partial y^{2}}=S \frac{\partial h}{\partial t} \tag{54}
\end{equation*}
$$

In the alternating direction implicit method, a row equation is solved with $x$ implicit and $y$ explicit. The terms $\frac{\partial^{2} h}{\partial x^{2}}$ and $\frac{\partial^{2} h}{\partial y^{2}}$ are approximated with the
central difference form. Let $k-1$ be the previous timestep. Equation (54) becomes:

$$
\begin{align*}
\frac{h_{i+1, j, k}-2 h_{i, j, k}+h_{i-1, j, k}}{(\Delta x)^{2}} & +\frac{h_{i, j+1, k-1}-2 h_{i, j, k+1}+h_{i, j-1, k-1}}{(\Delta y)^{2}} \\
& =\frac{S}{\Delta t}\left(h_{i, j, k}-h_{i, j, k-1}\right) \tag{55}
\end{align*}
$$

For convenience, let

$$
(\Delta x)^{2}=(\Delta y)^{2}=\frac{S}{\Delta t}=1
$$

Rewriting equation (55) with the unknown head values on the left hand side, equation (55) yields

$$
\begin{align*}
h_{i-1, j, k}-3 h_{i, j, k}+h_{i+1, j, k} & =-h_{i, j-1, k-1}+h_{i, j, k-1} \\
& -h_{i, j+1, k-1} \tag{56}
\end{align*}
$$

A boundary equation has only two unknown head values in order to solve the system of tridiagonal equation of the row. Let the first node in the row be a flow boundary node. To reduce the number of unknowns to two, the head value $h_{i-1, j, k}$ is approximated (in the model) with the $R$-method (ratio method) as follows:

$$
\begin{equation*}
h_{i-1, j, k}=h_{i, j, k}-R\left(h_{i+1, j, k}-h_{i, j, k}\right) \tag{57}
\end{equation*}
$$

in which

$$
R=\frac{\left(h_{i, j, k-1}-h_{i-1, j, k}\right)}{\left(h_{i+1, j, k-1}-h_{i, j, k-1}\right)}
$$

Equation (57) becomes:

$$
\begin{equation*}
h_{i-1, j, k}=(R+1) h_{i, j, k}-R\left(h_{i+1, j, k}\right) \tag{58}
\end{equation*}
$$

Let $D$ represent the right hand side of equation (56). With expression (58) substituted, equation (56) becomes a flow boundary equation with two unknowns:

$$
\begin{equation*}
(R-2) h_{i, j, k}+(R+1) h_{i+1, j, k}=D \tag{59}
\end{equation*}
$$

There is another solution to the flow boundary and is a logical consequence of the R-method. This is called the FD-method (forward difference method). Instead of estimating $h_{i-1, j, k}$ with the $R$-method, $h_{i-1, j, k}$ may be approximated by solving

$$
\frac{\partial^{2} h}{\partial x^{2}}+\frac{\partial^{2} h}{\partial y}=S \frac{\partial h}{\partial t} \quad \underline{\text { explicitly }} \text { for node }(i-1, j)
$$

$x$ and $y$ are then explicit. $\frac{\partial^{2} h}{2}$ is approximated with a forward difference form, and $\frac{\partial^{2} y}{\partial x^{2}}$ with the central difference form. $\frac{\partial^{2} h}{\partial x^{2}}$ yields:

$$
\begin{equation*}
\left(\frac{\partial^{2} h}{\partial x^{2}}\right)_{i-1}=h_{i+1, j, k-1}-2 h_{i, j, k-1}+h_{i-1, j, k-1}+O(\Delta x) \tag{60}
\end{equation*}
$$

With expression (60) substituted, equation (54) becomes (with $(\Delta x)^{2}=$ $\left.(\Delta y)^{2}=\frac{S}{\Delta t}=1\right)$

$$
\begin{align*}
h_{i+1, j, k-1}-2 h_{i, j, k-1}+h_{i-1, j, k-1} & +h_{i-1, j+1, k-1} \\
-2 h_{i-1, j, k-1}+h_{i-1, j-1, k-1} & =h_{i-1, j, k}-h_{i-1, j, k-1} \tag{61}
\end{align*}
$$

solving for $h_{i-1, j, k}$ gives

$$
\begin{align*}
h_{i-1, j, k}=-2 h_{i, j, k-1}+ & h_{i+1, j, k-1}+
\end{aligned} \begin{aligned}
& h_{i-1, j+1, k-1} \\
& +h_{i-1, j-1, k-1} \tag{62}
\end{align*}
$$

With expression (62) substituted, equation (56) becomes a flow boundary equation with also two unknowns:

$$
\begin{align*}
-3 h_{i, j, k}+h_{i+1, j, k}=D+2 h_{i, j, k-1} & -h_{i+1, j, k-1} \\
& -h_{i-1, j+1, k-1}-h_{i-1, j-1, k-1} \tag{63}
\end{align*}
$$

It would be interesting to investigate how these two methods to solve a flow boundary differ. Lack of time prohibited this investigation. Both
methods utilize some procedure to estimate $h_{i-1, j, k}$.
The R-method is based partly on the implicit procedure with error $O\left[(\Delta x)^{2}\right]$, partly on the "explicit" ratio $R$, which is a function of $h$ at $t=k-1$ (Equation 59)).

The FD-method is based on the explicit method with error $O(\Delta x)$. (Equation 62)). Both methods use explicit procedures which influence the stability and convergence of the solution so that some kind of control necessary. The FD-method will probably be more general but may be more difficult to control.

The unsteady state calibration must be systematically approached. One method is to record the difference in calculated water table elevation and historical water table elevation at significant timesteps. The difference for each node is printed out in an array. Agreement of the computed and the historical value will appear as a zero. This is done for the maximum water table elevation, the minimum water table elevation, and two intermediate elevations. The computer scans through these four "difference" arrays and changes certain parameters according to the magnitude of the difference. These new parameter values are entered in the program and a new simulation is run. This is done until maximum agreement between computed and historic values is reached. If still noticeable changes exist, the same procedure is followed, but then other parameters are changed.

Another method is the method of trend surface analysis. Also here deviations from the trend surface cause change of parameter values.

Other parameter optimization techniques are yet to be considered. Most should provide the correct combination of parameter values for the calibration of a model.

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## NOTATION



| $\nabla$ | $=$ | gradient - delta operator. |
| :---: | :---: | :---: |
| H | = | potential as energy/weight. |
| $\mathrm{H}_{0}$ | $=$ | initial hydraulic head. |
| $\mathrm{H}_{\mathrm{i}}$ | = | initial hydraulic head. |
| h | = | hydraulic head. |
| $h c_{i, j}$ | $=$ | hydraulic head in confining layer at node i,j. |
| $\mathrm{h}_{\mathrm{i}}$ | $=$ | elevation of contour line i. |
| K | $=$ | hydraulic conductivity. |
| $K v_{i, j}$ | $=$ | vertical hydraulic conductivity of confining layer of node i,j. |
| Kx | $=$ | hydraulic conductivity in $x$-direction. |
| Ky | $=$ | hydraulic conductivity in y -direction. |
| Kz | $=$ | hydraulic conductivity in z-direction. |
| k | $=$ | permeability of porous medium. |
| Le ${ }_{i}$ | $=$ | length of contour line i. |
| $\operatorname{Lea}_{i}$ | $=$ | leakage in one mile wide area between contour lines i and $\mathrm{i}+1$. |
| $\nabla^{2}$ | $=$ | Laplacian operator. |
| n | $=$ | porosity of porous medium. |
| p | $=$ | pressure intensity. |
| p* | $=$ | potential as energy/volume. |
| Qi | $=$ | surface water flow across contour line i. |
| $Q(i, j, k)$ | ) $=$ | source term or input term |
| q | $=$ | groundwater flow across contour line i. |
| ¢ | = | potential as energy/mass |
| $\phi_{\mathrm{d}}$ | $=$ | drainage porosity*. |
| R | $=$ | radius of circular island. |
| $\mathrm{R}_{\mathrm{n}}$ | $=$ | daily net radiation. |
| r | $=$ | well radius. |

```
\rho = density of medium.
S = storage coefficient.
Ss = specific storage coefficient.
T = transmissibility.
Tx = transmissibility in x-direction.
Ty = transmissibility in y-direction.
u = velocity in x-direction.
u = viscosity.
v = velocity in y-direction
l}\begin{array}{l}{Vx}\\{Vy}\\{Vz}\end{array}}=\mathrm{ seepage velocities in }x,y,\mathrm{ and z-directions.
v = total velocity vector.
W = total daily windrun at height of 2 meters in miles.
W(x,y,t) = source term.
W'(x,y,t)= source term
W}\mp@subsup{W}{}{O}(i,j,k+l/2)= source term
w = velocity in z-direction.
\frac{\Delta}{\Delta+\gamma}}\mathrm{ and }\frac{\Delta}{\Delta+\gamma}\mathrm{ are mean air temperature weighting factors whose sum is l.
```



APPEṄDICES

## APPENDIX A

DISCUSSION AND SOLUTION OF THE FLOW BOUNDARY CONTROLS

## Discussion and Solution of the Flow Boundary Controls

If the estimates of the hydraulic parameters (conductivity, storage coefficients or other) are not accurate near the flow boundary, the flow boundary will not yield the correction solution because of the existence of a highly unsteady state condition in which the water table will change rapidly to match the input parameters. This highly unsteady state condition is prevented if this matching happens more gradually. Therefore, a check of the ( $\alpha 2 / \alpha 1$ ) ratio is introduced to control the rate of change of the hydraulic slope. Also upper limits for the head difference between succeeding nodepoints are incorporated in the program. As for these controls a distinction has to be made between two cases, dependent on the location of the flow boundary.
I. Flow Boundary is on the beginning of a row (Figure 48).

Let $\left(\frac{\alpha^{2}}{\alpha 1}\right)_{\max }=\operatorname{RMAX} \quad$ Let $\left(\frac{\alpha 2}{\alpha 1}\right)_{\min }=$ RMIN.
RMAX and RMIN are representative fixed values chosen according to the aquifer properties. $\alpha 2$ and $\alpha 1$ are calculated from previous half timestep head values.

$$
\text { 1. If }\left(\frac{\alpha 2}{\alpha 1}\right)>\operatorname{RMAX} \text { or }\left(\frac{\alpha 2}{\alpha 1}\right)<\operatorname{RMIN}
$$

$\left(\frac{\alpha 2}{\alpha l}\right)$ is made equal to a fixed 'average' value AANl.

- 2. The new calculated head values $(t=k) h 1, h 2$, and h3 are then used to calculate the new value for $h_{i b-1, j, k}$. Before doing so the differences in head values between nodes ( $\mathrm{ib}, \mathrm{j}$ ) ( $\mathrm{ib}+1, \mathrm{j}$ ) and ( $\mathrm{ib}+2, \mathrm{j}$ ) are checked.

Let DIF be the arbitraryly maximum difference.
3. If (h3-h2) >DIF, (h3-h2) is set to BA3. If (h2-h1) >DIF, then (h2-hl) is set to BA4. BA3 and BA4 are fixed 'average' values.


Figure 48. Schematic Representation of the Hydraulic Head in the Vicinity of the Flow Boundary Located at the Beginning of a Row.


Figure 49. Schematic Representation of the Hydraulic Head in the Vicinity of the Flow Boundary Located at the End of a Row.
II. Flow boundary is on the end of a row (Figure 49).

1. If $\left(\frac{\alpha 2}{\alpha 1}\right)>\operatorname{RMAX} \quad$ or $\left(\frac{\alpha 2}{\alpha 1}\right)<\operatorname{RMIN}$
$\left(\frac{\alpha 2}{\alpha 1}\right)$ is set to a fixed 'average' value AAN2.
2. The new calculated head values ( $t=k$ ) h0, h1, and h3 are then used to calculate the new value for $h_{i b+1, j, k}$. Before doing so the differences in head between nodes ( $\mathrm{ib}-2, \mathrm{j}$ ), ( $\mathrm{ib}-1, \mathrm{j}$ ) and ( $\mathrm{ib}, \mathrm{j}$ ) are checked.

Again, let DIF be the arbitrarily chosen maximum difference.
3. If (h0-h1) >DIF, (h0-h1) is set to $\mathrm{DE}_{3}$. If (h1-h2) >DIF, (h1-h2) is set to $\mathrm{DE}_{4}$. $\mathrm{DE}_{3}$ and $\mathrm{DE}_{4}$ are fixed 'average' values.

The procedure described in I. and II. above is executed for every half timestep. Similar procedures apply to the calculation of the columns. Reference to the set of variables, containing RMAX, RMIN, DIF, AAN1, AAN2, BA3, BA4, DE3 and DE4 will occur under the name CONTROL CONSTANTS.

## APPENDIX B

DISCUSSION AND TREATMENT OF THE VARIABLES, REJATED TO THE LOCATION OF THE FLOW BOUNDARY IN THE GRID SYSTEM

## Discussion and Treatment of the Variables, Related to the Location of the Flow Boundary in the Grid System

The variables to be discussed are related to the location of the flow boundary in the grid system. In order to understand the need for these variables the calculation of three consecutive half timesteps for the nodes near the flow boundary is considered, as shown in Figures 50, 51 and 52.

Figure 50 represents the calculation of the nodepoints columnwise at $t=k$.

Figure 51 represents the calculation of the nodepoints rowwise at $t=k+1$.

Figure 52 represents the calculation of the nodepoints columnwise at $t=k+2$ 。

The flow boundary is denoted as a wave line ( $\sim \sim \sim \sim$ ). The impermeable boundary is denoted as (2H1NHWNWN). The figures are explained as follows:

Figure 50 In this half timestep the head values in the columns are calculated. For example if the $i^{\text {th }}$ column is considered all nodepoints (vertical) are calculated including outside point ( $i, j, b-l, k$ )

Figure 51 In the next half timestep the rows (horizontal) are calculated $(t=k+1)$. The first row that is calculated, however, is the $j b^{\text {th }}$ row, the second th
is the $(j b+1)$ row, etc. No new values are calculated for nodes ( $\mathrm{i}, \mathrm{jb}-1$ ), ( $i+1, j b-1$ ). $h_{i, j b-1, k+1}$, $h_{i+1, j b-1 ; k+1}$, etc. are automatically set to zero by the computer. This will appear to cause problems in the next calculation.



Figure 5l. Calculation of the Nodes Rowwise at $t=k+1$. No New Values are Calculated for Row (jb-l), the Outside Boundary Nodes for $t=k+l$.


Figure 52. Calculation of the Nodes Columnwise at $t=k+2$. Outside Boundary Nodes Will be Calculated with the Ratio Technique. This Technique, However, Utilizes Values for (i, jb-l), (i+1, jb-l) etc. Evaluated at $t=k+1$, Which Have Not Been Calculated.

Figure 52 In the next half timestep ( $\mathrm{t}=\mathrm{k}+2$ ), again the columns are calculated. Because the column starts with a flow boundary, the outside nodepoint ( $\mathrm{i}, \mathrm{jb}-1, k+2$ ) is approximated by utilizing previous half timestep values, i.e., $h_{i, j b+1, \underline{k+1}}, h_{i, j b, \underline{k+1}}$, and $h_{i, j b-1, \underline{k+1}}$ in the ratio $(\partial 2 / \partial 1)$. However $h_{i, j b-1, k+1}$ has never been calculated!

In order to solve this problem, an extra routine is taken $u$ in the program to calculate these values $h_{i, j b-1, k+1}$. Going back to Figure 51, the calculation of rows; After the $j b^{\text {th }}$, the $(j b+1)^{\text {th }}$ and all further rows are calculated for $t=k+1$. The extra routine calculates the $(\underline{j b-1})^{\text {th }}$ row. For instance: nodepoint $(i, j b-1, k+1)$ is calculated via a second order polonomial using nodes (i,jb,k+1), (i,jb+l,k+1) and (i,jb+2,k+1).

In general this problem occurs if the flow boundary is parallel with either column or row direction. In order to recognize this, introduction is made of input variables locating these 'problem' points. In above described case, the flow boundary is parallel to row. (See Figure 53.)

In the computer program, after calculation of the rows, a check is made of the value NY. If NY> 0 then outside boundary nodes, parallel to the flow boundary, are calculated. They are the nodes with (PHI(I, NY-1,K), $I=N X S, ~ . ~ . ~ . ~ ., ~ N X F) . ~ . ~$

If $N Y=0$ the problem does not exist and the routine is skipped. In this case the flow boundary was on the beginning of the columns. Similar solution applies if the flow boundary is located on the end of the column. If the flow boundary is parallel to the column and on the beginning or the end of a row, asimilar solution applies.


Figure 53. Flow Boundary Is Parallel to Row and on the Beginning of the Columns.



Figure 56. Flow Boundary Is Parallel to Column and on Beginning of Row.


Figure 57. Flow Boundary Is Parallel to Column and on End of Row.

IN SCHEME:
a. Flow boundary parallel to row, checking succeeds routine that calculates the rows.

## 1. flow boundary on beginning of column. (Figure 54)

Variables: NY, NXS, NXF
Calculated is:
PHI(I,NY-1,K) for $I=N X S, ~ . ~ ., ~ N X F . ~$
2. flow boundary on end of column. (Figure 55)

Variables: NYY, NXSS, NXFF
Calculated is:
PHI(I,NYY+1, K) for I = NXSS, . . ., NXFF.
b. Flow boundary parallel to column, checking succeeds routine that calculates the columns.

1. flow boundary on beginning of row. (Figure 56)

Variables: NX, NYS, NSF
Calculated is:
PHI(NX-1,J,K) for J = NYS, . . . . NYF
2. flow boundary on end of columns. (Figure 57)

Variables: NXX, NYXX, NYFF
Calculated is:

$$
\operatorname{PHI}(N X X+1, K, J) \text { for } J=N Y S S, ~ . ~ . ~ ., ~ N Y F F ~
$$

Variables which locate termination of flow boundaries are:
NX, NXX, NYS, NYF, NYSS, NYFF, NY, NYY, NXS, NXF, NXSS
and NXFF.

C-1. FLOWCHART OF THE MAIN PROGRAM THE GROUNDWATER MODEL C-2. LISTING OF THE MAIN PROGRAM


#### Abstract

C-1. Flowchart of the Main Program The Groundwater Model


Page 149 shows a simple flow diagram of the model. This flow diagram of the model is composed of five sections and two boxes. These sections and boxes are shown in more detail on pages l50through 160

Section I is shown on page 150 and 151.
II is shown on page 152 and 153.
III is shown on page 154 and 155.
IV is shown on page 155 and 156.
V is shown on page 157and 158.
Box $\quad 1$ is shown on page 159.
2 is shown on page 160 .
In the flow diagram, the notation for the equations in the thesis, and translation in computer laguage are used interchangeably. Reference to this notation can be found in Appendix A and B and Chapter III page 39 through 46.













| -- | C | PROGRAM : DIGITAL MODEL OF GROUNDWATER AQUIFER |
| :---: | :---: | :---: |
|  | C | USING A FINITE DIFFERENCE SCHEME:J.L.J. DE SONNEVILLE |
| - | C | OUTPUT AS A PRINTOUT AND/OR CONTOURPLOT ON MICROFILM |
|  | C | THE HYDRAULIC HEAD PHI IS DIMENSIONED: $\operatorname{PHI}(25,18,3)$ |
| $\cdots$ |  | REAL MU |
| - |  | DIMENSION NCX $(25,20), \operatorname{NCY}(25,20), \mathrm{S}(25,20), \mathrm{KX}(25,20), \mathrm{KY}(25,20)$, |
|  |  | * $\mathrm{Z}(25,20), \mathrm{H}(25,20), \mathrm{BE}(45), \mathrm{G}(45), \mathrm{TX}(25,20), \mathrm{TY}(25,20), \operatorname{PHI}(25,20,3)$, |
|  |  | * $\operatorname{SURF}(25,20), \operatorname{QC}(25,20), \operatorname{FAC}(25,20), \operatorname{PSI}(25,20), \operatorname{NN}(25,20)$, |
| mam |  | *SUWA $(25,20), \mathrm{TCl}(25,20), \mathrm{TC} 2(25,20), \mathrm{TC} 3(25,20), \mathrm{TC} 4(25,20), \mathrm{FU}(3)$, |
| ma |  | * $\operatorname{BAl}$ (2), DEl (2), AN 1 (5), AN2 (5), DELH (45) |
|  |  | DIMENSION HY( 25,20 , X $(25), \mathrm{Y}(20), \mathrm{Q}(25,20,43)$ |
|  |  | DIMENSION TTXX $(25,20)$ |
| $\cdots$ |  | COMMON/CONTRL/XLNTH, YLNTH |
|  |  | M7 $=5$ |
|  |  | NEER=0 |
| ... | C | THE SOURCE TERM Q (I, J, K) CAN BE DIMENSIONED IN TWO |
|  | C | DIFFERENT WAYS :IF FOR ANY NODEPOINT (I,J) THE SOURCE TERM |
|  | C | VARIES FOR EACH TIMESTEP, THE SOURCE TERM +(I, J, K) HAS AS |
|  | C | K-DIMENSION THE NUMBER OF TIMESTEPS(NTS):THEN THE DUMMY VARIABLE |
|  | C | 'MORE' HAS THE SAME VALUE AS NTS (NTS>3) |
| - | C | IF THE SOURCE TERM IS DIFFERENT FOR EACH NODEPOINT BUT CONSTANT |
|  | C | FOR EACH TIMESTEP, THE SOURCE TERM HAS THE DIMENSION $\mathrm{Q}(\mathrm{I}, \mathrm{J}, 3)$ |
| * | C | THEN THE DUMMY VARABLE HAS THE VALUE 3 |
|  |  | READ (M7, 410) MICX, MICY, NPX, NPY, NQ , NIM , NSER, MORE |
|  |  | READ (M7, 415) XA, YA, AMPL, FLUX, BA3, BA 4, DE3, DE 4, AAN 1, AAN2 |
| $\cdots$ | 415 | FORMATE (2F5.1,F10.0, E10.4, 4F5.1,2F5.2) |
|  |  | READ (M7, 410) NX, NXX, NYS , NYF, NYSS, NYFF, NY, NYY, NXS , NXF , NXSS , NXFF |
|  |  | READ (M7, 736)RMAX, RMIN, DIF, VER, NFLUX |
|  | 736 | FORMAT (3F5.2,F5.1, I5) |
| $\cdots$ |  | READ (M7, 405) NCOL, NROW, DELT, DELX, DELY, LTS, NTS, NS U 1, NSU2 , NSU3 |
|  | 405 | FORMAT (2I5, 3F5.0, 5I5) |
|  |  | READ (M7, 504)(DELH ${ }^{(K)}$, $\mathrm{K}=1, \mathrm{NTS}$ ) |
| ** |  | READ (M7,1001)FFF |
|  | 1001 | FORMAT(F5.2) |
| -- | 410 | FORMAT(12I5) |
| ********) |  | XLNTH=XA |
|  |  | YLNTH=YA |
|  |  | DELl=(DELX**2)/(DELY**2) |
|  |  | DEL2=1.0/DEL1 |
| \% |  | $\mathrm{Rl}=\mathrm{DELX} / \mathrm{DELY}$ |
|  |  | R2=DELY/DELX |
|  |  | DO $800 \mathrm{~K}=1$, MORE |
| - |  | DO $800 \mathrm{~J}=1, \mathrm{NCOL}$ |
|  |  | DO $800 \mathrm{I}=1, \mathrm{NROW}$ |
|  | 800 | $Q(\mathrm{I}, \mathrm{J}, \mathrm{K})=0.0$ |
|  |  | $\mathrm{X}(\mathrm{l})=1$ |
| - |  | DO $900 \mathrm{I}=2$, NROW |
| - | 900 | $\mathrm{X}(\mathrm{I})=\mathrm{X}(\mathrm{I}-1)+1$ |
|  |  | $\mathrm{Y}(1)=1$ |
| $\cdots$ |  | DO $910 \mathrm{I}=2, \mathrm{NCOL}$ |

```
    910 Y(J.) =Y(I-1)+1
    DO l K=1,NTS
    1 BE(K)=0.0
    NNCOL=NCOL-1
    NNROW=NROW-1
    NCUL=NCOL+l
    DO 401 J=1,NCOL
    401 READ(M7,500)(NCX(I,NCUL-J),I=1,NROW)
    DO 402 J=1,NCOL
    402 READ (M7,500)(NCY(I,NCUL-J),I=1,NROW)
    DO 403 J=1,NCOL
    403 READ(M7,500)(NN(I,NCUL-J),I=1,NROW)
    DO 404 J=1,NCOL
404 READ (M7,500)(KX(I,NCUL-J),I=1 ,NROW)
    DO 406 J=1,NCOL
    406 READ (M7,500)(KY(I,NCUL-J), I=1 ,NROW)
500 FORMAT (25I3)
    DO 407 J=1,NCOL
407 READ(M7,501)(Z(I,NCUL-J) ,I=1 ,NROW)
    DO 408 J=1,NCOL
    408 READ(M7,811)(PHI(I,NCUL-J,1),I=1,NROW)
    811 FORMAT(16F5.0)
    DO 409 J=1,NCOL
    4 0 9 ~ R E A D ( M 7 , 1 0 0 0 ) ( P S I ( I , N C U L - J ) , I = 1 , N R O W )
1000 FORMAT(25F3.0)
501 FORMAT(16F5.0)
    DO 4l2 J=1,NCOL
    412 READ(M7,501)(SURF(I,NCUL-J),I=1,NROW)
    DO 6l0 J=1,NCOL
    610 READ(M7,502)(S(I,NCUL-J),I=1,NROW)
    502 FORMAT(25F3.3)
        DO 411 J=1,NCOL
    411 READ(M7,503)(FAC(I,NCUL-J),I=1,NROW)
503 FORMAT(25F3.3)
504 FORMAT(25F3.1)
    IF NQ=1 ,Q(I,J,K) WILL BE READ IN FROM FILE
    IF NQ=2 , Q(I,J,K)=FLUX
    IF NQ=3 , Q(I,J,K)=0.0
    GO TO (2,3,13),NQ
    3 DO 8 K=2,NFLUX
        DO }8\textrm{J}=2,NNCO
        DO }8\textrm{I}=2,NNRO
    8 Q(I,J,K)=FLUX
    GO TO 13
    2.DO 12 K=2,MORE
    DO 11 J=1,NCOL
    11 READ(9)(Q(I, J,K),I=1,NNROW)
    12 CONTINUE
    13 CONTINUE
        DO 920 K=1,3
        DO 925 J=1,NCOL
        DO 925 I=1,NROW
```

```
925 PHI(I, J, K)=PHI(I, J, l)
920 CO NTINUE
    DO 930 J=1,NCOL
    DO 930 I=1,NROW
930 FAC(I,J)=FAC(I,J)/10.0
    DO 4 J=1,NCOL
    DO 4 I=1,NROW
    KX(I,J)=KX(I,J)*AMPL
    4 KY(I,J)=KY(I,J)*AMPL
        DO 6 J=1,NCOL
        DO 6 I=1,NROW
        H(I,J)=PHI(I,T,1)-Z(I,T)
        TX(I,J)=KX(I,J)*H(I,J)
        TY(I,J=KY(I,J)*H(I,J)
    6 TTXX(I,J)=TX(I,J)/(100.0*AMPL)
        PRINT 700,MICX,MICY,NPX,NPY,NQ,NIM,NSER,MORE
700 FORMAT(1H1,4X,'MICX=',I3,4X,'MICY=',I3,4X,'NPX =',I3,4X,'NPY =',I3
    *,4X'NQ =,'I3,4X,'NIM =',I3,4X,'NSER=',I3,4X,'MORE=',I3)
        PRINT 701
701 FORMAT (1H0)
    PRINT 702,XA,YA,AMPL,FLUX,BA3,BA4,DE3,DE4,AAN1,AAN2
702 FORMAT(lH0,4X,'XA =',F5.l,4X,'XY = ',F5.1,2X,'AMPL=',F5.0,2X,'FLUX=
    *',F10.0,3X,'BA3=',F5.2,4X,'BA4=',F5.2,4X,'DE3=',F5.2,4X,'DE4=',F5.
    *2,4X,'AAN l=',F5.2,1X,'AAN2=',F5.2)
        PRINT }74
742 FORMAT(1H0)
        PRINT 743,NX,NXX,NYS,NYF,NYSS,NYFF,NY,NYY,NXS,NSF,NXSS,NXFF
743 FORMAT(1H0,3X,'NX=',I3,3X,'NXX=',I3,3X,'NYS=',I3,3X,'NYF=',I3,3X,'
    *NYSS=' , I3, 3X,'NYFF=',I3,3X,'NY=' ,I3,3X,'NYY=' , I3,3X,'NXS=' , I3, 3X,'
    *NXF=',I3,3X,'NXSS=',I3,3X,'NXFF=',I3)
        PRINT }73
735 FORMAT (1 H0)
        PRINT 737,RMAX,RMIN,DIF,VER,NFLUX
737 FORMAT(1H0,4X,'RMAX=',F5.2,4X,'RMIN=',F5.2,4X,'DIF=',F5.2,4X,'VER=
    *',F5.1,4X,'NFLUX=',I5)
        PRINT 703
703 FORMAT(1H0)
        PRINT 704,NCOL,NROW,DELT,DELX,LTS,NTS,NSU1,NSU2,NSU3
704 FORMAT(1H0,4X,'NCOL=',I3,4X,'NROW=',I3,4X,'DELT=',F5.0,4X,'DELX=',
    *F5,0,4X,'LTS=',I3,4X,'NTS=',I3,4X,'NSUl=',I3,4X,'NSU2=',I3,4X,'NSU
    * 3=', I3)
        PRINT }70
706 FORMAT(1H1,55X, TABLE OF NCX(I,J)')
        PRINT 707,(I,I=1,NROW)
707 FORMAT(1H0,' J/I',25I5)
        DO 708 J=1,NCOL
        NJ=NCUL-\
    708 WRITE (6,709(NT),(NCX(I,NCUL-J),I= l,NROW)
709 FORMAT(1H0,I3,1X,25I5)
        PRINT 710
710 FORMAT(1H1,55X,'TABLE OF NCY(I,J)')
        PRINT 707,(I,I=1,NROW)
        DO 7ll J=l,NCOL
        NJ=NCUL-I
```

```
711 WRITE(6,709(NJ,(NCY(I,NCUL-J),I=1,NROW)
    PRINT }71
712 FORMAT(1H1,55.X,'TABLE OF NN(I,J)')
    PRINT 707,(I,I=1,NROW)
    DO 713 J=1,NCOL
    NJ=NCUL-J
713 WRITE(6,709)NJ,(NN(I,NCUL-J),I=1,NROW)
    PRINT }71
714 FORMAT(1H1,55X,'TABLE OF PERMEABILITY KX(I,J)')
    PRINT 707,(I,I=1,NROW)
    DO 715 J=1,NCOL
    NJ=NCUL-J
715 WRITE(6,709(NJ,(KX(I,NCUL-J),I=1,NROW)
    PRINT }71
716 FORMAT(1H1,55X,'TABLE OF PERMEABILITY KY(I,J)')
    PRINT 707,(I,I=1,NROW)
    DO 717 J=1,NCOL
    NJ=NCUL-J
717 WRITE (6,709(NJ,(KY(I,NCUL-J),I=1,NROW)
    PRINT }71
718 FORMAT(1H1,45X,'TABLE OF AQUIFERBOTTOM ELEVATION Z(I,J)')
    PRINT 707,(I,I=1,NROW)
    DO 719 J=1,NCOL
    NJ=NCUL-J
719 WRITE(6,720)NJ,(Z(I,NCUL-J),I=1,NROW)
720 FORMAT(1H0,I3,3X,25F5.0)
    PRINT }72
721 FORMAT(1H1,55X,'INITIAL WATERTABLE PHI(I,J,1)')
    PRINT 707,(I,I=1,NROW)
    DO 722 J=1,NCOL
    NJ=NCUL-J
722 WRITE (6,720)NJ,(PHI(I,NCUL-J,1),I=1,NROW)
    PRINT }72
723 FORMAT(1H1,35X,'TABLE OF HYDRAULIC HEAD DIFFERENCE OF LEAKY
    *AQUIFER')
        PRINT 707,(I,I=1,NROW)
    DO 724 J=1,NCOL
    NJ=NCUL-J
724 WRITE(6,720)NJ,(PSI(I,NCUL-J),I=1,NROW)
    PRINT }72
725 FORMAT(1H1,50X,'TABLE OF LANDSURFACE ELEVATION SURF(I,J)')
    PRINT 707,(I,I=1,NROW)
    DO 726 J=1,NCOL
    NJ =NCUL-J
726 WRITE(6,720)NJ,(SURF(I,NCUL-J),I=1,NROW)
    PRINT }72
727 FORMAT(1Hl,45X,'TABLE OF STORAGECOEFFICIENT S(I,T)')
        PRINT 707,(I,I=1,NROW)
        DO 728 J=1,NCOL
        NJ=NCUL-J
728 WRITE(6,729)NJ,(S(I,NCUL-J),I=1,NROW)
729 FORMAT(1H0,I3,2X,25F5.2)
    PRINT 730
```

```
730 FORMAT(1H1,40X,'TABLE OF IMPEDANCE OF LEAKY AQUIFER FAC(I,J)')
    PRINT 707,(I,I=l,NROW)
    DO 731 J=1,NCOL
    NJ=NCUL-T
731 WRITE(6.740)NJ,(FAC(I,NCUL-J),I=1,NROW)
740 FORMAT(1H0,I3,2X,25F5.4)
    PRINT }73
732 FORMAT(1H1,35X,'TABLE OF INITIAL TRANSMISSIBILITY TTXX(I,J
    *)/100.0*AMPL')
    PRINT 707,(I,I=1,NROW)
    DO 733 J=1,NCOL
    NJ=NCUL-J
733 WRITE (6,734)NJ,(TTXX(I,NCUL-J),I=1,NROW)
734 FORMAT(1H0,I3,2X,25F5.0)
    DO 1002 J=1,NCOL
    DO 1002 I=1,NROW
    PSI(I,J)=PHI(I,J,l)-PSI(I,J)
1002 QC(I,J)=2.0*FAC(I,J)*PSI(I,J)*DELX*DELY
    DO 7 J=2,NNCOL
    DO }7\mathrm{ I=2,NNROW
    TCl(I, J)=(TX(I-1,J)+TX(I,J))/2.0
    TC2(I,J)=(TX(I,J)+TX(I+1,J))/2.0
    TC3(I,J)=(TY (I,J-1)+TY(I,J))/2.0
    7 TC4(I:J)=(TY(I,J)+TY(I.J+1))/2.0
        KL=2
4 0 0 ~ D O ~ 3 2 5 ~ K L L = 2 , N T S , 2
    IF(MORE.GE.4) KL=KLL
    NEER=NEER +1
    K=2
    MU=2.0*FFF/DELT
    DO 5 J=1,NCOL
    DO 5 I=1,NROW
    H(I,J)=PHI(I,J,K-1)-Z(I,J)
    TX(I,J)=KX(I,J)*H(I,J)
    5 TY(I,J)=KY(I,J)*H(I,I)
    DO 140 J=1,NCOL
    DO 135 I=1,NROW
    K4=NCX(I,J)
    GAl={(2,0*S(I,J)/DELT)+FAC(I,T))*(DELX**2)
    GA2=((-2.0*S (I,J)/DELT)+FAC(I,J))*(DELX**2)
    IF (NCX(I,J).LE.l) GO TO 135
    IF(NN(I,J))15.15,10
10 Tl=TCl(I.J)
    T2=TC2(I,J)
    T3=TC3(I, J)
    T4=TC4(I,J)
    GO TO 16
    15 Tl=(TX(I-1,J)+TX(I,J))/2.0
    T2=(TX(I,J)+TX(I+1,J))/2.0
    T3=(TY(I,J-1) +TY(I,J))/2.0
    T4=(TY(I.J) +TY(I,J+1))/2.0
```

```
16 IF(KX(I-1,J).EQ.0) Tl=0.0
    IF(KX(I+1,J).EQ.0) T2=0.0
    IF(KY(T,J-1).EQ.0) T3=0.0
    IF(KY(I,J+1).EQ.0) T4=0.0
    GO TO (135,70,75,80,20,48,85,90),K4
4 8 \mathrm { D } = - \mathrm { DEL } \mathrm { 1*T3*PHI(I,J-1,K-1)+(DELI*(T3+T4)+GA2)*PHI(I,J,K-1)-DEL1*T4* }
    *PHI(I,J+l,K-1)+R1*(MU*Q(I,J,KL)-QC(I,J))
50 A=T1
    B=-Tl-T2-GAl
5 5 \mathrm { W } = \mathrm { B } - \mathrm { A } * \mathrm { BE } ( \mathrm { I } - 1 )
    G(I)=(D-A*G(I-l))/W
    NR4=1
    GO TO 125
20 NRl=I
    Tl=0.0
70 D= - DET, % *T3*PHI(I.J-1,K-1)+(DEL.1* (T3+T4) +GA2)*PHI(I,J,K-1)-DEL1*T4*
    *PHI(I,j+1,K-1)+Rl*(MU*Q(I,J,KL)-QC(I, J))
    A=T1
72 B=0T1-T2-GA1
    C=T2
73 W=B -A*BE (I-1)
    BE (I) +C/W
    G(I)=D-A*G(I-1))/W
    GO TO 135
75 PHI(I-1,J,K)+PHI(I-1,J,1)+DELH(KL)
    D=-DEL1*T3*PHI(I,J-1,K-1)+(DEL1*(T3+T4) +GA2)*PHI(I,J,K-1)-DELI*
    *T4*PHI(F,J+1,K-1)-Tl*PHI(I-1,J,K)+R1*(MU*Q(I,J,KL)-QC(I,J))
    A=0.0
    NRI=I
    GO TO }7
80 PHI(I +1, J, K) = PHI (I +1,J,1) +DELH(KL)
    D=-DEL1*T3*PHI(I,J-1,K-1)+(DEL1*(T3+T4)+GA2)*PHI (I,J,K-1)-DEL **
    *T4*PHI (T,J+1,K-1)-\mathbb{R *PHI}(\mathbf{I}+1,J,K)+R1*(MU*Q(I,J,KL)-QC(I,J))
    GO TO 50
```



```
    *T4*PHI(I,J+1,K-1)+R1*(MU*Q(I,J,KI) -QC(I,J))
        NR1=1
        IF(PHI(NRl+1,J,K-1).EQ.PHI(NR1,J,K-1)) GO TO }8
        AN1(K-1)=(PHI(NR1,J,K-1)-PHI(NR1-1,J,K-1))/(PHI(NR1+1,J,K-1)
    *-PHI(NRl,J,K-1))
        IF(ANl(K-1).LT.RMIN.OR.AN1(K-1).GT.RMAX) GO TO }8
        GO TO 87
86 ANl(K-1)=AANl
87 A=0.0
    B=ANl(K-1)*Tl-T2-GAl
    C=T2 -Tl*ANl(K-1)
    GO TO 73
90 D=-DEL1*T3*PHI(I,J-1,K-1) +(DEL1*(T3+T4)+GA2)*PHI(I,J,K-1)-DEL1*
    *T4*PHI(I,J+l,K-1)+Rl*(MU*Q(I,J,KL)-QC(I,J))
    NR4=I
    IF(PHI(NR4,J,K-1).EQ.PHI(NR4-1,J,K-1)) GO TO 91
    AN2(K-1)=(PHI(NR4+1,J,K-1)-PHI(NR4,J,K-1))/(PHI(NR4,J,K-1)
```

```
    * -PHI(NR4-1,J,K-1))
    IF(AN2(K-1).IT.RMIN.OR.ANI (K-1).GT.RMAX) GO TO 91
    GO TO 92
91 AN2 (K-1)=AAN2
92 A=T1-T2*AN2(K-1)
    B=T2*AN2(K-1)-T1-GA1
    GO TO 55
C SOLVE SYSTEM OF TRIDIAGONAL EQNS,PUT SOLUTION INTO PHI(I,J,K)
125 PHI(NR4,J,K)=G(NR4)
    IAST=NR4-NTRI
    DO 130 KK=1,LAST
    II=NR4-KK
130 PHI(II, J, K)=G (II) - BE (II)* PHI (II +1, J,K)
    IF(NCX(NR1,J).EQ.7) GO TO 131
    GO TO 132
131 FU(1)=PHI(NRl,J,K)
    FU(2)=PHI(NRl +1, I,K)
    FU(3)=PHI(NRI +2, J,K)
    IF(FU(3)-FU(2).GT.FU(2)-FU(1)) GO TO }62
    IF(FU(3).LT.FU(2)) GO TO 620
    IF((FU(3)-FU(2)).GT.DIF.OR.(FJ(2)-FU(1)).GT.DIF)GO TO 620
    GO TO 621
620 CONTINUE
    FU(2)=FU(3)-BA3
    FU(1)=FU(2)-BA4
    PHI(NRl+1,J,K)=FU(2)
    PHI(NRI,J,K)=FU(l)
6 2 1 ~ B A l ( 1 ) = F U ( 3 ) - F U ( 2 )
    BAl (2)=FU(2)-FU(1)
    BA2=BAl(1)-BA1 (2)
    PHI(NRI-1, J, K)=FU(3)-3.0*BA1 (1) +3.0*BA2
132 IF(NCX(NR4,I).EQ.8) GO TO 133
    GO TO 135
133 FU(1)=PHI(NR4-2,J,K)
    FU(2)=PHI(NR4-1,J,K)
    FU(3)=PHI(NR4, J,K)
    IF(FUU(1)-FU(2).GT.FU(2)-FU(3)) GO TO }62
    IF (FU(l).LT.FU(2)) GO TO }62
    IF((FU(1)-FU(2)).GT.DIF.OR.(FU(2)-FU(3)).GT.DIF) GO TO }62
    GO TO 623
6 2 2 ~ C O N T I N U E ~
    FU(2)=FU(1)-DE3
    FU(3)=FU(2)-DE 4
    PHI(NR4-1,J,K)=FU (2)
    PHI(NR4,J,K)=FU(3)
623 DEl(1)=FU(2)-FU(1)
    DEI (2)=FU(3)-FU(2)
    DE2=DEl(2)-DEl(1)
    PHI(NR4+1,J,K)=FU(1)+3.0*DEI (1) +3.0*DE2
135 CONTINUE
140 CONTINUE
```

```
    IF(NY.EQ.0) GO TO 150
    DO l45 I=NXS,NXF
    FU(1)=PHI(I,NY,K)
    FU(2)=PHI(I,NY+1,K)
    FU(3)=PHI(I,NY+2,K)
    IF(FU(3)-FU(2).GT.FU(2)-FU(1)) GO TO 624
    IF(FU(3).LT.FU(2)) GO TO 624
    IF((FU(3)-FU(2)).GT.DIF.OR.(FU(2)-FU(1)).GT.DIF) GO TO }62
    GO TO 625
6 2 4 ~ C O N T I N U E
    FU(2)=FU(3)-BA3
    FU(1)=FU(2)}<\textrm{BA}
    PHI(I,NY+1,K)=FU(2)
    PHI(I,NY,K)=FU(1)
6 2 5 \mathrm { BAl } ( 1 ) = F U ( 3 ) - F U ( 2 )
    BAl (2)=FU(2)-FU(1)
    BA2=BAl (1)-BA1 (2)
145 PHl(I,NY-1,K)=FU(3)-3.0*BAl(1)+3.0*BA2
150 CONTINUE
152 IF(NYY.EQ.0) GO TO 160
    DO 155 I=NXSS,NXFF
    FU(1)=PHI(1,NYY-2,K)
    FU(2)=PHI(1,NYY-1,K)
    FU(3)=PHI(1,NYY,K)
    IF(FU(1)-FU(2).GT.FU(2)-FU(3)) GO TO 630
    IF(FU(1).rT.FU(2)) GO TO 630
    IF((FU(1)-FU(2)).GT.DIF.OR.(FU(2)-FU(3)).GT.DIF) GO TO }63
    GO TO 631
6 3 0 ~ C O N T I N U E ~
    FU(2)=FU(1)-DE3
    FU(3)=FU(2)-DE 4
    PHI(I,NYY-1,K)=FU(2)
    PHI(I,NYY, K)=FU(3)
6 3 1 ~ D E l ( 1 ) = F U ( 2 ) - F U ( 1 )
    DE1(2)=FU(3)-FU(2)
    DE2=DEl(2)-DEl(1)
I55 PHI(I,NYY+1,K)=FU(1)+3.0*DE1(1)+3.0*DE2
160 CONTINUE
162 CONTINUE
    NCOUNT=KLL
    IF(NPX.LT.l) GO TO 377
    PRINT 163,NCOUNT,DELT,Q(8,10,KL)
163 FORMAT(1H1,4X,'NUMBER OF STEP=',I3,4X,'DELT=',F5.1,4X,'Q=',El0.4)
    PRINT 167
    PRINT 167
167 FORMAT(1H0)
    PRINT 707.(I,I=1,NROW)
    PRINT 167
    DO 164 J=1,NCOL
    NJ=NCUL-J
164 WRITE(6,I66)NJ,(PHI(I,NCUL-J,K) I=1,NROW)
166 FORMAT(1 Fi0,13,3X,25F5.0)
```

```
    377 IF(MICX.LT.1) GO TO 378
        DO 375 J=1,NCOL
        DO 374 I=1,NROW
    374 HY(I,NCUL-J)=PHI(I,J,K)
    375 CONTINUE
    CALL CONPLT(X,NROW,Y,NCOL,HY,NROW,NCOL,NIM,1,NROW,1,NCOL,'X'
    *,l,'Y',l,'WATER TABLE STUDY',17)
    378 CONTINUE
        NEER=NEER+1
        DO 165 I=1,NROW
        DO 165 J=1,NCOL
        H(I,J)=PHI(I,J,K)-Z(I,J)
        TY(I,J)=KY(I,J)*H(I,J)
    165 TX(I,J)=KX(I,J)*H(I,J)
C CALCULATE THE HEAD PHI(I,J,K+1) AT EACH POINT OF THE MATRIX
C FOR THE COIUMNS (Y-IMPLICIT)
    DO 305 I=1,NROW
    DO 300 J=1,NCOL
    K5=NCY (I, J)
    GA3=((2.0*S(I,T)/DELT)+FAC(I,J))*(DELY**2)
    GA4=((-2.0*S(I,J)/DELT)+FAC(I,J))*(DELY**2)
    IF (NCY(I,J).LE.l) GO TO 300
    IF(NN(I,J))175,175,170
170 Tl=TCl(I,J)
    T2=TC2(I,J)
    T3=TC3(I, J)
    T4=TC4(I,J)
    GO TO 176
175 Tl=(TX(I-1,J)+TX(I,J))/2.0
    T2=(TX(I,J)+TX(I+1,J))/2.0
    T3=(TY (I,T-1) +TY(I,T))/2.0
    T4=(TY(I, J) +TY(I,T+1))/2.0
176 IF(KX(I:-1,J).EQ.0) Tl=0.0
    IF(KX(T+1,J).EQ.0) T2 =0.0
    IF(KY(I,J-1).EQ.0) T3=0.0
    IF(KY(I,J-1).EQ.0) T4=0.0
    GO TO (300,230,240,245,220,195,250,255),K5
195 D=-DEL2*T1*PHI(I-1,J,K)+(DEL2*(T1+T2)+GA4)*PHI(I,J,K)-DEL2*T2*PHI(
    *I+1,J,K)+R2*(MU*Q(I, J,KL+1)-QC(I,J))
196 A=T3
    B=-T3-T4-GA3
200 W=B-A*BE(J-1)
    G(J)=(D-A*G(I-1))/W
    NR4=T
    GO TO 290
220 NR1=J
    T3=0.0
230 D=-DEL2*T1*PHI(I-1,J,K)+(DEL2*(T1+T2)+GA4)*PHI(I,J,K)-DEL2*T2*PHI(
    *I+1,J,K)+R2*(MU*Q(I,J,KL+1)-QC(I,J))
    A=T3
232 B=-T3-T4-GA3
    C=T4
```

```
235 W=B-A*BE(J-1)
    BE (J)=C/W
    G(J)=(D-A*G(J-1))/W
    GO TO 300
240 PHI(I,J-1,K+1)=PHI(I,J-1,K)+DELH(KL+1)
    D=-DEL2*T1 *PHI(I-1,J,K)+(DEL2*(Tl+T2) +GA4)*PHI(I,J,K)-DEL2*T2*PHI(
    *I +1,J,K)-T3*PHI(I,J-1,K+1)+R2*(MU*Q(I,J,KL+1)-QC(I, J))
    A=0.0
    NRl=J
    GO TO 232
245 PHI(I,J+1,K+1)=PHI(I,J+1,K) +DELH(KL+1)
    D=-DEL2*T1*PHI(I-1,J,K) +DEL2*(T1 +T2) +GA4)*PHI(I,J,K) -DEL2*T2*PHI(
    *I+1,J,K)-T4*PHI(I,J+1,K+1)+R2*(MU*Q(I,J,KL+1)-QC(I,J))
    GO TO 196
250 D=-DEL2*T1*PHI(I-1,J,K)+(DEL2*(T1+T2) +GA4)*PHI(I,J,K)-DEL2*T2*PHI(
    *I H1,T,K)+R2*(MU*Q(I,J,KL+1)-QC(I,J))
    NRl=J
    IF(PHI(I,NRl+1,K).EQ.PHI(I,NR1,K)) GO TO 252
    ANl(K)=(PHI(I,NR1,K)-PHI(I,NRl-1,K))/(PHI(I,NRl+1,K)-PHI
    *(I,NR1,K))
    IF(ANl (K).LT.RMIN.OR.AN1(K).GT.RMAX) GO TO 252
    GO TO 253
    252 ANl (K)=AANl
    253 A=0.0
        B=ANl(K)*T3-T4-GA3
        C=T4-T3*AN1(K)
        GO TO 235
    255 D=-DEL2*T1*PHI(I,J+1,K)+(DEL2*(T1+T2)+GA4)*PHI(I,J,K)-DEL2*T2*PHI(
    *I+1,J,K)+R2*(MU*Q(I,J,KL+1)-QC(I,J))
    NR4=J
    IF(PHI(I,NR4,K).EQ.PHI(I,NR4-1,K)) GO TO 257
    AN2 (K) =(PHI(I,NR4+1, K)-PHI(I,NR4,K))/(PHI(I,NR4,K)-PHI
    *(I,NR4-1,K))
        IF(AN2(K).LT.RMIN.OR.AN2 (K).GT.RMAX) GO TO 257
        GO TO 258
257 AN2(K)=AAN2
258 A=T3-T4*AN2(K)
    B=T4*AN2 (K)-T3-GA3
    GO TO 200
C SOLVE SYSTEM OF TRIDIAGONAL EQNS,PUT SOLUTION INTO PHI(I,J,K+1)
290 PHI(T,NR4,K+1)=G(NR4)
    LAST=NR4-NR1
    DO 295 KK=1,LAST
    JJ=NR4-KK
295 PHI(I,JJ,K+1)=G(JJ)-BE(JJ)*PHI(I,JJ+1,K+1)
    IF(NCY(I,NR1).EQ.7) GO TO 296
    GO TO 297
296 FU(1)=PHI(I,NR1,K+1)
    FU(2)=PHI(I,NR1+1, K+1)
    FU(3)=PHI(I,NR1+2,K+1)
    IF(FU(3)-FU(2).GT.FU(2)-FU(1)) GO TO }64
    IF(FU(3).LT.FU(2)) GO TO 640
    IF((FU(3)-FU(2)).GT.DIF.OR.(FU(2)-FU(1)).GT.DIF) GO TO 640
    GO TO 64I
```

```
6 4 0 ~ C O N T I N U E ~
    FU(2)=FU(3)-BA3
    FU(1)=FU(2)-BA4
    PHI(I,NRl +1, K+1)-FU(2)
    PHI(I,NR1,K+1)=FU(l)
6 4 1 ~ B A I ( 1 ) = F U ( 3 ) - F U ( 2 )
    BAl (2)=FU(2)-FU(1)
    BA2=BAl(1)-BAl (2)
    PHI(I,NR1-1,K+1)=FU(3)-3.0*BA1 (1) +3.0*BA2
297 IF(NCY'(I,NR4).EQ.8) GO TO 298
    GO TO 300
298 F U'(1)=PHI(I,NR4-2,K+1)
    FU(2)=PHI(I,NR4-1,K+1)
    FU(3)=PHI(I,NR4,K+1)
    IF(FU(1)-FU(2).GT.FU(2)-FU(3)) GO TO 642
    IF(FU(1).LT.FU(2)) GO TO 642
    IF((FU(1)-FU(2)).GT.DIF.OR.(FU(2)-FU(3)).GT.DIF) GO TO 642
    GO TO 643
6 4 2 ~ C O N T I N U E ~
    FU(2)=FU(1)-DE3
    FU(3)=FU(2)-DE4
    PHI(1,NR4-1,K+1)=FU(2)
    PHI(I,NR4,K+1)=FU(3)
643 DEl(l)=FU(2)-FU(1)
    DEl(2)=FU(3)-FU(2)
    DE2=DE1(2)-DEl(1)
    PHI(I,NR4+1,K+1)=FU(1)+3.0*DE1(1)+3.0*DE2
300 CONTINUE
305 CONTINUE
    IF(NX.EQ.0) GO TO }32
    DO 315 J=NYS,NYF
    FU(l)=PHI(NX,J,K+1)
    FU(2)=PHI}(NX+1,J,K+1
    FU(3)=PHI}(NX+2,J,K+1
    IF(FU(3)-FU(2).GT.FU(2)-FU(1)) GO TO 650
    IF(FU(3).LT.FU(2)) GO TO 650
    IF((FU(3)-FU(2)).GT.DIF.OR.(FU(2)-FU(1)).GT.DIF) GO TO }65
    GO TO 651
650 CONTINUE
    FU(2)=FU(3)-BA3
    FU(1)=FU(2)-BA4
    PHI(NX+l,J,K+l)=FU(2)
    PHI (NX,J,K+1)=FU(1)
651 BAl(1)=FU(3)-FU(2)
    BAl (2)=FU(2)-FU(1)
    BA2=BAl(1)-BA1 (2)
315 PHI(NX-1,J,K+1)=FU(3)-3.0*BA1 (1)+3.0*BA2
320 CONTINUE
322 IF(NXX.EQ.0) GO TO 324
    DO 323 ]=NYSS,NYFF
    FU(1)=PHI(NXX-2,J,K+1)
    FU(2)=PHI(NXX-1,J,K+1)
    FU(3)=PHI(NXX,T,K+1)
```

$$
383 \text { IF'(MICY.LT.1) GO TO } 384
$$

1F NSER=1 , TIMESTEPS $3,5,7,9,11, \ldots$ ETC. WILL BE PLOTTED(CONTOUR)
If NSER=2 ,TIMESTEPS $3,7,11,15, \ldots . . E T C$ WILL BE PLOTTED(CONTOUR)
GO TO $(306,308)$, NSER
308 IF(NCOUNT.EQ.3) GO TO 306
IF (NEER.EQ.4) GO TO 307
(G) TO 384

307 NEER=0
$306 \mathrm{DO} 381 \mathrm{~J}=1, \mathrm{NCOL}$
DO $380 \mathrm{I}=1$, NROW
$380 \mathrm{HY}(\mathrm{I} . \mathrm{NCUL}-\mathrm{J})=\mathrm{PHI}(\mathrm{I}, \mathrm{J}, \mathrm{K}+1)$
381 CONTINUE
CALL CONPLT(X, NROW, Y, NCOL, HY,NROW, NCOL,NIM, $1, N R O W, 1, N C O L, ' X '$
*, $1, \mathrm{Y}^{\prime}, 1, '$ WATER TABLE STUDY',17)
384 CONTINUE
IF (NCOUNT'EQ.LTS) DELT=2. 0 *DELT
IF (NCOUNT.EQ.NSU1) GO TO 351
IF'(NCOUNT.EQ.NSU2) GO TO 351
If (NCOUNT.EQ.NSU3) GO TO 351
GO TO 329
351 DO $352 \mathrm{~J}=1$, NCOL
DO $352 \mathrm{I}=1$, NROW
SUWA (I, J) $=\operatorname{SURF}(\mathrm{I}, \mathrm{J})-\operatorname{PHI}(\mathrm{I}, \mathrm{J}, \mathrm{K}+1)$
352 CONTINUE
PRINT 168, NCOUNT
168 FORMAT(1H1,4X,'NUMBER OF STEP=',I3,' SURFACE ELEVATION-
*(GROUNDWATER ELEVATION')
PRINT 167
PRINT 167
PRINT 707, (I, I=1,NROW)
PRINT 167
DO $353 \mathrm{~J}=1, \mathrm{NCOL}$
NJ=NCUL-J
353 WRITE $(6,812)$ NJ, (SUWA $(I, N C U L-J), I=1$,NROW)
812 FORMAT(1H0,13,3X,25F5.0)
329 DO $327 \mathrm{~J}=1, \mathrm{NCOL}$
DO $327 \mathrm{I}=1$, NROW
$327 \operatorname{PHI}(\mathrm{I}, \mathrm{J}, \mathrm{K}(-\dot{i})=\mathrm{PHI}(\mathrm{I}, \mathrm{J}, \mathrm{K}+1)$
325 CONTINUE
99 STOP
END

## APPENDIX D

D-1. DISCUSSION AND DETERMINATION OF THE INPUT ARRAYS PSI(I,J) AND FAC(I,J) D-2. DISCUSSION AND DETERMINATION OF THE INPUT ARRAYS KX(I,J) AND KY(I,J)

## D-1. Discussion and Determination of the Input Arrays PSI $(i, j)$ and FAC(i,j)

The hydraulic head of the confining layer (leaky aquifer) is assumed to be constant in time for each nodepoint. The array is called PSI $(i, j)$. Variation in aquifer head $\mathrm{PHI}(i, j, k)$ causes variation in leakage. The impedance of the leaky aquifer is denoted by the array $\operatorname{FAC}(\mathrm{i}, \mathrm{j})$. Average values for $\operatorname{PSI}(i, j)$ and $\operatorname{FAC}(i, j)$ can be obtained by using overall data for water in-and outflow in the area, recorded and computed in the summer of 1971. (Progress Report ${ }^{22}$ ). A water budget is set up utilizing overall values for the growing season ( 180 days $=$ May-November). Consider a simplified map of the study area, (Figure 58).

No flow boundary

Snake River

Flow boundary
Impermeable boundary

Figure 58. Simplified Map of the Study Area Showing Groundwater Leaving the Area.

A balance can be set up as follows:

$$
Q_{d i}+Q_{s o}+Q_{r s}+Q_{s l a}+Q_{e v}+Q_{g o}+Q_{l e}=0
$$

$Q_{d i}=$ Total net diversion of irrigation canals into area. A total of $1,507,000$ acre feet was measured.
$Q_{\text {so }}=$ Total surface water which is transported out of the area. A total of 100,000 acre feet was measured.
$Q_{r s}=$ Total flow of surface water which returns to the Snake River during the season. A total of 48,000 acre feet was measured.
$Q_{\text {sla }}=$ Total losses in the Snake River (seepage) which returns to area during the season. Current metering measurements revealed losses of 808 cfs to 208 cfs with an average of 408 cfs. Based on a loss of 408 cfs , the total flow counts for 40,000 acre feet for the whole season.
$Q_{e v}=$ Total loss due to evapotranspiration. In Input Program I。 the total evapotranspiration is calculated at a value of 2 feet for the season. This means a 162,000 acre feet for the whole area.
$Q_{\text {go }}=$ Total flow of ground water which leaves the area. As can be seen in the map, part of the area is bounded by no flow or impermeable boundaries. Most of the area is bounded by the Snake River which, according to the general course of the equipotential lines of the groundwater table, can be considered as a no flow boundary at least from point (1) to point (2) in Figure 58. From point (1) to point (3) a possibility of flow under the Snake River exists although very little is known.

The area of interest is the southwest corner of the study area which is a flow boundary. Here, the greatest part of the groundwater flow leaves the area and the flow is generally in southwest direction. The width of this
"funnel" is about 13 miles. The aquifer depth is 300 feet and the average hydraulic slope is $6 \mathrm{ft} / \mathrm{mile}$. If a certain value for the hydraulic conductivity $K$ is assumed, an estimate of the total groundwater flow for 180 days can be made. Per day a flow of 900 cfs leaves the area. This amounts to 320,000 acre feet for the whole season.
$Q_{l e}=$ Total leakage out of the area. Since all other terms of the equation are known, $Q_{l e}$ can be calculated. $Q_{1 e} \approx 900,000$ acre feet for total season.

This is for one day 5,000 acre feet. The study area approximates about 128,000 acres so that the leakage (average) per day is calculated as:

Daily Leakage $=\frac{5,000}{128,000}=0.04 \mathrm{ft}$ or 0.33 inches.
This is the average leakage during the season with the assumption that the leaky aquifer underlies the total area.

Leakage $=\left(\frac{K V}{b}\right)\left(h_{c}-h\right)$
$\frac{K v}{b}=$ impedance $=\operatorname{FAC}(i, j)$
Let the average head difference be 20 ft , then:
$0.04=\left(\frac{\mathrm{Kv}}{\mathrm{b}}\right) 20 . \rightarrow$ Impedance $=0.002[1 / \mathrm{day}]$.
From these values a starting value for the array $\operatorname{PSI}(i, j)$ may be derived. The values of the average impedance indicate the order of magnitude for the leakage. Well tests showed that in the eastern part of the study area, the aquifer is underlain by a thick clay layer so that the impedance here is zero. To balance the budget, the values for impedance elsewhere are made slightly higher than the calculated average. This provides the two arrays $\operatorname{PSI}(i, j)$ and $\operatorname{FAC}(i, j)$.

## D-2. Discussion and Determination of the Input Arrays $K X(i, j)$ and $K Y(i, j)$

In order to get approximate values for the hydraulic conductivity, a transmissibility computation was set up. Since again little is known about the area, the assumption is made that $K X(i, j)=K Y(i, j)$. A balance was set up for the Snake River Fan.

The flow across 5 contour lines is calculated, going from contour line 4,950 to contour line 4,700 (Figures 59 and 60). If no change in storage is assumed, the following balance can be set up (Figure 60).

$$
\begin{align*}
& q_{1}+Q_{1}-E_{1}-\text { Lea }_{1}=q_{2}+Q_{2}+\Delta S_{1}^{\circ}  \tag{64}\\
& q_{2}+Q_{2}-E_{2}-\text { Lea }_{2}=q_{3}+Q_{3}+\Delta S_{2}^{\circ} \tag{65}
\end{align*}
$$

$$
q_{i}=\frac{\operatorname{Kid}_{i}\left(h_{i+1}-h_{i}\right) L e_{i}}{L_{i}} \quad \text { with } \quad \operatorname{Kid}_{i}=T i \text {, or, }
$$

$$
q_{i}=\frac{T i\left(h_{i+1}-h_{i}\right) L e_{i}}{L_{i}}
$$

where,

$$
\begin{array}{ll}
\mathrm{Q}_{\mathrm{i}}= & \text { surface water flow across contour line } \mathrm{i} \\
\mathrm{q}_{\mathrm{i}}= & \text { groundwater flow across contour line } \mathrm{i} \\
\mathrm{E}_{\mathrm{i}}=\quad & \text { evapotranspiration in acres, one mile } \\
& \text { wide between contour line } \mathrm{i} \text { and } \mathrm{i}+1
\end{array}
$$



Figure 59. Simplified Map of the Study Area. Figure 60 Shows a Cross-Section as Indicated by I-I.


Figure 60. Cross-Section of Study Area. I-I as Indicated in Figure 59.

| $L_{i}$ | $=$ | average distance between two contour lines $i$ and $i+1$ |
| :---: | :---: | :---: |
| $\mathrm{h}_{\mathrm{i}}$ | $=$ | elevation of contour line i |
| $A_{i}$ | $=$ | average surface between contour lines i and i+l |
| $d_{i}$ | = | average depth of aquifer at contour line i |
| $L e_{i}$ | $=$ | length of contour line i |
| Lea $_{i}$ | $=$ | leakage in area, one mile wide between contour line i and $\mathrm{i}+1$ |

In August, 1971, extensive current metering was carried out along the five contour lines: From these measurements the approximate values for $Q_{i}$ are calculated (Progress Report ${ }^{22}$ ).

In order to calculate $E_{i}$, the assumption is made that the average evapotranspiration is 0.75 inches/day. Summarized, the data for this balance compuation are:

$$
\begin{array}{lllll}
\mathrm{h}_{1}=4950 \mathrm{ft} . & \mathrm{Q}_{1}=4846 \mathrm{cfs} & \mathrm{Le}_{1}=2.8 \mathrm{mi} . & \mathrm{L}_{1}=2.5 \mathrm{mi} \\
\mathrm{~h}_{2}=4900 \mathrm{ft} . & \mathrm{Q}_{2}=4383 \mathrm{cfs} & \mathrm{Le}_{2}=6.7 \mathrm{mi} . & \mathrm{L}_{2}=2.8 \mathrm{mi} . \\
\mathrm{h}_{3}=4850 \mathrm{ft} . & \mathrm{Q}_{3}=3775 \mathrm{cfs} & \mathrm{Le}_{3}=10.5 \mathrm{mi} . & \mathrm{L}_{3}=3.5 \mathrm{mi} . \\
\mathrm{h}_{4}=4800 \mathrm{ft} . & \mathrm{Q}_{4}=3140 \mathrm{cfs} & \mathrm{Le}_{4}=14.8 \mathrm{mi} . & \mathrm{L}_{4}=2.1 \mathrm{mi} . \\
\mathrm{h}_{5}=4750 \mathrm{ft} . & \mathrm{Q}_{5}=2605 \mathrm{cfs} & \mathrm{Le}_{5}=17.4 \mathrm{mi} . & \mathrm{L}_{5}=1.5 \mathrm{mi} .
\end{array}
$$

The assumption is made that K at 4,950 contour line $(\mathrm{Kl})=10^{5} \mathrm{gpd} /$ $\mathrm{ft}^{2}$ (Meinzers) and $\mathrm{d}_{1}=150 \mathrm{ft} . \quad \mathrm{Tl}=1.510^{7} \mathrm{gpd} / \mathrm{ft}$.

Equation (64) becomes:

$$
\mathrm{T} 1\left(\frac{\mathrm{~h}_{2}-\mathrm{h}_{1}}{\mathrm{~L}_{1}}\right) \mathrm{Le} e_{1}+\mathrm{Q}_{1}-\mathrm{E}_{1}-\mathrm{Lea}_{1}=\mathrm{T} 2\left(\frac{\mathrm{~h}_{3}-\mathrm{h}_{2}}{\mathrm{~L}_{2}}\right) \mathrm{Le} e_{2}+\mathrm{Q}_{2}
$$

in which all terms except $T 2$ are known. $T 2$ is calculated and successively Equation (65) is solved for T3. This yields to:

$$
\begin{aligned}
\mathrm{T} 1 & =12.310^{4} \mathrm{cfs} / \mathrm{miles}=15.1 \quad 10^{6} \mathrm{gpd} / \mathrm{ft}=2.0210^{6} \mathrm{ft}^{2} / \mathrm{day} \\
\mathrm{~T} 2 & =7.610^{4} \mathrm{cfs} / \mathrm{miles}=9.3 \quad 10^{6} \mathrm{gpd} / \mathrm{ft}=1.2410^{6} \mathrm{ft}^{2} / \mathrm{day} \\
\mathrm{~T} 3 & =8.110^{4} \mathrm{cfs} / \mathrm{miles}=9.810^{6} \mathrm{gpd} / \mathrm{ft}=1.3110^{6} \mathrm{ft}^{2} / \mathrm{day} \\
\mathrm{~T} 4 & =4.310^{4} \mathrm{cfs} / \mathrm{miles}=5.2610^{6} \mathrm{gpd} / \mathrm{ft}=0.7110^{6} \mathrm{ft}^{2} / \mathrm{day} \\
\mathrm{~T} 5 & =3.110^{4} \mathrm{cfs} / \mathrm{miles}=3.8010^{6} \mathrm{gpd} / \mathrm{ft}=0.5110^{6} \mathrm{ft}^{2} / \mathrm{day}
\end{aligned}
$$

$$
\text { if } \mathrm{d}_{1}=150 \mathrm{ft} \text { then } \mathrm{Kl}=13,000 \mathrm{ft} / \text { day }
$$

$$
\mathrm{d}_{2}=180 \mathrm{ft} \text { then } \mathrm{K} 2=6,900 \mathrm{ft} / \text { day }
$$

$$
\mathrm{d}_{3}=250 \mathrm{ft} \text { then } \mathrm{K} 3=5,200 \mathrm{ft} / \text { day }
$$

$$
\mathrm{d}_{4}=300 \mathrm{ft} \text { then } \mathrm{K} 4=2,300 \mathrm{ft} / \mathrm{day}
$$

$$
\mathrm{d}_{5}=400 \mathrm{ft} \text { then } \mathrm{K} 5=1,300 \mathrm{ft} / \text { day }
$$

Between the contour lines values for $K X(i, j)$ and $K Y(i, j)$ are interpolated using above derived values.

$$
\begin{aligned}
& A_{1}=9.5 \mathrm{sq} . \mathrm{mi} . \quad E_{1}=\mathrm{Al} \times \mathrm{l} / 4 \mathrm{H} / \mathrm{Le}_{\mathrm{i}} \approx 13 \mathrm{cfs} . \\
& A_{2}=19.13 \mathrm{sq} \cdot \mathrm{mi}, \quad E_{2}=A 2 \times 1 / 4 " / L e_{i} \approx 15 \mathrm{cfs} . \\
& A_{3}=41.15 \mathrm{sq} . \mathrm{mi} . \quad E_{3}=\mathrm{A} 3 \times 1 / 41 / L e_{i} \approx 15 \mathrm{cfs} \\
& A_{4}=22.5 \text { sq. mi. } \quad E_{4}=A 4 \times 1 / 41 / \mathrm{Le}_{\mathrm{i}} \approx 15 \mathrm{cfs} \\
& \text { Lea }_{1}=\mathrm{Al} \times 1 / 3^{\prime \prime} / \mathrm{Le}_{\mathrm{i}} \approx 17 \mathrm{cfs} \\
& \text { Lea }_{2}=A 2 \times 1 / 3 \prime / \operatorname{Le}_{i} \approx 20 \mathrm{cfs} \\
& \operatorname{Lea}_{3}=A 3 \times 1 / 3 " / \operatorname{Le}_{i} \approx 20 \mathrm{cfs} \\
& \operatorname{Lea}_{4}=A 4 \times 1 / 3 " / \operatorname{Le}_{i} \approx 20 \mathrm{cfs}
\end{aligned}
$$

## APPENDIX E <br> E-l. FLOWCHART OF INPUT PROGRAM I E-2. LISTING OF INPUT PROGRAM I

E-1. Elowchart of Input Program I

INPUT PROGRAM I





## E-2. Listing of Input Program I

ENl=FUNCTION TO CONVERT TEMPERATURE TO VAPOR PRESSURE FUNCTION ENI (V)
EN $1=1.522+7.1 * V / 10 * * 2+1.431 * V * * 2 / 10 * * 3+2.219 * V * * 3 / 10 * * 5+$ $\& 6.916 * \mathrm{~V} * * 4 / 10 * * 8+1.343 * \mathrm{~V} * * 5 / 10 * * 9$
RETURN
END
DIMENSION $\mathrm{MO}(225), \mathrm{ND}(225), \mathrm{TX}(225), \mathrm{TN}(225), \mathrm{TD}(225), \mathrm{W}(225), \operatorname{RS}(225)$,
*RSO(225) , PRECIP (225) , VPD (225) , TAX(225) , TEN(225), RLO(225) , RL(225)
*, RN(225), TAVG (225), T1 (225), T2 (225) , GEST(225) , VDEF (225) , PET(225),
${ }^{\operatorname{EVVAP}(225)}, \operatorname{VP}(225), \mathrm{OO}(10), \mathrm{C}(10,8), \operatorname{ER}(225), \operatorname{CE}(225), \mathrm{E}(10,45), \operatorname{PRE}(45)$
M6=5
M7 $=8$
C NOWN=DUMMY VARIABLE UF NOWN=0, INPUT DATA HAVE RS VALUES FROM
MA=NUMBER OF DAYS AFTER START MODEL ON WHICH ALFALFA HAS SEC.

* CUT

READ (M6,3) NDAYS,NOWN,LAST, MA,DELX
FIRST DAY OF COMPUTING PET IS FIRST DAY OF DIGITAL MODEL.NDAYS
IS TOTAL NUMBER OF DAYS OF CROP SEASON
DO $50 \mathrm{I}=1$, NDAYS
READ (M6, 33) MO(I) , ND (I) ,TX(I) , TD (I) , W(I) , RS (I) , RSO(I) , PRECIP (I)
$A=T X(I)$
EXS=EN1 (A)
$B=T N(I)$
ENS=EN1 (B)
$\mathrm{D}=\mathrm{TD}$ (I)
$\operatorname{VPD}(\mathrm{I})=E N 1$ (D)
$\mathrm{VP}(\mathrm{I})=(E X S+E N S) * 0.5$
C COMPUTE NOW RN(I)
$\operatorname{TAX}(\mathrm{I})=((\mathrm{TX}(\mathrm{I})-32) / 1.8+273) / 100$
$\operatorname{TEN}(\mathrm{I})=((\operatorname{TN}(\mathrm{I})-32) / 1.8+273) / 100$
SIG MA $=(8.132 / 10 * * 3) * 1440$
$\mathrm{X}=\mathrm{VPD}$ (I)
IF (NOWN.EQ.0)RS(I) $=37.015+0.86433 * R S(I)$
$E M T=0.325+0.045 * \operatorname{SIN}((30 *(M O(I)+N D(I) / 30.0)-1.5) * 3.1416 / 180$.
RLO(I) $=((E M T-0.044 * S Q R T(X))) * S I G M A *(T A X(I) * * 4+T E N(I) * * 4) * .5$
$\mathrm{RL}(\mathrm{I})=(1.22 * \mathrm{RS}(\mathrm{I}) / \mathrm{RSO}(\mathrm{I})-0.18) * \mathrm{RLO}(\mathrm{I})$
$\operatorname{RN}(\mathrm{I})=0.77 * R S(\mathrm{I})-\mathrm{RL}(\mathrm{I})$
$\operatorname{TAVG}(\mathrm{I})=(\mathrm{TX}(\mathrm{I})+\mathrm{TN}(\mathrm{I})) * 0.5$
$\mathrm{Tl}(\mathrm{I})=0.041+0.0125 * \mathrm{TAVG}(\mathrm{I})-4.534 * T A V G(\mathrm{I}) * * 2 / 10 * * 5$
$\mathrm{T} 2(\mathrm{I})=0.959-0.0125 * \mathrm{TAVG}(\mathrm{I})+4.545 * \mathrm{TAVG}(\mathrm{I}) * * 2 / 10 * * 5$
$\operatorname{IF}(\mathrm{I}-3) 30,30,31$
$30 \operatorname{GEST}(\mathrm{I})=0.0$
GO TO 35
$31 \operatorname{GEST}(\mathrm{I})=(\mathrm{TAVG}(\mathrm{I})-(\mathrm{TAVG}(\mathrm{I}-1)+\mathrm{TAVG}(\mathrm{I}-2)+\mathrm{TAVG}(\mathrm{I}-3)) / 3.0) * 5$

```
    35 VDEF(I)=VP(I)-VPD(I)
        GEST(1)=GEST(2)
        PET(I)=(Tl (I)*(RN (I) -GEST(I))+T2(I)*15.36*(1.0+0.0092*W(I))
    **VDEF(I))/1485.9
    50 CONTINUE
    10 CONTINUE (OO(I), I=1,LAST)
    5 FORMAT(9X,10F5.2)
    11 FORMAT(8E10.4)
        DO 165 NR=1,LAST
    READ IN TOTAL DAYS BETWEEN PLANTING AND EFFECTIVE COVER(DAYS)
    COMPUTE HOW MANY PERCENT IS ONE DAY OF THE TOTAL(PCT),READ IN
    HOW MANY UNITS HAVE PASSED TILL DAY WHICH IS THE START OFPRO
    GRAM OF DIGITAL MODEL(UNM),IF DAY OF PLANTING IS AFTER THE STA
    RT. DAY OF THE DIG. MODEL,READ IN NUMBER OF DAYS FROM START
    TILL SPECIFIC NUMBER OF DAYS AFTER WHICH THE CROP HAS THE LAST
    CUT(LF).READ IN TOTAL NUMBER OF HALF TIME STEPS,AFTER WHICH
    DELT IS DOUBLED(LTS).READIN HOW MANY DAYS HAVE PAST IF
    PLANTING DATE IS AFTER FIRST DAY OF DIG. MODEL. (NPL)
    READ(M6,16) DAYS,UNM,NPL,LTS,LF
    16 FORMAT(10X,2F5.0,315)
    FLOAT=DAYS-UNM
    IFL=FLOAT
    PCT=100.0/DAYS
    IF(NPL(28,28,20
    20 DO 21 I=1,NPL
    21 ER(I)=0.12*PET(I)
    IST=FIRST DAY THAT EVAP. IS COMPUTED WITH POLONOMIAL
    IST=NPL+1
    NNDAYS=DAYS
    IFL=NNDAYS +IST
    AA=PCT
    GO TO 29
    28 AA=PCT*(UNM+1.0)
    IST=1
    29 DO 100 M=IST,IFL
    CE(M)=C(NR,1)+C(NR,2)*AA+C(NR,3)*AA**2+C(NR,4)*AA**3
    IF(NR.EQ.3) GO TO 40
    IF(NR.EQ.4) GO TO 42
    GO TO 45
    40 IF(CE(M).GT.1.0) CE(M)=1.0
    GO TO 45
    42 IF(CE(M).GT.(0.87)) CE(M)=0.87
    45 ER(M)=CE(M)*PET(M)
    ER(M)=CE(M)*PET(M)
    AA=AA+PCT
    100 CONTINUE
    IFL=IFL+1
    AA=1.0
    DO 110 M=IFL,LF
    CE(M)=C(NR,5)+C(NR,6)*AA+C(NR,7)*AA**2+C(NR,8)*AA**3
    AA=AA+1,0
```

```
    IF(NR.EQ.3) GO TO 105
    IF(NR.EQ.4) GO TO 106
    GO TO 110
    105 IF(CE(M).GT.1.0) CE(M)=1.0
    MM=M
    IF(MM.EQ.MA) AA=1.0
    GO TO 110
    106 IF(CE(M).GT.(0.87)) CE(M)=0.87
    110 ER(M)=CE(M)*PET(M)
        LL=2
        DO 150 I=7,LF,7
        E(NR,LL) =(ER(I) +ER(I-1) +ER(I-2) +ER(I-3) +ER (I-4) +ER(I-5)+ER (I-6))*
    *OO)NR)*DELX*DELX/12.0
    LL=LL+1
150 CONTINUE
    DO 160 I= LL,ITS
    LE=(I-1)*7-3
160 E(NR,I)=0.15*PET(IE)*7.0
165 CONTINUE
    DO 168 I-2,LTS
    EVAP}(\textrm{I})=(\textrm{E}(1,I)+E(2,I)+E(3,I)+E(4,I)
168 CONTINUE
    LL=2
    MIN l=NDAYS-1
    DO 170 I=1,MIN1,7
    PRE(LL) =(PRECIP (I) +PRECIP (I+1) +PRECIP (I +2) +PRECIP (I +3) +PRECIP(
    *I+4)+PRECIP(I+5)+PRECIP(I+6))*DELX*DELX/12.0
    LL=LL+1
170 CONTINUE
    3 FORMAT(4X,4I3,F5.0)
    33 FORMAT(3X,2I3,6F5.0,5X,F5.2)
        WRITE (6,180) (K,EVAP (K) , PRE (K) , K=2,LTS)
    180 FORMAT(5X,I3,5X,E16.6,5X,E16.6)
        STOP
        END
```


## APPENDIX F

IRRIGATION CANALS DIVERTING FROM THE SNAKE RIVER AND THE GREAT FEEDER

|  | Irrigation Canals Diverting From The |
| :--- | :--- |
| Snake River And The Great Feeder |  |

APPENDIX G
G-1. FLOWCHART OF INPUT PROGRAM II
G-2. LISTING OF THE INPUT DATA OF INPUT PROGRAM II G-3. LISTING OF INPUT PROGRAM II





|  | $\operatorname{EVAP}(\mathrm{k})$ | PRECIP(k) |  |  |
| ---: | ---: | ---: | :--- | :--- |
| k |  |  | 0.418176 E | 06 |
| 2 | 0.703359 E | 06 | 0.123130 E | 07 |
| 3 | 0.639374 E | 06 | 0.153331 E | 07 |
| 4 | 0.128263 E | 07 | 0.139392 E | 06 |
| 5 | 0.157448 E | 07 | 0.0 |  |
| 6 | 0.207327 E | 07 | 0.164947 E | 07 |
| 7 | 0.195963 E | 07 | 0.132422 E | 07 |
| 8 | 0.218087 E | 07 | 0.0 |  |
| 9 | 0.289279 E | 07 | 0.302016 E | 06 |
| 10 | 0.338375 E | 07 | 0.464640 E | 05 |
| 11 | 0.365337 E | 07 | 0.185856 E | 06 |
| 12 | 0.383102 E | 07 | 0.302016 E | 06 |
| 13 | 0.433273 E | 07 | 0.209088 E | 06 |
| 14 | 0.335461 E | 07 | 0.0 |  |
| 15 | 0.317775 E | 07 | 0.0 |  |
| 16 | 0.303268 E | 07 | 0.255552 E | 06 |
| 17 | 0.244142 E | 07 | 0.906048 E | 06 |
| 18 | 0.248198 E | 07 | 0.111514 E | 07 |
| 19 | 0.243768 E | 07 | 0.302016 E | 06 |
| 20 | 0.190877 E | 07 | 0.0 |  |
| 21 | 0.146575 E | 07 | 0.0 |  |
| 22 | 0.125672 E | 07 | 0.232320 E | 05 |
| 23 | 0.132217 E | 07 | 0.255552 E | 06 |
| 24 | 0.138240 E | 07 | 0.418 I 76 E | 06 |
| 25 | 0.623727 E | 06 | 0.255552 E | 06 |
| 26 | 0.559801 E | 06 | 0.929280 E | 06 |
| 27 | 0.313632 E | 06 | 0.0 | 0 |
| 28 | 0.376695 E | 06 | 06 | 0.325248 E |
| 29 | 0.224311 E | 06 | 06 |  |
| 30 | 0.123741 E | 00 | 0.144038 E | 07 |
| 31 | 0.683015 E | -01 | 00 |  |

$\operatorname{EVAP}(k)=$ total evapotranspiration per nodepoint in 「ft ${ }^{3} /$ half timestepl for timestep $k$.
$\operatorname{PRECIP}(k)=$ total precipitation per nodepoint in $\left\lceil\mathrm{ft}^{3} /\right.$ half timestepl for timestep $k$.

# TOTAL NET DIVERSION PER IRRIGATION DISTRICT AND PER TIMESTEP 

[cfs-days/half timestep]
IRRIGATION DISTRICT NO.

| $\begin{aligned} & \text { TIME } \\ & \text { STEP } \end{aligned}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 0 。 | 0 . | 525. | 1416. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 98. | 0. | 230. | 0. | 64. |
| 3 | 0 . | 0 。 | 525. | 1673. | 162. | 0 . | 0. | 0 . | 0. | 226. | 0. | 175. | 423. | 0. | 1085. | 1049. | 300. |
| 4 | 305. | 0. | 1350. | 3168. | 335. | 30. | 24. | 120. | 0. | 655. | 112. | 1819. | 553. | 210. | 1266. | 1340. | 352. |
| 5 | 1267. | 521. | 2847. | 5264. | 605. | 140. | 168. | 420. | 42. | 1231. | 427. | 2435. | 568. | 409. | 2072. | 1454. | 575. |
| 6 | 2867. | 1929. | 4645. | 6785. | 1289. | 180. | 285. | 572. | 297. | 1413. | 599. | 2656. | 1006. | 393. | 3567. | 1461. | 980. |
| 7 | 6475. | 2765. | 4818. | 7421. | 1522. | 210. | 441 | 648. | 531 | 1652. | 720. | 3065. | 1034. | 457. | 4537. | 1425. | 1260. |
| 8 | 3830. | 1949. | 2850. | 6830. | 1200. | 140. | 321. | 553. | 499. | 1431. | 672. | 2497. | 1008. | 447. | 4122. | 1342. | 1330. |
| 9 | 6495 | 2739. | 4795 | 6994. | 1594. | 180. | 321. | 604. | 426. | 1493. | 718. | 2950. | 1205. | 44 | 4331. | 1383. | 1200. |
| 10 | 7678. | 2343. | 3423. | 6390. | 1365. | 284. | 343. | 672. | 494. | 1593. | 833. | 2636. | 1188. | 470. | 3998. | 1220. | 1110. |
| 11 | 8013. | 2297. | 4336. | 6393. | 1323. | 235. | 370. | 708. | 396. | 1279 | 802. | 2422. | 1071 | 421. | 3857. | 1305. | 1070. |
| 12 | 7309. | 2292. | 4093. | 6554. | 1325. | 233. | 380. | 527. | 351. | 1243. | 715. | 2363. | 869. | 356. | 3468. | 1315. | 1010. |
| 13 | 8854. | 2822. | 4908. | 7229. | 1557. | 253 | 379. | 529. | 408. | 157 | 58 | 3105. | 950. | 455. | 3933. | 1496. | 1100. |
| 14 | 8745. | 2484. | 4666. | 7262. | 1594. | 204 | 326. | 629. | 394 | 1419. | 604. | 2997. | 867. | . 392. | 3824. | 1545. | 1060. |
| 15 | 8651 | 2451. | 4787. | 7153. | 1540. | 222. | 404. | 673. | 372. | 1343. | 591. | 2714. | 832. | 410. | 3258. | 1469. | 900. |
| 16 | 7751 | 1980 | 4527. | 6830. | 1566. | 206 | 407. | 672. | 407. | 1324. | 582. | 2526. | 860. | 406. | 3848 . | 1462. | 1070. |
| 17 | 7473 | 2107. | 4436 | 6474. | 1297. | 206 | 334. | 636. | 404. | 1008. | 540. | 2442. | 913. | . 384. | 3635. | 1407. | 1000. |
| 18 | 6629. | 1281. | 3305. | 6361. | 1103. | 193. | 303. | 435. | 301. | 902. | 536. | 2227. | 784. | . 389. | 3445. | 1352. | 960. |
| 19 | 6498. | 1421. | 3192. | 5970. | 884. | 187. | 320. | 567. | 268. | 854. | 576. | 2100. | 730. | . 309. | 3296. | 1190. | 915. |
| 20 | 4678. | 1171. | 1636. | 4482. | 685. | 74 | 163. | 350. | 232. | 582. | 438. | 1606. | 630. | . 312. | 2855. | 1064. | 790. |
| 21 | 4462. | 1127. | 1908. | 3937. | 611. | 16. | 114. | 248. | 225. | 589. | 411. | 1349. | 564. | . 292. | 2925. | 934. | 815. |
| 22 | 4596. | 1739. | 2177. | 4074. | 656. | 55 | 136. | 336. | 265. | 620. | 413. | 1441. | 523. | . 264. | 2837. | 943. | 785. |
| 23 | 3792. | 1857. | 2130. | 3915. | 711. |  | 134. | 335. | 281. | 621. | 395. | 1401. | 511. | 261. | 2684. | 955. | 745. |
| 24 | 3432. | 1635. | 2508. | 3762. | 679. | 42. | 128. | 217. | 256. | 709. | 310. | 1302. | 425. | . 235. | 2323. | 840. | 646. |
| 25 | 3130. | 1316. | 2100. | 3307. | 624. |  | 120. | 199. | 212. | 635. | 255. | 1214. | 355. | 229. | 2054. | 751. | 570. |
| 26 | 2630. | 995 | 1692. | 2898. |  |  | 111 | 181. | 168. | 562. | 199. | , 1125. | 284. | . 221. | . 1831. | 661. | 510. |

$\dot{\sim} \dot{\sim} \dot{\sim}_{\sim}^{\infty} \dot{\sim}$

$\begin{array}{lllllll}\dot{\infty} & \dot{0} & \infty & \infty & \infty & 0 & 0 \\ \underset{\sim}{\infty} & \underset{\sim}{\infty} & \infty & \infty & \cdots & 0 & 0 \\ -1 & -1 & - & -1 & \infty & & \end{array}$





$\begin{array}{lllllll}0 & \dot{\sim} & \dot{N} & \dot{\sigma} & \dot{\circ} & \dot{0} & \dot{0}\end{array}$

$\underset{\sim}{\dot{-}} \dot{\sim} \dot{\sim} \dot{\sim} \dot{\sim} \dot{\sim} \dot{\sim}$







| $\pm$ | $\underset{\underset{i}{N}}{N}$ |
| :---: | :---: |
| - | $\dot{\circ}$ |

$$
\begin{array}{r}
\dot{\sim} \\
\dot{\sim} \\
\dot{\sim} \\
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\end{array}
$$

$$
\begin{aligned}
& \text { hown below: } \\
& 10=\text { Rigby } \\
& 11=\text { Clark-Edwards } \\
& 12=\text { Parks Lewisville } \\
& 13=\text { East Labelle } \\
& 14=\text { North Rigby }
\end{aligned}
$$

$$
15=\text { West Labelle-Long Island }
$$

These

$$
16=\text { Island }
$$

$$
20=\text { non-irrigation district }
$$

|  | $\begin{aligned} & \underset{\sim}{\prime} \\ & \underset{N}{2} \end{aligned}$ | $\bigcirc$ | $\infty$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\stackrel{\sim}{N}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\omega$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | 0 |
|  | $\underset{\sim}{N}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $N$ | N | $\rightarrow$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | $\bigcirc$ | $\bigcirc$ |
|  | $\vec{\sim}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $N$ | $\bigcirc$ | N | $\cdots$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | $\bigcirc$ | 0 | 0 |
| $\boxminus$ | $\begin{aligned} & \circ \\ & \sim \end{aligned}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | G） | $\sigma$ | 10 | $\square$ | N | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| $\stackrel{\text { 号 }}{Z}$ | $\underset{\pi}{\sigma}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $\cdots$ | 10 | $m$ | 10 | 15 | N | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| $\begin{aligned} & \text { Q } \\ & \text { E-1 } \\ & U \end{aligned}$ | $\underset{\sim}{\infty}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\underset{\sim}{n}$ | $\underset{\rightarrow}{m}$ | $\xrightarrow{-}$ | $\infty$ | 10 | 1 | N | $\bigcirc$ | 0 | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| $\begin{aligned} & \underset{\sim}{1} \\ & \mathrm{E} \\ & \underset{y}{0} \end{aligned}$ | $\stackrel{N}{\sim}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $\stackrel{\oplus}{\sim}$ | $\underset{\sim}{n}$ | $\vec{\sim}$ | $\nabla$ | $\infty$ | $m$ | N | N | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| A | $\underset{-1}{\oplus}$ | 0 | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\underset{\sim}{\underset{-1}{0}}$ | $\underset{\sim}{\infty}$ | $\checkmark$ | $\circledast$ | $\infty$ | $\cdots$ | N | N | $\checkmark$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| $\stackrel{\underset{1}{\sim}}{\stackrel{1}{n}}$ | $\stackrel{n}{\square}$ | $\bigcirc$ | 0 | 0 | 0 | $\underset{-1}{0}$ | $\underset{\sim}{\sim}$ | $\stackrel{\sim}{\square}$ | $\underset{\sim}{\circ}$ | $\nabla$ | $\infty$ | $\infty$ | $\cdots$ | $m$ | N | 0 | 0 | 0 | 0 | 0 | 0 |
| $\stackrel{\leftrightarrow}{\bullet} \stackrel{\rightharpoonup}{\ominus}$ | $\stackrel{+}{\square}$ | $\bigcirc$ | $\bigcirc$ | 0 | $\underset{\sim}{6}$ | $\underset{\sim}{\oplus}$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\sim}$ | $\underset{-1}{0}$ | $\nabla$ | $\infty$ | $\infty$ | $\cdots$ | $m$ | N | $\leftharpoondown$ | $\checkmark$ | $\checkmark$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| $\begin{aligned} & \stackrel{\rightharpoonup}{\alpha} \\ & \stackrel{1}{\lessgtr} \end{aligned}$ | $\cdots$ | $\bigcirc$ | $\bigcirc$ | $\underset{\sim}{6}$ | $\underset{-}{6}$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\oplus}$ | $\stackrel{\downarrow}{\hookleftarrow}$ | $\underset{-}{0}$ | $\nabla$ | $\nabla$ | $\infty$ | $\cdots$ | $\cdots$ | N | $\square$ | $\leftharpoondown$ | $\checkmark$ | $\checkmark$ | $\square$ | $\bigcirc$ |
| $\stackrel{H}{H}$ | $\underset{\sim}{\sim}$ | $\bigcirc$ | $\bigcirc$ | $\stackrel{\square}{\square}$ | $\underset{-}{6}$ | $\underset{\sim}{\square}$ | $\underset{\sim}{\oplus}$ | $\underset{-}{\bullet}$ | $\underset{-1}{0}$ | $\nabla$ | サ＇ | $m$ | $\cdots$ | $m$ | N | $\cdots$ | $\square$ | $\square$ | $\square$ | $\checkmark$ | $\bigcirc$ |
| $\stackrel{\Gamma_{1}}{\mathrm{O}}$ | $\underset{\sim}{F}$ | $\bigcirc$ | $\bigcirc$ | 0 | $\stackrel{n}{7}$ | $\underset{\sim}{\bullet}$ | $\underset{\sim}{ヵ}$ | $\underset{\sim}{N}$ | $\underset{-}{0}$ | $\nabla$ | W | $\cdots$ | $m$ | $\nabla$ | N | N | $\cdots$ | $\leftharpoondown$ | $\checkmark$ | $\checkmark$ | $\bigcirc$ |
| 2 $O$ -1 -1 | $\underset{\sim}{0}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\stackrel{\square}{\rightarrow}$ | $\stackrel{\oplus}{\square}$ | $\stackrel{\square}{\square}$ | $\underset{\sim}{N}$ | $\underset{\sim}{\sim}$ | $\nabla^{\prime}$ | F＇ | $\cdots$ | $m$ | 『＇ | N | N | $\cdots$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | 0 |
| $\stackrel{H}{\mathbb{G}}$ | $\sigma$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\stackrel{\sim}{\sim}$ | $\stackrel{\square}{\square}$ | 0 | $\underset{\sim}{N}$ | $\underset{\sim}{\sim}$ | $\nabla$ | $\nabla$ | サ | $\nabla$ | $m$ | $m$ | $\square$ | $\checkmark$ | $\checkmark$ | $\cdots$ | $\cdots$ | 0 |
| ［ | $\infty$ | $\bigcirc$ | $\bigcirc$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\sim}$ | $\stackrel{n}{\sim}$ | $\bigcirc$ | $\underset{\sim}{\sim}$ | $\nabla$ | $\nabla$ | $\nabla$ | $\nabla$ | $\nabla$ | $m$ | $\cdots$ | $\checkmark$ | $\checkmark$ | $\square$ | $\checkmark$ | $\square$ | 0 |
| E－1 | $N$ | $\bigcirc$ | 0 | $\stackrel{\sim}{\sim}$ | $\underset{\sim}{\infty}$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\sim}$ | $\bigcirc$ | $\nabla$ | $\nabla$ | $\nabla$ | サ | $\nabla$ | $\nabla$ | $\square$ | $\checkmark$ | $\cdots$ | 1 | $\rightarrow$ | $\bigcirc$ |
| 品 | $\omega$ | $\bigcirc$ | $\bigcirc$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{n}$ | $\stackrel{\sim}{\square}$ | $\underset{\sim}{\sim}$ | $\xrightarrow{\sim}$ | $\bigcirc$ | $\bigcirc$ | サ＇ | $\nabla$ | $\nabla$ | $\nabla$ | 『 | $\checkmark$ | $\checkmark$ | $\checkmark$ | 1 | $\checkmark$ | $\bigcirc$ |
| $\underset{\mathrm{E}}{\mathrm{E}}$ | 10 | $\bigcirc$ | 0 | $\underset{\sim}{\wedge}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\stackrel{\odot}{\sim}$ | $\begin{aligned} & \odot \\ & \sim \end{aligned}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\sim}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\sim}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\underset{\sim}{0}$ | $\stackrel{-}{\sim}$ | $\bigcirc$ |
|  | $\nabla$ | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | $\bigcirc$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\begin{aligned} & \mathrm{O} \\ & \mathrm{~N} \end{aligned}$ | ○ | $\stackrel{\odot}{\circ}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\mathrm{O}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
|  | $\cdots$ | 0 | 0 | 0 | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\begin{aligned} & \odot \\ & \sim \end{aligned}$ | $\begin{aligned} & \odot \\ & \uparrow \end{aligned}$ | $\stackrel{\odot}{\odot}$ | $\begin{aligned} & \mathrm{O} \\ & \mathrm{~N} \end{aligned}$ | $\stackrel{\odot}{\odot}$ | $\stackrel{\odot}{\odot}$ | $\stackrel{\odot}{\mathrm{N}}$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
|  | N | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | 0 | 0 | $\bigcirc$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\mathrm{N}}$ | $\stackrel{\odot}{\sim}$ | $\stackrel{\odot}{\sim}$ | $\stackrel{\odot}{\sim}$ | 0 | 0 | 0 | 0 | 0 |
|  | $\checkmark$ | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
|  | $\stackrel{\mapsto}{\curvearrowright}$ | $\begin{aligned} & \bigcirc \\ & \sim \end{aligned}$ | $\stackrel{\square}{\square}$ | $\underset{\sim}{\infty}$ | $\underset{\sim}{\sim}$ | $\underset{-}{\omega}$ | $\stackrel{\pi}{\sim}$ | $\stackrel{\text { 『 }}{\sim}$ | $\cdots$ | $\underset{\sim}{\sim}$ | $\underset{\sim}{\boldsymbol{r}}$ | $\underset{\sim}{\circ}$ | $\infty$ | $\infty$ | N | $\omega$ | $\omega$ | $\nabla$ | $m$ | N | $\square$ |

$\qquad$















게 $\dot{O}$ $\underset{\sim}{\infty} \dot{\circ} \dot{\circ} \quad \dot{\circ}$ $\approx \dot{N}$




 $\dot{\sim}$ 닝









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$\underset{\sim}{\circ} \dot{\circ} \quad \dot{\circ} \quad \dot{\circ} \quad \dot{\circ}$ $\cdots \cdots \dot{N} \quad \dot{\sim}$


 $\underset{\sim}{\underset{\sim}{\sim}} \boldsymbol{\sim}$ $\begin{array}{llllllllllllllllllll}\infty & \dot{O} & \dot{0} & \dot{0} & \dot{0} & \dot{N} & \dot{0} & \dot{\sim} & \dot{\sim} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} \\ \dot{0} & \dot{0}\end{array}$



 $\cdots \dot{\sim} \dot{\sim} \dot{\sim}$ $\underset{\sim}{\underset{\sim}{J}} \cong \stackrel{\sim}{\sim}$




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 $-1 \begin{array}{llllllllllllllllllll}\dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0}\end{array}$


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J/I 1 1 2 2 3 [14 4
20 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
19 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
18 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
17 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
16 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
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14 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
13 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5 3.5
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 $\underset{\sim}{\text { 岂 }} 0 \dot{\circ} \dot{\circ} \underset{\sim}{\dot{\sim}} \underset{\sim}{\sim} \underset{\sim}{\sim} \underset{\sim}{\sim} \dot{\sim}$
 $\begin{array}{cccccccccccccccccccc}- & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & c & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} \\ \mathrm{~m} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{O} & \dot{O} & \dot{O} & \mathrm{c} & \dot{O} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} & \dot{0} \\ \dot{0}\end{array}$




## G-3. Listing of Input Program II



DATA ZT/500*0.0/
NCUL=NCOL+1
DO $182 \mathrm{~J}=1, \mathrm{NCOL}$
$182 \operatorname{READ}(\mathrm{M} 6,600)(\mathrm{NIR}(1, N C U L-J), I=1, N R O W)$
600 FORMAT (24I3)
DO $185 \mathrm{~J}=1, \mathrm{NCOL}$
$185 \operatorname{READ}(\mathrm{M} 6,179)(\operatorname{ARFOR}(\mathrm{I}, \mathrm{NCUL}-\mathrm{J}), \mathrm{I}=1, \mathrm{NROW})$
DO $186 \mathrm{~J}=1, \mathrm{NCOL}$
$186 \operatorname{READ}(\mathrm{M} 6,179)(\operatorname{ARTOT}(\mathrm{I}, \mathrm{NCUL}-\mathrm{J}), \mathrm{I}=1, \mathrm{NROW})$
DO $187 \mathrm{~J}=1$, NCOL
$187 \operatorname{READ}(\mathrm{M} 6,179)(\operatorname{ARW}(\mathrm{I}, \mathrm{NCUL}-\mathrm{J}), \mathrm{I}=1$, NROW)
DO $188 \mathrm{~J}=1, \mathrm{NCOL}$
$188 \operatorname{READ}(\mathrm{M} 6,181)($ RATE ( $\mathrm{I}, \mathrm{NCUL}-\mathrm{j}), \mathrm{I}=1, \mathrm{NROW})$
181 FORMAT (3X, 25F3.1)
DO $189 \mathrm{~J}=1, \mathrm{NCOL}$
$189 \operatorname{READ}(\mathrm{M} 6,179)$ (DAOWN (I, NCUL-J), I=1, NROW)
DO $190 \mathrm{~J}=1, \mathrm{NCOL}$
190 READ (M6,179) (DAFOR(I,NCUL-J) , I=1, NROW)
179 FORMAT(5X,12F6.1/5X,12F6.1)
DO $193 \mathrm{~J}=1, \mathrm{NCOL}$
DO 193 I=1, NROW
$193 \operatorname{AROWN}(\mathrm{I}, \mathrm{J})=\operatorname{ARTOT}(\mathrm{I}, \mathrm{J})-\operatorname{ARFOR}(\mathrm{I}, \mathrm{J})$
PRINT 730
730 FORMAT(1H1)
WRITE $(6,750)$ (DELM $(\mathrm{K}), \mathrm{K}=2,10)$
750 FORMAT(1HO,5X,'DELM $(K)=', 9 F 5.0)$
PRINT 760
760 FORMAT(1 HO)
PRINT 765,NROW,NCOL,DELT,DELX,NTS,LTS,NI,NCHOIC, CH
765 FORMAT(1HO,3X,'NROW=', I5,3X,'NCOL=', I5,3X,'DELT=',F5.0,3X,'DELX='
*F5.0,3X,'NTS=', I3,3X,'LTS=', I3, 3X,'NI=', I3, 3X,'NCHOIC =', I3, 3X,'CH
*=', F3. 0)
PRINT 730
DO $710 \mathrm{~K}=2, \operatorname{LTS}$
WRITE $(6,715) \operatorname{EVAP}(\mathrm{K}), \operatorname{PRE}(\mathrm{K})$
710 CONTINUE
715 FORMAT(5X,E16.6,5X,E16.6)
PRINT 730
PRINT760
PRINT 770
770 FORMAT(1HO,23X,'TOTAL NETDIVERSION PER IRR.DISTR AND PER TIMESTEP *')
PRINT 775
775 FORMAT(1HO,1X,'TIME',28X,'IRRIGATIONDISTRICT NO:')
PRINT 776, (I, $\mathrm{I}=1,17$ )
776 FORMAT(1H ,1X,'STEP',I6,16I 7)
PRINT 760
DO 780 I=2,LTS
WRITE $(6,785) \mathrm{I},(\mathrm{TA}(\mathrm{L}, \mathrm{I}), \mathrm{L}=1,17)$
785 FORMAT(1HO,14,2X,17(2X,F5.0))
780 CONTINUE
WRITE $(6,790)(\mathrm{D}(\mathrm{I}), \mathrm{I}=1, \mathrm{NI})$
790 FORMAT(1H1,3X,'D(I)=',17F4.0)

WRITE (6, 791) COE1, COE2, COE3
791 FORMAT(1HO,3X,'COE1=',F5.2,3X,'COE2=',F5.2,3X,'COE3=',F5.2) PRINT 800
800 FORMAT(1H1,40X,'TABLE OF THE LOCATION OF THE VARIOUS IRR.

* DISTRICTS $\left.\operatorname{NIR}(\mathrm{I}, \mathrm{J}){ }^{\prime}\right)$

PRINT 810, ( $\mathrm{I}, \mathrm{I}=1, \mathrm{NROW}$ )
810 FORMAT(1HO, 'J/I', 25I5)
DO $815 \mathrm{~J}=1$, NCOL NJ=NCUL- -
$815 \operatorname{WRITE}(6,820) \mathrm{NJ},(\operatorname{NIR}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$
820 FORMAT(1HO,13,25I5) PRINT 825
825 FORMAT(H1, 40X,'TABLE OF FOREIGN WETTED AREA/NODE ARFOR(I, J)') PRINT 810, (I,I=1,NROW)
DO $830 \mathrm{~J}=1, \mathrm{NCOL}$
NJ $=$ NCUL-J
830 WRTTE (6,835)NT, (ARFOR(I,NT), I=1,NROW)
835 FOKIMAT(1HO, $13,1 \mathrm{X}, 25 \mathrm{~F} 5.0$ )
PRINT 840
840 FORMAT(1H1,40X,'TABLE OF TOTAL WETTED AREA/NODE ARTOT(I,J)')
PRINT 810, (I, I=1,NROW)
DO $845 \mathrm{~J}=1, \mathrm{NCOL}$
$\mathrm{NJ}=\mathrm{NCUL}-\mathrm{]}$
$845 \operatorname{WRITE}(6,835) \mathrm{NJ},(\operatorname{ARTOT}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$ PRINT 850
850 FORMAT(1H1,40X, TABLE OF WETTED AREA/NODE OF ANNUAL STREAMS *ARW (I, J)')
PRINT 810,( $\mathrm{I}, \mathrm{I}=1, \mathrm{NROW})$
DO $855 \mathrm{~J}=1, \mathrm{NCOL}$ NJ=NCUL-J
$855 \operatorname{WRITE}(6,835) \mathrm{NJ},(\operatorname{ARW}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$ PRINT 870
870 FORMAT(1Hl,40X,'TABLE OF AVERAGE SEEPAGE RATE/NODE RATE(I,J)') PRINT 8l0,(I,I=l,NROW)
DO $875 \mathrm{~J}=1$, NCOL
$\mathrm{NJ}=\mathrm{NCUL}-\mathrm{J}$
$875 \operatorname{WRITE}(6,880) \operatorname{NJ},(\operatorname{RATE}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$
880 FORMAT(1HO, I3, 1X, 25F5.1) PRINT 885
885 FORMAT(1Hl,40X,'TABLE OF CHANGE OF OWN WETTED AREA/NODE
*DAOWN (I, J)')
PRINT 810,(I, I=1,NROW)
DO $890 \mathrm{~J}=\mathrm{l}, \mathrm{NCOL}$ $\mathrm{NJ}=\mathrm{NCUL}-\mathrm{J}$
$890 \operatorname{WRITE}(6,835) \mathrm{NJ},(\mathrm{DAOWN}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$ PRINT 895
895 FORMAT(1 Hl, 40X, 'TABLE OF CHANGE OF FOREIGN WETTED AREA/NODE *DAFOR (I, J)') PRINT 810,(I,I=1,NROW)
DO $900 \mathrm{~J}=1, \mathrm{NCOI}$, NJ=NCUL--J
$900 \operatorname{WRITE}(6,835) \mathrm{NJ},(\operatorname{DAFOR}(\mathrm{I}, \mathrm{NJ}), \mathrm{I}=1, \mathrm{NROW})$ PRINT 860

```
860 FORMAT(1Hl,40X,'TABLE OF OWN WETTED AREA/NODE AROWN(I,J)')
    PRINT 810.(I,I=1,NROW)
    DO }865\textrm{J}=1,NCO
    NJ=NCUL-J
865 WRITE (6,835)NJ,(AROWN (I,NJ),I=1,NROW)
    COS=DELT*DELX*DELX/2.0
    WCGS=2.0*COS
    DO 194 J=l,NCOL
    DO 194 I=1,NROW
194 MULT(I,J)=500.0*RATE(I,J)*DELT
    DO 100 K=1,NI
    SSEEP(K)=0.0
    DO 105 J=1,NCOL
    DO 105 I=l,NROW
    IF(NIR(I,J).EQ.K)GO TO 103
    GO TO 105
103 SSEEP (K)=SSEEP (K)+MULT(I,J)*AROWN (I,J)
105 CONTINUE
    IF(K.EQ.1) SSEEP(1)=1.2* SSEEP(1)
100 CONTINUE
    DIFA=3600.0*24.0*7.0
    DO 195 J=1,NCOL
    DO 195 I=1,NROW
195 RESEEP(I,J)=MULT(I,J)*(AROWN(I,J)+ARFOR (I,J)+ARW (I,J)
    *-DAOWN(I,J) -DAFOR(I,J))
        GO TO (210,230),NCHOIC
210 DO 240 M=2,LTS
    DO 215 K=1,NI
    A(K)=((DIFA/7.0)*TA (K,M)*CH-SSEEP(K))/D(K)
215 IF(TA(K,M).LE.O.1) A(K)=0.0
    DO 220 J=l,NCOL
    DO 220 I=1,NROW
    KK=NIR(I,J)
    IF(KK.EQ.0)GO TO 217
    IF(KK.EQ.20) GO TO 216
    IF(TA(KK,M).LE.(0.1)) RESEEP(I,J)=MULT(I,J)*ARW(I,J)
    AMO(I,J)=DELM(M)*DELX*DELX*ZT(I,J)/100.0
    Q(I,J,M)=-A(KK)-RESEEP (I,J) +AMO(I,J) +EVAP(M)-PRE(M) -QL(I,J)*
    *COS+DIFA*PU(I,J)
    GO TO 220
217 AMO(I,J)=DELM(M)*DELX*DELX*ZT(I, J)/100.0
    Q(I,J,M)=-MULT(I,J)* (ARFOR(I,J) +ARW (I ,J)-DAFOR(I,J))
    * +AMO(I,J) +EVAP(M)-PRE(M)-QL(I,J)*COS+DIFA*PU(I,J)
        IF(M.GT.31) Q(I,J,M)=Q(I,J,M)+MULT(I,J)* (ARFOR(I,J)-DAFOR(I,J))
        GO TO 220
216 AMO(I,J)=DELM(M)*DELX*DELX*ZT(I,J)/100.0
    Q(I,J,M)=+AMO(I,J)-QL(I,J)*COS+DIFA*PU(I,J)
220 CONTINUE
240 CONTINUE
    GO TO 260
230 DO 250 M=2,LTS
    DO 245 J=1,NCOL
    DO 245 I=1,NROW
```

```
        AMO(I,J)=DELM(M)*DELX*DELX*ZT(I,J)/100.0
        Q(I,J,M)=-EVAP(M)* (1.0/COE1 +COE 2+COE3)-RESEEP(I,J)+AMO(I,J)+EV
    *AP(M)-PRE(M)-QL(I, J)*COS+DIFA*PU(I,J)
245 CONTINUE
25b CONTINUE
260 LL=LTS+1
    DO 300 J=1,NCOL
    DO 300 I=1,NROW
    FF=-2.0*MULT(I,J)*ARW(I,J)-WCOS*QL(I,J) +2.0*DIFA*PU(I,J)
    DO 290 M=LL,NTS
290 Q(I,J,M)=FF
300 CONTINUE
    DO 310 K=2,NTS
    PRINT 311
    311 FORMAT(1H1)
    DO 305 J=1,NCOL
    305 WRITE (6,315)(Q(I,NCUL-J,K),I = 1,NROW)
    310 CONTINUE
    315 FORMAT(9(2X,E12.5)/9(2X,E12.5)/6(2X,El2.5))
    173 FORMAT(5X,17F4.1)
    174 FORMAT(5X,9F8.1/5X,9F8.1)
        STOP
        END
```

APPENDIX H
LISTING OF THE INPUT DATA OF THE MAIN PROGRAM FOR THE STEADY STATE CALIBRATION

## TABLE OF $\operatorname{NCX}(\mathrm{I}, \mathrm{T})$

| J/I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 19 | 1 | 1 | 1 | 1 | 1 | 3 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| . 18 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 4 | 1 | 1 | 1 | . 3 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 17 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 16 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 15 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 14 | 1 | 1 | 1 | 1 | 1 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13 | 1 | 1 | 1 | 1 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 |
| 10 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 |
| 9 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 6 | 1 | 1 |
| 8 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 6 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

TABLE OF NCY(I, J)

| J/I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 19 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 18 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 4 | 1 | 1 | 1 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 17 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 16 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 15 | 1 | 1 | 1 | 1 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 1 | 1 | 1 | 1. | 1 | 1 | 1 | 1 |
| 14 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13 | 1 | 1 | 1 | 1 | 6 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 1 | 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 | 6 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 6 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 | 1 |
| 10 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 1 | 1 |
| 9 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 5 | 5 | 5 | 1 | 1 |
| 8 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 5 | 5 | 5 | 5 | 1 | 1 | 1 | 1 | 1 |
| 7 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1. | 1 |
| 6 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 2 | 2 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 2 | 2 | 2 | 5. | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 7 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |










 $\begin{array}{llllllllllllllllllllll}\text { 上 心 } & \text { 上 } & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$





 $\infty \quad \sigma \quad \sigma \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$











| J/I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 2425 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 0. | 0. | 0. | 0. |  | 4470.4 | 4480. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0. |
| 19 | 0. | 0. | 0. |  | 40.4 | 4460.4 | 4474. | 90. | 0. | 0. | 0. | 548. | 4585. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0. |
| 18 | 0. | 0. | 0.44 | 415. | 5.4 | 450.4 | 65. | 30 | 00. | 05. | 10 | 4545 | 596 | 90. | 0 | 0 | 0. | 0. | 0. | 0. | 0 . | 0. | 0. | 0.0 . |
| 17 | 0. | 0. | 0.44 | 415. | 23.4 | 44 | 4461 | 8 | 00 | 05 | 15. | 5 | 597 | 45 | 657. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 16 | 0. | 0. | 0.44 | 15.44 | 415.4 | 4442.4 | 59. | 481. | 497.4 | 10.4 | 522. | 560. | 4605.4 | 4650.4 | 4665. | 4675.4 | 7700. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0 . |
| 15 | 0. | 0. | 0.44 | 410.4 | 412.4 | 4438.4 | 4455.4 | 72.4 | 4490.4 | 10.4 | 525. | 560. | 4603.4 | 645. | 4660 | 4680 | 4705. | 4725. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 14 | 0. | 0. |  | 410. | 15 | 413 | 51 | 33 | 89 | 15 | 25 | 535 | 577 | 20 | 45 | 80 | 05 | 735 |  | 810. | 0. | 0. | 0. | 0.0 . |
| 13 | 0. | 0. | 0.44 | 400. | 36 | 50 | 48 | 461 | 80 | 08 | 4525 | 565 | 602 | 40 | 75 | 990. | 710 | 738 | 777 | 15 | 50. | 0. | 0. | 0.0 . |
| 12 | 0. | 0. |  | 265. | 33 | 4391 | 20 | 423.4 | 66.4 | 4498 |  |  | 66 |  | 0 | 90 | 10 | 74 | 85 | 20 | 50 | 870. | 0. | 0.0. |
| 11 | 0. | 0.42 | 65.427 | 270.4 | 270. | 303.4 | 4346. | 90 | 33 | 78 | 20 | 40 | 570 | 20 | 55 | 00 | 16 | 55 | 792 | 25 | 60 | 4870 | 885 | 0.0. |
| 10 | 0.426 | 0.42 | 5. | 5. | 65 | 265 | 08 | 348. | 400.4 | 43 | 91 | 530 | 55 | 97. | 35 | 5 | 17 | 55 | 95 | 35 | 70 | 85 | 92 | 05. 0. |
| 9 | 0.426 | 60.4 | 0. | 5 | 275.4 | 58 | 65 | 311. | 356.4 | 4405.4 | 4456 | 503 | 45 | 595.4 | 635 | 675. | 713. | 50 | 775 | 32 | 870 | 885 | 05. | 5. 0. |
| 8 | 0. | 0.4 | 0. | 5. | 45 | 50 | 57. | 72 | 13 | 66 | 15 | 63 | 511 | 65 | 15 | 60 | 710. | 25 | 50 | 00. | 0. | 0. | 0. | 0. 0. |
| 7 | 0. | 0. | 0.4 | 40.42 | 245. | 250.4 | 4255.4 | 270. | 283.4 | 16 | 76 | 25 | 476.4 | 26.4 | 575 | 630. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0 . |
| 6 | 0. | 0. | 0. |  | 240.4 | 4250.4 | 4260.4 | 260.4 | 4277.4 | 4293.4 | 4334. | 4391. | 4440.4 | 4488. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 5 | 0. | 0. | 0. | 0. |  | 4240.4 | . 4240.4 | 260.4 | 4275.4 | 4287.4 | 4305. | 4354. | 4491.4 | 4440. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0. |
| 4 | 0. | 0. | 0. | 0. | 0. | 0.4 | 4245.4 | 255.4 | 4267.4 | 4280.4 | 4295. | 4305. | 4350.4 | 4400. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 3 | 0. | 0. | 0. | 0. | 0. | 0. |  | 245.4 | 4255. | 4270.4 | 4285. | 4285. | 4290.4 | 4350. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0. |
| 2 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.4 | 4250.4 | 4265. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0 . |


| INITIAL WATERTABLE PHI(I, J, 1) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| J/I 1 | 2 | 3 | 4 | 5 | 56 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |  | 25 |
| 200. | 0. | 0. | 0. |  | 0.4776 .477 | 4779. | 0. | 0. | 0. |  |  | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | - 0. |
| 190. | 0. | 0. |  | 772. | . 4776.4 | 4779.4 | 787. | 0. | 0. | 0. | 4821. | 4830. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 180. | 0. |  | 66.4 | 68.4 | .4775.477 | 4779.4 | 89. |  | 806.4 | 4812. | 4819.4 | 4830.4 |  | 0 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | - 0 . |
| 170. | 0. |  | 766.4 | 764.4 | . 4774.4 | 4778.4 | 784. | 795 | 804.4 | 4810. | 4815.4 | 4827. | 840.4 | 855. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 160. | 0. |  | 58.4 | 60.4 | . 4770.4 | 777.4 | 86. | 92 | 801.4 | 306. | 11.482 | 822. | 37.4 | 50 | 770. | 881. | 0. | 0. | 0. | 0. | 0. | 0. |  | - 0 . |
| 153. | 0. |  | 54.4 | 54.4 | . 4764.4 | 4775.478 | 87. | 88 | 795.4 | 4802 | 808. | 17 | 30 | 43 | 60. | 881 | 892. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 140. | 0. |  | 0. | 0.4 | 0.4715 .47 | 7770.47 | 76. | 84. | 789.4 | 797. | 803.4 | 811. | 22.4 | 837 | 850. | 881 | 895 | 911 | 19. | 0. | 0. | 0. |  | . 0. |
| 130. | 0. | 0. |  | 645.4 | . 4675 | 4715 | 70 | 77 | 784 | 789 | 98 | 04 | 15 | 827. | 43 | 65 | 90 | 08 | 2 | 5. | 0. | 0. |  | . 0. |
| 120. | 0. |  | 30.4 | 0.4 | . 4645.4 | 675. | 5. | 0 | 77.4 | 84 | 91 | 93 | 06 | , | 33 |  | 85 | 8 | 22 | 44 | 1. | 0. |  | . 0. |
| 110. |  | 20.4 | 25.4 | 30.4 | . 4640 | 645 | 5 | 115. | 70 | 77 | 4783.4 | 4791. | 798. | 810. | 4 | 838 | 860 | 0 | 22. | 43 | 61 | 0. |  | . 0. |
| 100.4 |  | $15 .$ | , | 5.4 | 5.4630 | 40 | $545 .$ |  | $15$ | $70$ | 77 | 84 | , |  | 816. |  |  | , |  | 40 | 59. | 1. | 85 | . 0 |
| 90.46 | 05 | 10 | 15 | 620.4 | . 4625.4 | 4630 | 41 | 645 | 675.4 | 15 | 70 | 77 | 84 | 790 | 03. | 22 | 37 | 58 | 0. | 23 | 45. | 8 |  | . 0. |
| 80. |  | 05.4 | 10.4 | 15.4 | . 4620.4 | 4627.4 | 35.4 | 642 | 645.4 | 4675.4 | 715.4 | 770. | 77.4 | 784 | 791. | 05. | 34. | 850. |  | 0. | 0. | 0. |  | . 0. |
| 70. | 0. |  | 605.4 | 610.4 | .4616.4 | 4622.4 | 628. | 640. | 4643.4 | 4645.4 | 4675.47 | 4715. | 770.4 | 777. | 784. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 60. | 0. | 0. |  | 608.4 | . 4612.4 | 618.4 | 624. | 630. | 4640.4 | 4643.4 | 4645.4 | 4675.4 | 715. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 50. | 0. | 0. | 0. | 0.4 | . 4610.4 | 615.4 | 623.4 | 627. | 4633.4 | 4640.4 | 4643.4 | 4645.4 | 675. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 40. | 0. | 0. | 0. | 0. | . 0.4 | 4617.4 | 622. | 626. | 4623.4 | 4636.4 | 4640.4 | . 4643.4 | 645. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 30. | 0. | 0. | 0. | 0. | . 0 . |  | 622.4 | 626. | 4633.4 | 4634.4 | 4638.4 | 4642.4 | 645. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 20. | 0. | 0. | 0. | 0. | . 0. | 0. | 0.4 | 621. | 4628. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |
| 10. | 0. | 0. | 0. | 0. | . 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |  | . 0. |



嵒 $\quad$ N $\dot{\circ} \quad \dot{\circ}$


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 $\sim \quad \dot{\circ}$



| J/I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | $24 \quad 25$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 0. | 0. | 0. | 0. |  | 4790.4 | 4790. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 19 | 0. | 0. | 0. |  | 4780.4 | 4780.4 | 4786.47 | 790 | 0. | 0. |  | 4818.4 |  | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 18 | 0. | 0. | 0.47 | 75.4 | 4775.4 | 4780.4 | 85. | 90. | 00.4 | 4805.4 | 10 | 820 | 31 | 40. | 0 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 17 | 0. | 0. |  | 5.4 | 775.4 | 4780.4 | 785.47 | 90.4 | 00.4 | 4805.4 | 15. | 825. | 32. | 45. | 4857. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 16 | 0. | 0. |  | 75. | 775.4 | 4780. | 885. | 92 | 02.4 | 4810. | 22 | 330 | 40 | 50 | 65 | 75 | 385. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 15 | 0. | 0. |  | 0.4 | 772.4 | 4778.4 | 785.4 | 92.4 | 00.4 | 4810.4 | 25. | 330. | 43 | 55. | 865.4 | 4880.4 | 4890. | 4900. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 14. | 0. | 0. |  | 0. | 75.4 | 4780. | 5. | 95 | 00 | 815 | 825. | 35 | 847 | 860.4 | 70 | 80 | 00 | 10 | 15 | 35. | 0. | 0. | 0. | 0. 0 . |
| 13 | 0. | 0. |  | 60.4 | 770.4 | 4850.4 | 5. | 5. | 00.4 | 4815. | 25. | 40. | 52 | 65 | 375. | 890 | 905 | 913 | 927 | 40 |  | 0. | 0. | 0. 0 . |
| 12 | 0. | C. |  | 5. | 50.4 | 4825. | 0. | 790. | 800.4 | 815 | 825.4 | 40 | 351. | 65.4 | 75 | 90 | 4910.4 | 23 | 35 | 45 | 50 |  | 0. | 0.0 . |
| 11 | 0. | C. 481 | 8.47 | 0.4 | 0.4 | 4770. | 0. | 0 | 00.4 | 4812. | 25. | 40 | 55 | 70. | 30 | 00 | 915 | 30 | 42 | 0 | 60 |  |  | 0.0. |
| 10 | 0.484 | 11.484 | 0.4 | 0. | 5.4 | 4765. | 5. | , | . 4 | 4810 | , | , | 5 | , | 85. | 00. | 17 | 30. | 45 | 6 | 70 | 5 | 2 | 5. 0. |
| 9 | 0.485 | 50.484 | 45.47 | 90. | 75.4 | 4758. | 65. | 8. | 90.4 | 4805. | 23. | 37. | 55 | 70. | 885. | 9900.4 | 913. | 4925. | 40 | 7. | 50 | 2. | 05. | 5. 0. |
| 8 | 0. | 0.481 | 1.48 | 00.4 | 775.47 | 4750.47 | 757.4 | 772. | 90.4 | 4800.48 | 875. | 830. | 45. | 65. | 75. | 993. | 10 | 225 | 4950 | 40. | 0. | 0. | 0. | 0. 0. |
| 7 | 0. | 0. | 0.47 | 75.48 | 825.47 | 4750.47 | 755.47 | 770.4 | 783.4 | 4793.48 | 810.4 | . 4825.4 | 843.4 | 860.4 | 4875.4 | 4890. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 6 | 0. | 0. | 0. |  | 790.47 | . 7775.47 | 760.47 | 760. | 777.4 | 4793.48 | 810.4 | 4825. | 840. | 855.4 | 4950. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 5 | 0. | 0. | 0. | 0. |  | 740.47 | 740.47 | 760.4 | 775.4 | 4787.48 | 805.4 | 4820.4 | 835.4 | 930. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 4 | 0. | 0. | 0. | 0. | 0. |  | 745.47 | 755.4 | 667.4 | 4780.47 | 795.4 | 4805.4 | 860.4 | 950. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. 0 . |
| 3 | 0. | 0. | 0. | 0. | 0. | 0. | 0.47 | 745. |  | 4770. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |
| 2 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0. |
| 1 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.0 . |







table of storagecoerficient $\mathrm{S}(\mathrm{I}, \mathrm{J})$
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F





[^0]:    *For the Snake River Fan the aquifer had to be extended very far in South-West direction to obtain such a constant head boundary. This implied a great enlargement of the study area. Because of this reason, this solution was not applied.

