IRRIGATION DEMAND CALCULATOR: Spreadsheet Tool for Estimating Economic Demand for Irrigation Water

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PROBLEM STATEMENT

Water in the western United States is essentially fully allocated; thus water for new demands must come from reallocation of existing supplies. Since over 80% of the water withdrawals in the west are for agriculture, potential reallocation mechanisms and the economic impact of reallocation require knowledge of the economic demand for irrigation water. Knowledge of the economic demand for irrigation water is also needed to evaluate:

1) Water-use impacts of shifts in evapotranspiration and precipitation resulting from climate change.
2) Water-supply engineering projects such as building new storage or constructing aquifer storage and recovery (ASR) infrastructure.
3) Water-management options such as:
    a) water marketing
    b) ground-water banking
    c) buyout of irrigation water rights
    d) pricing mechanisms designed to encourage conservation
4) Water-use impact of the current unprecedented increase in agricultural commodity prices.

The spreadsheet tool *IRRIGATION DEMAND CALCULATOR* is designed for economic practitioners who need to obtain an equation for aggregate demand for irrigation water, under conditions of limited time and/or data and resources. The Tool finds a middle ground between approaches that require large amounts of data and approaches that have restrictive assumptions.

DEMAND FUNCTIONS

Description

A demand function is a price-quantity relationship, with price decreasing as the quantity consumed increases, yielding a downward sloping demand function. Another label for demand is “willingness-to-pay”. For instance, residential water demand is the willingness-to-pay for a shower, a car wash etc. Like most consumer goods, its demand is derived from the satisfaction or utility that the consumers obtain from using the good; how much pleasure they receive from showering or how much social status and enjoyment they get from having a clean automobile. In irrigated agriculture, water is an input to a production process. Therefore, agricultural demand is a derived demand, derived from the value of the crops that can be grown with that irrigation water. In the western United States, the price is typically measured in dollars per acre foot (AF) and quantity is measured as AF of withdrawn water.

In virtually all agricultural settings, the amount of water withdrawn from the source exceeds the amount consumptively used. "Consumptive Use" measures the evapotranspiration (transpiration from the plant plus canopy and soil-surface evaporation in the field) plus other losses in the delivery and return paths. "Withdrawals" measures the gross amount of water diverted from a river, reservoir, or stream, or pumped from an aquifer. The remainder (withdrawals minus consumptive use) typically returns to a surface-water body or percolates to an aquifer, and in either case is available for re-diversion by other users.

In some jurisdictions, a permanent water-right transfer (a change in one of the elements of the water right, such as the point of diversion, place of use, season or purpose of use) operates on the consumptively-used portion of the water right. In that case, one could consider water demand to be measured by the consumptive-use amount. A temporary market acquisition of water, such as a one-season purchase of a given quantity of reservoir storage water, is usually quantified in terms of gross diversion volume. As irrigators make water-purchase and water-application decisions, their decisions are couched in terms of gross volume. In these cases, the appropriate measure of demand would be willingness to pay for a volume of withdrawn water. In either case, as transactions are contemplated or modeling is undertaken, investigators must be very careful to understand the linkages between withdrawals, consumptive use, return flows, and re-use of returns by other diverters. In considering returns, one must be very careful to distinguish between returns to the original source (surface returns of water that had been diverted from the surface-water body; deep percolation of water that had been pumped from aquifers) and returns to a different source (runoff into a stream from pumped groundwater; deep percolation of diverted surface water). The distinction is important because the population of secondary users is often very different depending on the water body to which non-consumed water is returned. An additional complexity that is not directly embodied in short-term demand curves is that irrigator *behavior* is based on price per quantity diverted, but impact upon the system and therefore upon other users depends upon quantity consumptively used. In a case where water prices were raised as a policy tool to reduce water use, a rational irrigator behavior in the long run would be to invest in more efficient infrastructure. This could have the unfortunate effect of simultaneously *reducing* gross diversions and *increasing* net consumptive use. Even if the average price were not changed, a change from an annual-fee mechanism to a marginal-cost mechanism could have a similar effect.[1] As the net consumptive use is the proper measure of impact upon the system, these efforts could have the perverse effect of actually reducing, rather than increasing, the availability of water for other uses.

[1] Anecdotal evidence of this principle is the fact that ground-water systems, where the cost faced by the irrigator is a marginal energy cost based on volume pumped, are typically observed to convert to higher-irrigation-efficiency technology much sooner than surface-water systems, where the cost faced by the irrigator is typically a flat per-acre annual fee with essentially zero marginal cost.

Demand for water varies by form, place and time and thus a demand function must be qualified in those terms. For example, demand for irrigation water at the farmer's head gate is obviously not comparable to demand for domestic water treated and delivered to the individual household tap. If the process of estimating a demand function for a particular water use focuses on treated and delivered water, treatment and delivery costs must be subtracted from the estimated willingness to pay, in order to yield demand for raw water. Alternately, treatment and delivery could be included in the supply price, and calculations could be based on the willingness-to-pay for conditioned, delivered water.

Derivation of Demand

The short-term economic demand for a variable production input such as irrigation water can be derived from the production function of the input and the value of the product. This is based on the fact that "a variable input can, in the absence of capital restraints and under perfect knowledge, be profitably used up to the point where the *MVP* [marginal value product][2] of that input equals its price" (Hexem and Heady, 1978).

For an individual crop, the quantity of water demanded at a given price is a function of the commodity price, area harvested and crop yield. Crop yield is a function of agronomic factors including ET, and ET is a function factors including crop vigor and variety, amount of water applied, irrigation efficiency, and salinity of irrigation water.

Demand and elasticity have been approximated with mathematical programming, regression analysis of parametric data, and water production function estimates from field experiments (Scheierling, Loomis and Young, 2004). Another approach is an optimization/crop-production modeling procedure using a detailed crop-production simulation algorithm such as EPIC (Williams and others, 1989). These approaches are necessarily data intensive, time consuming and costly. Field experiments and crop-production modeling produce site-specific results that may not be generally applicable. Demand may also be estimated by assuming a constant-elasticity demand curve and applying a given elasticity to a single known price/quantity relationships (Cook and others, 2004). This approach requires few data but is entirely dependent upon the assumption of constant elasticity and the estimated elasticity used. Estimation of elasticity is not a trivial exercise and estimates vary widely (Scheierling, Loomis and Young, 2004).

The complexity and data requirements for derivation of applied-water production functions, along with the precondition of constant elasticity and the uncertainty of elasticity analyses, point to a need for a method of generating a mathematical demand function that can be adjusted to any location with only a few readily observable data. The spreadsheet tool *IRRIGATION DEMAND CALCULATOR* is designed to meet this need. It relies upon universally-applicable physical relationships between yield,

[2] The value of the marginal production associated with a single unit of additional input.

evapotranspiration (ET), irrigation application and irrigation efficiency. The tool can be deployed with just a few data that are generally obtainable, and it includes guidance for estimating appropriate values where data are not available.

Aggregate Demand

Where multiple crops compete for irrigation water, the aggregated demand may be represented by a horizontal summation of demand for individual crops (if demand is represented as quantity on the horizontal axis and price on the vertical axis). Figure 1 illustrates a hypothetical aggregate demand curve derived from three individual crop demand curves. At price zero, all three crops demand irrigation water and the total demand is approximately 19 volume units. At a price of 3.5, the marginal cost of water exceeds the MVP for crop C1 and it is not irrigated.[3] About 2.5 units of water are demanded by crop C2 and 1.8 by crop C3. At a price of six, only crop C3 will be irrigated, with approximately one unit of water. These are significantly less than the zero-price quantities.

By thus achieving equal MVP, the operation of horizontal summation to find the aggregate demand curve obtains the optimum allocation of water between competing components of demand (i.e. between different crops). Whether reductions in demanded quantity of water are through deficit irrigation on the full parcel, full irrigation on reduced areas, or some combination of these, is immaterial for the derivation of demand. Which it will be depends on the amount of rainfall (i.e. the non-irrigated yields), the underlying agronomy of the particular crops, product-quality concerns, and the information and decision-making skill available to the irrigator. If all potential crops are included in the horizontal summation, the shifting of water from one crop to another will automatically be indicated by the operation of supply interacting with individual and aggregate demand.

[3] In areas of higher rainfall, crop C1 would be produced as a rain-fed or dryland crop. In areas of lower rainfall, the crop would not be produced.

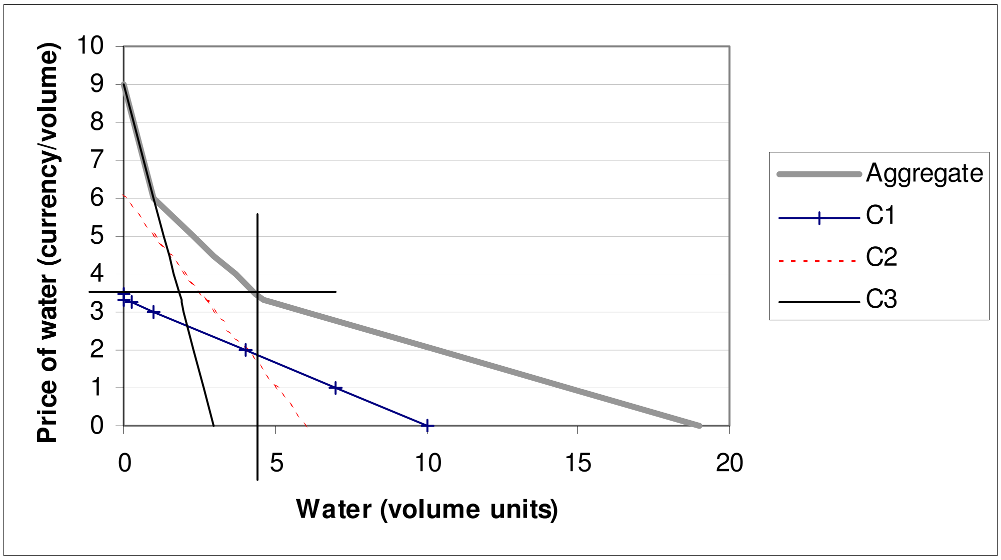

Figure 1. Three individual-crop demand curves and aggregate demand curve obtained by horizontal summation

EVAPOTRANSPIRATION, IRRIGATION EFFICIENCY AND APPLIED-WATER PRODUCTION FUNCTIONS

As mentioned, the demand for a production input may be derived from its production function and the price of the produced commodity. Water production functions can be considered on the basis of *evapotranspiration* or *consumptive use* (the water evaporated from soil and leaf surfaces and transpired from leaf stomates) or on the basis of *applied irrigation water.* Since *applied water* is the unit managed by irrigators and under their direct control, it is the focus of *IRRIGATION DEMAND CALCULATOR.*

Doorenbos and others (1979) describe generally linear relationships between ET and yield, up to a maximum determined by crop physiology and available energy to drive ET. Non-linear ET/Yield relationships are described by English and Dvoskin (1977) and Liu and others (2002), and also mentioned by Doorenbos. Even so, the non-linear equations tend to plot nearly linearly. A linear relationship is implicit in the equation by Martin and others (1989) which is the basis for this Spreadsheet Tool and for Ward and Dagnino's modifications (2008). There is universal agreement that when irrigation efficiency is less than 100% (that is, in virtually all real-world irrigation settings), the irrigation/yield relationship (as opposed to ET/yield) shows a non-linear relationship with decreasing marginal returns. Figure 2 illustrates a general linear ET/Yield relationship and a general Irrigation/Yield relationship.

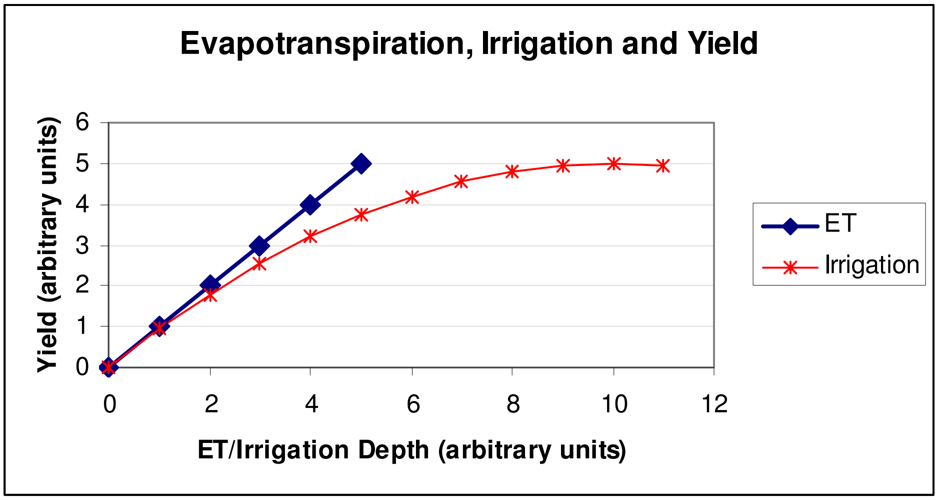

Figure 2. Linear Yield/ET relationship with corresponding Irrigation/ET relationship.

ET/Yield relationships are driven by plant physiology and are broadly applicable across geographic and climatic regions (Doorenbos and others, 1979). Irrigation/Yield relationships are driven not only by plant physiology but by location-specific soil, climate, management and irrigation factors. *IRRIGATION DEMAND CALCULATOR* follows the lead of Martin and others (1989) in using ET production functions, in order to allow broad geographic applicability.

The horizontal distance between the ET and Irrigation curves in Figure 2 represents water applied to the crop but not utilized for ET. The rest evaporates, percolates to ground water, returns to surface-water bodies or is transported to other lands.[4] One definition of irrigation efficiency is the ratio of ET to applied water. It is a function not only of the irrigation methods and management, but also of the depth of application relative to crop requirements. Conceptually one can see that if only a small amount of water were delivered to a crop, nearly all of it would satisfy ET and efficiency would be nearly 100%. However, as ET begins to be fully satisfied, additional application cannot all supply ET and some must supply one of the loss streams; efficiency will decrease at higher application depths. In Figure 2, it appears that efficiency is about 50% at an irrigation depth of 10 units, but exceeds 75% at a depth of four units and approaches 100% at a depth of one unit. Irrigation efficiency, then, varies with the fraction of full supply made available.

Sepaskhah and Ghahraman (2004) describe one approach to variable irrigation efficiency with adequacy of irrigation. *IRRIGATION DEMAND CALCULATOR* relies on the approach of Martin and others (1989), where an exponent related to efficiency is applied to the irrigation deficit. Their production-function equation (with altered notation) is shown as equation (1):

[4] Part of the portion that percolates to ground water or returns to surface water may be required to control salinity. On a quantity basis it is still potentially available for subsequent uses.

$$Y = Y_d + (Y_m - Y_d)[1 - (1 - I/I_m)^{(1/B)}] \quad (1)$$

Where

Y	= crop yield (yield units/area)
Y_m	= crop yield at full irrigation (same units as Y)
Y_d	= non-irrigated (dry land) crop irrigation (same units as Y)
I	= irrigation depth (length)
I_m	= irrigation depth at full irrigation (same units as I)
ET_m	= evapotranspiration at Y_m (same units as I)
ET_d	= evapotranspiration at Y_d (same units as I)
B	= $(ET_m - ET_d)/I_m$ (unitless)[5]

It is the use of exponent 1/B that bridges the gap between the linear ET/Yield production function implicit in term $(Y_m - Y_d)$ and the general non-linear form of Irrigation/Yield production functions. As applied in *IRRIGATION DEMAND CALCULATOR*, this avoids the need for location-specific empirical derivation of a production function for each crop and irrigation method.

For *IRRIGATION DEMAND CALCULATOR*, (I_m) is assumed to include any leaching requirement. Essentially, this is a linear simplification based on an assumption that in the presence of salinity and deficit irrigation, "some of the [infiltrated water] will be partitioned into ET and some into [leaching fraction] to maintain, on average, an equilibrium between the ET reduction (necessary to create some [leaching fraction]) and the [salinity] that is responsible for ET reduction" (Allen, 2007).

SPREADSHEET *IRRIGATION DEMAND CALCULATOR*

Description

IRRIGATION DEMAND CALCULATOR is contained in file "200803-2.xls," with an acres/feet version in file "200803-3.xls." Note that files may download from the IWRRI website with an altered filename, such as "200803-2[1].xls." Both versions contain the following worksheets:

1) Readme. This describes the tool and gives operating suggestions.
2) 1_Crop. This allows analysis of demand for a single crop. It produces an exact demand equation based on the derivation above, with derivatives and elasticities for a user-input water price.
3) 6_Crop. This allows aggregation of demand for up to six crops, with horizontal summation as described above to give a graphical representation of aggregate

[5] Parameter "B" is closely related to irrigation efficiency, depending on the particular definition of efficiency. In general efficiency may be defined as the ratio of (water required)/(water applied). If salinity is not an issue, so that leaching requirement is zero, (water required) is essentially $(ET_m - ET_d)$ and (B) as defined here is irrigation efficiency.

demand. It also offers the opportunity to hand-calibrate parameters for two different approximation equations for aggregate demand (equation (9) and equation (10)). As described in the "Readme" worksheet, the user can also extract the tabular summation data for use in regression analysis. Note, however, that spreadsheet regression analysis does not always produce satisfactory results.

4) Climate. The input variables in equation (1) through equation (8) are not independent. Attempting to estimate the effects of climate change, or adapt data to a different climate for hypothetical or planning purposes, requires that only independent variables be manipulated. Worksheet "Climate" calculates the underlying independent variables from a user-input set of crop data, and allows the user to vary other independent variables to simulate demand at a different climate condition. This can be used to extrapolate to areas where crop data are not available but climate data are, or to investigate potential effects of climate change. Appendix 5 contains the derivation of the independent variables used in this worksheet.
5) Im_Guidance_IrrEff_1. This worksheet contains guidance on using irrigation efficiency to derive the value I_m (irrigation depth required for full yield).
6) Im_Guidance_IrrEff_2, Im_Guidance_IrrEff_3. These contain additional guidance for determining I_m.
7) ETm_Guidance_1, ETm_Guidance_2. These worksheets provide guidance on determining ET_m, the evapotranspiration depth at full yield.
8) Ym_Guidance. This sheet contains guidance on determining the full-irrigation crop yield.
9) Yd_Guidance. In this worksheet is guidance and discussion for determining the rain-fed (dryland) yield.
10) SW_Marg_Cost. This worksheet contains brief discussion of the appropriate marginal cost of surface water to consider as the supply price seen by water users.
11) GW_Marg_Cost. In this worksheet are pre-built equations that allow the user to derive the marginal cost of pumping ground water, based on work by the US Geological Survey (Goodell, 1988).
12) Derivation. The derivations of the demand equations and the independent-variable equations are reported in the derivation worksheet. They are also included in Appendix 4 and Appendix 5 to this report.
13) Bibliography.

Because crop acreage and crop mix are exogenous variables input by the user, *IRRIGATION DEMAND CALCULATOR* produces a short-term demand function. That is, it is assumed that the derived demand function is applicable over a time period during which irrigators will not have opportunity to change factors generally considered fixed in the short run such as infrastructure or technology, acreage base,[6] management skill, access to capital, or marketing opportunities. Commodity prices are assumed to be independent of local production decisions. Additional discussion is found in Appendix 1.

[6] Within the assumed acreage base, acres may or may not be irrigated as indicated by the interaction between water demand and supply. The maximum full-irrigation acreage is constrained by user input.

Full specification of demand requires discussion of the *form, place and time* of water use. The demand functions produced by *IRRIGATION DEMAND CALCULATOR* assume the following:

1) The *FORM* of water is applied[7] liquid water of the temperature and chemical characteristics for which the input yield, irrigation and evapotranspiration values apply. This could include water of less-than-ideal quality, if that is the form of water for which a demand function is of interest.
2) If the *IRRIGATION DEMAND CALCULATOR* is used to analyze a single crop or parcel, the *PLACE* is the point of delivery to crop plants, for the irrigation technology for which the yield, irrigation and evapotranspiration values apply.[8] This means that variable (marginal) delivery and pumping costs must be included in supply prices when analyses are performed.
3) The *IRRIGATION DEMAND CALCULATOR* could also be used for whole-farm, district or basin analysis. In this case, the *PLACE* would be the point of delivery to the farm, irrigation district or basin, typically the main-canal headgate or a measurement point where the supply conduit enters the district or region. If re-use of runoff occurs within the geographic region to which the analysis applies, the volumes used to calculate application depth (I_m) must be adjusted so that across all parcels ($\Sigma(I_m \times area_{crop})$ = (delivery volume for geographic region)).
4) The *TIME* considered is a single irrigation season. For dry climates in the Northern Hemisphere this may be March through November. For wet climates with need only for supplemental irrigation (for instance, early-summer irrigation of fall-seeded small grains) this may be only a few weeks. This definition of time requires two assumptions:
   a) Timeliness of delivery is adequate for the considered crop mix
      i) Typical temporal patterns of runoff are appropriate; or
      ii) Sufficient surface-water storage and delivery infrastructure exist to allow irrigators to choose the time of application; or
      iii) Irrigation is from wells where water levels are amenable to pumping for the entire irrigation season
   b) Irrigators have the management skill to deliver irrigation water at appropriate times for full crop production.

The demand function estimated by *IRRIGATION DEMAND CALCULATOR* is an ordinary (Marshallian) demand function, which includes both substitution and income effects. Demand is estimated for price per volume of water (currency units per 10,000 m^3 or currency units per acre foot).

Limitations and Adaptations

[7] For some studies the demand for *consumptively-used* water is required. The production function equation underlying *IRRIGATION DEMAND CALCULATOR* is not amenable to 100% irrigation efficiency, which would be required to derive demand for consumptively-used water.
[8] The delivery point is the crop canopy (sprinkler irrigation), soil surface (gravity or drip irrigation) or root zone (drip irrigation) for a single-parcel analysis. For a regional analysis it is the delivery point for which ($\Sigma(I_m \times area_{crop})$ = (delivery volume for geographic region)).

The Spreadsheet Tool has potential limitations. Several of these are listed below, followed by a more detailed discussion and possible adaptations that users may consider:

1) Linear Yield/ET relationship.
2) Fixed acreage.
3) Fixed allocation of acreage between crops.
4) Only six crops are represented.
5) Manual parameterization of aggregated equations.
6) The Spreadsheet Tool gives short-term demand (due to the first two limitations).
7) Only full-irrigation-season demand is considered.

The linear yield/ET relationship is partly responsible for the elegant simplicity of equation (1). It is a reasonable assumption, even if the true relationships are slightly non-linear. The only real possible adaptation would be to rebuild the Tool with a different base production function.

There are three responses to the fixed acreage limitation. The first is that in most analyses of real systems, the status quo will be at an allocation of water at very low marginal price (often zero in surface-water systems; the typical assessment is a fixed per-season charge). In that case the observed acreage is not practically constrained by water price, but implicitly represents the effects of all the other constraints. The operation of horizontal summation will allow lower-revenue crops to "drop out" of the mix as marginal price of water increases. Physically this can occur from deficit irrigation (reduced irrigation depth on full acres) or reduction in acreage. To the extent that the yield/ET relationship is linear it will not matter (for purposes of computing demand for irrigation water) which choice irrigators make.

The second response to the fixed-acreage limitation applies to analyses that include representation of demand for new irrigation if water supply were to shift outward. It is assumed that the higher-valued crops, since they do not currently command all available acres, are already constrained by factors other than land availability. Therefore, additional lands brought into production would attract lower-revenue crops.[9] The adjustment would be to estimate the yields that could be produced on the (presumably) less-productive lands that would enter production, and the prices that the crops produced on these lands would command (which may be lower than the prices for the same crops on originally-irrigated lands, due to quality impacts of less-productive land). In the spreadsheet, additional "crops" (which may represent the same nominal crop varieties already in the analysis, but with different yield, water use and production parameters) would be added based on these estimates, and would appear in the horizontal summation as additional component curves with low zero-quantity intercepts on the price-of-water axis.

[9]Physically one might observe high-value crops on the new lands, but we assert that these would be migration of crops from other previously-planted lands and that the change in acreage in a region would be composed of low-value crops, unless other constraints were simultaneously relaxed (see "third response" discussion).

The third response is that one could assume that development of new supply would invite new investment to a region, both in terms of capital and management. The adjustment would be similar to adjustment number two, except that the hypothetical new crops could include representation of additional higher-valued crops.

The fixed allocation of acreage to crops discussion is similar to the fixed-acreage discussion. As long as the assumption of linear yield/ET relationships is approximately true and the “base” partitioning of acres to crops is derived from observed behavior, the operation of horizontal summation implicitly allocates water optimally between crops by achieving equal marginal production value. This is true whether changes are made by varying application depth or planted acres. For purposes of estimating demand for irrigation water it is unimportant which mechanism prevails.

Users can adapt to the inclusion of only six crops for analysis by manually rebuilding the spreadsheet, or by using repeated applications of the one-crop worksheet to populate a new spreadsheet for manual horizontal summation.

As discussed in the “Readme” worksheet, the manual parameterization of aggregate equations can be overcome by extracting the horizontal summation of demand from the six-crop worksheet for further analysis. In a separate worksheet or other software application, more sophisticated regression can then be performed on the summarized demand series.

At this time we have not identified a ready adjustment for the limitation that the underlying equations, and the Spreadsheet Tool, are based on full-season demand for irrigation water.

As a general approach, if an investigator finds an effect that is important for a particular analysis but not incorporated in the spreadsheet equations, the magnitude of this effect at various prices can be quantified and added to the summation prior to estimating aggregated demand equations.

Additional Work Needed

IRRIGATION DEMAND CALCULATOR could benefit from further work in several areas, including the following:

1) Exploration of optimization methods to more explicitly represent changes in total acreage and crop mix.[10] This will be especially valuable in analyzing hypothetical conditions associated with proposed changes in infrastructure or water supply.
2) Exploration of non-linear yield/ET production functions.
3) Verification and validation by applying the tool to data sets from existing studies.
4) Development of methods to estimate demand at various growth stages or at intermediate dates within a single irrigation season.

[10]Dr. Levan Elbakidze, Dr. Garth Taylor and Brett Schiller of University of Idaho are currently addressing this aspect.

5) Further exploration of elasticities, cross-elasticities, and their implications.

CONCLUSIONS

The spreadsheet tool *IRRIGATION DEMAND CALCULATOR* is designed for economic practitioners who need to obtain an equation for aggregate demand for irrigation water, under conditions of limited time and/or financial resources. The Tool finds a middle ground between the approaches that require large amounts of data and approaches that have restrictive assumptions. Optimization routines and whole-farm modeling are examples of the first approach, and fitting an assumed constant-elasticity demand curve to a single data point is an example of the last. The tool is intended for estimating demand for irrigation water on an irrigation-season basis, but short-term in the sense that technology, management skill, infrastructure and acreage base are assumed to be fixed. Its results should be considered most reliable with small departures from base conditions. Sample applications are illustrated in Appendix 3 to this report.

Required input data for the Spreadsheet Tool include crop prices, yields, evapotranspiration and irrigation requirements. These are typically available locally from producers or crop advisors. The tool also includes guidance to estimate values where data are not available.

For a single crop, the tool produces a demand curve and equation. For multiple crops it produces a graphic and tabular summary of aggregate demand by horizontal summation, and provides the ability to manually adjust parameters of two different approximation equations. The users' guide and report provide guidance for adapting the results to larger analyses, or in response to potential limitations of the Tool's design.

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APPENDICES

Appendix 1: Demand for irrigation water

Literature Review

Study of demand and price elasticity of irrigation water began in the early 1960s (see, for instance, Moore and Hedges, 1963). The focus of this research was to estimate price elasticity of irrigation water in order to inform pricing policy decisions. Demand and elasticity have been approximated with mathematical programming, regression analysis of parametric data, and water production function estimates from field experiments (Scheierling, Loomis and Young, 2004).

The following lists provide a sampling in various categories, with some works cited in more than one category:

1) Estimating and measuring crop ET, crop water use and irrigation efficiency.
   a) Williams and others (1989)
   b) Fereres, Orgaz and Villalobos (1993)
   c) United States Department of Agriculture (1993)
   d) Klamm and Brenner (1995)
   e) Allen and others (1998)
   f) Dechmi and others (2003)
   g) Samani and others (2005)
   h) Jayanthi and others (2007)
   i) United States Bureau of Reclamation (2008)

This research is focused on methods to guide planing and management by estimating water requirements, or else describe actual water use for administrative and modeling purposes. They generally assume that the goal is to provide full crop production.

2) Relationships between water application and crop yield (water production functions).
   a) English and Dvoskin (1977)
   b) Hexem and Heady (1978)
   c) Barrett and Skogerboe (1980)
   d) Vaux and Pruitt (1983)
   e) Martin, Watts and Gilley (1984)
   f) Williams and others (1989)
   g) Scheierling, Cardon and Young (1997)
   h) Liu and others (2002)
   i) Dechmi and others (2003)
   j) Brumbelow and Georgeakakos (2007)
   k) Ward and Dagnino (2008)

These sources describe physical relationships and methodology to understand and explain the crop production to be expected from a given level of water application, or the water application desired for a given level of crop production.

3) Relationships between ET and crop yield (ET production functions).
    a) Doorenbos and others (1979)
    b) Allen and others (1998)
    c) Liu and othersl (2002)
    d) Ward and Dagnino (2008)

Water production functions implicitly include both the physiological response of plants to applied irrigation water and the efficiency of methods to deliver water. ET production functions focus only on the physiological response, allowing users of the functions to calculate water production relationships for a wide range of water-application methods and levels of management skill and intensity.

4) Use of production functions to inform management or policy decisions.
    a) Kumar and Khepar (1980)
    b) Barrett and Skogerboe (1980)
    c) Martin, Watts and Gilley (1984)
    d) Hargreaves and Samani (1987)
    e) Martin and others (1989)
    f) Martin, van Brocklin and Wilmes (1989)
    g) Williams and others (1989)
    h) Willardson, Allen and Frederiksen (1994)
    i) Sunantata and Ramirez (1997)
    j) Ghahraman and Sepaskhah (1997)
    k) Allen and others (1998)
    l) Schneider and Howell (2001)
    m) Taha and others (2002)
    n) English and others (2002)
    o) Dechmi and others (2003)
    p) Sepaskhah and Ghahraman (2004)
    q) Peterson and Ding (2005)
    r) Smout and Gorantiwar (2006)
    s) Prasad, Umamahesh and Viswanath (2006)
    t) Fereres and Soriano (2007)
    u) Ward and Dagnino (2008)

This largest subcategory of literature explores methods to improve the information available to decision makers, to better approach the market requirement of perfect information. Research describes tools or methods to bring knowledge of plant physiology, ET requirements and irrigation methods to bear on water allocation and water use decisions. Risk, uncertainty, and water-user behavior are considered carefully and thoughtfully. This important area of research aims to improve the utilization of scarce resources by enabling administrative and production decisions that better allocate water resources to the production of goods and services required by society. The Spreadsheet Tool is not designed for use in optimization or decision-support activities.

5) Estimation of irrigation water demand
    a) Hexem and Heady (1978)
    b) Scheierling, Loomis and Young (2004)
    c) Cook and others (2004)

The specific purpose of the Spreadsheet Tool is to provide economic practitioners and researchers with a quick and low-cost estimate of economic demand for irrigation water. The fifth literature category is most closely aligned with this purpose, and the category where fewest references were found. Hexem and Heady (1978) outline the foundation of the underlying assumption of the Spreadsheet Tool, that economic demand of a production input can be derived from its marginal production value. Scheierling, Loomis and Young (2004) explore elasticity of demand. Cook and others (2004) show how a constant-elasticity demand function can be derived with a single price/quantity data point. The Spreadsheet Tool relies on an equation by Martin and others (1989), though that research does not directly address estimation of demand so it is not listed in item (5). Scheierling, Loomis and Young provide a more complete bibliography of irrigation water demand literature.

Derivation of Demand - Additional Discussion

Demand functions for production inputs may be derived from production functions and prices of outputs.[11] Applied-water production functions can be derived experimentally but field experiments are costly and the results site specific. Water production functions can also be based on crop-process models such as EPIC (Williams and others, 1989) but these are data intensive and the results also site specific. With an assumption of known constant elasticity, a demand function can be derived from a single observed price/quantity data point (Cook and others, 2004) but the constant-elasticity assumption is a strong precondition. Further, estimation of elasticity is itself a complex process, with results sensitive to the method of analysis (Scheierling, Loomis and Young, 2004).[12] Conceptually, one could consider four general approaches to estimation of demand for irrigation water:

1) Estimation from price/quantity data
2) Production-function modeling coupled with optimization-modeling of crop mix, total crop acreage and water application at various commodity-price/water-price combinations
3) Derivation of demand from one or two data and assumptions about the shape of the demand function (see Appendix 2).

[11] Additional discussions of demand and elasticity are included in Appendix 1, Appendix 2 and Appendix 6.
[12] Excerpts from Sheierling, Loomis and Young (2004) are included in Appendix 2.

4) Production-function derivation assuming that observed cropping patterns, acreages and water-use behavior implicitly include all constraints, user decisions and optimization within the limits of skill and information available to decision makers.

To obtain the required degrees of freedom for the first method, large numbers of data are required. These could be theoretically gathered from a large cross-section of farms or a long time series of data. Time series are usually discarded because the necessary assumption of 20 or 30 years of a stable demand function is considered unreasonable, and gathering adequate data from large numbers of farms is costly and time consuming. There is also a statistical-sampling danger in that there could be some correlation between willingness to participate and particular irrigator behaviors.

The second method could be based on one of the many references to optimizing the application of water to crops, with different objectives maximized, in the fourth literature category described above. In earlier literature crop yield tended to be the maximized objective, with profit or water-use efficiency emphasized in later literature. A full analysis of any region larger than a single farm field includes the opportunity to vary crop mix, total acreage planted, and water application to each crop. A wide range of constraints can be considered, including land (total acreage as well as productivity), management, capital, and attitude towards risk. This optimization approach essentially requires an evolution of increasing refinement by adding components to better reflect observed behaviors. Ward and Dagnino (2008), for instance, expand a water production function described by Martin, Watts and Gilley (1984) to include the influence of declining land quality as acreage is expanded. The intent of this modification is to allow "economic optimizing behavior to produce a mix of several crops often seen in larger irrigated regions," with the observation that otherwise "an optimization [produces] a single crop, namely that one crop that maximizes net income." One could further refine the approach by identifying and incorporating into the equation additional constraints on high-value crops, such as agronomic rotation requirements, management skill, capital, marketing-order allotments or access to sales contracts. Such an approach can be valuable in aiding understanding of all the constraints, and in coaching decision makers to better evaluate the implications of each. It will necessarily be complex, data intensive and location-specific.

The third approach is reliant upon the assumed shape of the demand curve. One possible assumption is a constant-elasticity curve, but the estimation of elasticity itself is not an easy task (Scheierling, Loomis and Young, 2004). Results will be entirely dependent upon the assumed shape and the single data point used.

The fourth general approach is to accept that observed cropping patterns and crop mix implicitly reflect all constraints and user decisions, and apply production functions to this mix of crops and acreage. An attraction of this approach is that it does include *all* constraints, even those the researcher may not have recognized or may not know how to characterize. Further, it implicitly represents the actual information environment in which allocation decisions are made, including the impact of less-than-optimum or less-than-perfectly-informed decisions. In this approach, one accepts that the maximum acreage observed of the highest-revenue crop reflects some combination of multiple constraints, without attempting to identify or quantify these constraints. Conceptually this is similar to accepting that observed prices implicitly reflect all the information available to all the decision makers in the marketplace. *IRRIGATION DEMAND CALCULATOR* adopts this approach for reasons of simplicity and broad applicability. Note that the operation of horizontal summation in the aggregation of demand automatically achieves equal marginal value product between crops and therefore does achieve the optimum allocation of water among crops.

Review of Demand Elasticity

Textbooks illustrate demand functions as straight lines but actual market demand functions are typically nonlinear. Demand functions are characterized in terms of price elasticity, which expresses the percentage change in quantity demanded for a percentage change in price. Price elasticities are typically negative, indicating that an increase in price corresponds to a reduction in use. If price elasticity (ε) is < -1 demand is elastic, if $\varepsilon > -1$ demand is inelastic and if $\varepsilon = -1$ then demand is unit elastic. Price elasticity is important for considering the impact that various events or policy decisions have upon use decisions. If demand is price-inelastic, then large increases in price (with correspondingly large burdens upon users) will be required to achieve modest reductions in use. Conversely, if demand is elastic, a modest price change can be used as a tool to achieve significant changes in quantity used.

A linear downward sloping demand curve (Figure A1 a) is elastic in the upper left and inelastic in the lower right, with a point of unit elasticity in between. A constant-elasticity downward sloping demand curve is convex to the origin (Figure A1 b), and undefined at a zero price. A constant elasticity function can be expressed as a power function such as $Q = c\,P^d$ with $c > 0$ and $d < 0$. The exponent d on price is equal to ε. A constant-elasticity demand curve in power form can be estimated by knowing ε and one price-quantity point along the curve. Because the curve does not cross either axis, the ends of the curve may be very sensitive to values of the particular point specified.

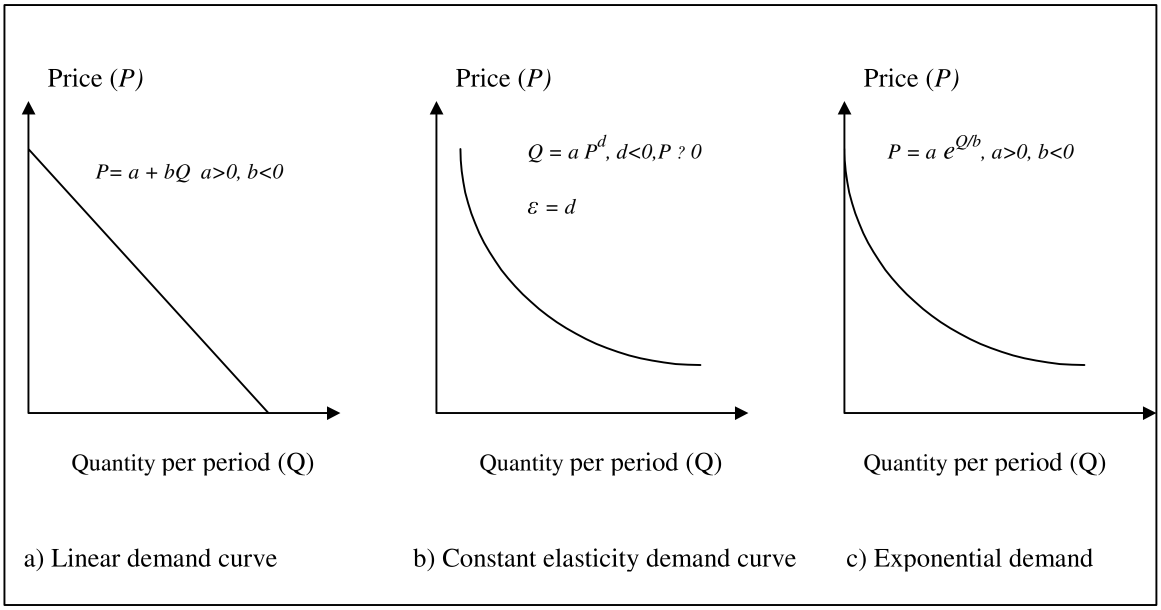

Figure A1. Illustrations of demand curves.

An attractive alternative to the power function is the exponential function $Q = g\, e^{P/h}$, with $g > 0$ and $h < 0$. This function can approximate a power function and has the additional feature that the curve crosses an axis (Figure A1 c). The demand function $Q = g\, e^{P/h}$ allows the demand curve to cross the quantity axis, indicating that satiation can be reached at price equals zero. The inverse demand function $P = a\, e^{Q/b}$ with $a > 0$ and $b < 0$, allows the curve to intercept the price axis indicating that consumption ceases if the price is high enough. Elasticity varies along an exponential function. The exponential function can be estimated by knowing two points along the curve. One point that may be known, or at least more easily assumed than others, is the price at which quantity demanded falls to zero. Call this price $P_{Q=0}$. Another convenient point is given by the quantity at which price equals the existing price P_e, call it $Q_{P=Pe}$. Given these two points, the intercept $a = P_{Q=0}$ and the exponent $b = Q_{P=Pe}/[ln(P_e) - ln(P_{Q=0})]$ of the inverse demand function $P = a\, e^{Q/b}$. The two-point approach to estimating an exponential function fails to use information about elasticity of demand that may be available.

The following discussion relies heavily on the work of Sheierling, Loomis and Young (2004), with much of it quoted nearly verbatim from their article. Reference citations are available in their original work.

Study of the price elasticity of irrigation water demand began in the early 1960s. The focus of this research was to estimate the price elasticity irrigation water in order to formulate irrigation price policies. A pricing policy where irrigation demand is inelastic means that large price increases are necessary to achieve small reductions in water use. Such prices increases could have damaging effects upon farmer's income. In contrast, a pricing policy to raise prices under conditions of elastic demand would be an effective incentive for farmers to reduce use of irrigation water, without huge detrimental effects on income.

Estimates of the demand function and price elasticity for irrigation water have been approximated with mathematical programming, especially linear programming. Early studies (e.g. Moore and Hedges) often tended to show that the demand is more price responsive (elastic) than generally believed, and that even for low prices it is not perfectly inelastic as the U.S. Bureau of Reclamation had claimed in the past. Later studies have constructed sub-regional or regional demand functions from models of representative farms, and commonly calculated responsiveness by either arc-elasticity estimates along the stepped demand curve or by calculating elasticities after fitting continuous regression equations to the parametric data. The results typically show either an inelastic estimate for the whole price range considered, or an inelastic estimate for the lower prices and a less inelastic or elastic estimate for the higher prices (Shumway).

Elasticities have also been estimated with econometric studies using data of actual farmer behavior (Frank and Beattie; Nieswiadomy; Ogg and Gollehon; Moore, Gollehon and Carey). Estimates calculated with econometric methods tend to be more inelastic than suggested by mathematical programming models, but in some cases they are also very elastic. Overall, elasticity estimates vary widely, not only between studies with different methods of analysis but also among them. A number of variables influencing the shape of the demand function as well as elasticity estimates have been identified in the literature, but there has been little systematic study on how and to what extent these variables influence the estimates and the policy recommendations based on them.

Scheierling, and others. used meta analysis to statistically investigate potential sources of variation in the available empirical estimates of the price elasticity of irrigation water demand. They note that studies on price elasticities of irrigation water demand distinguish themselves not only with regard to the particular methods they employ, but also with regard to the inclusion or exclusion of a wide range of factors as well as practical implementation issues, all of which may affect the elasticity estimates. Their work identifies theoretical considerations of the influence of a number of factors upon price elasticity:

1. <u>Method of Analysis</u>. We would expect that estimates from mathematical programming studies generally tend to be more elastic than those from econometric studies and in particular from field experiment studies. Ogg and Gollehon reasoned that these differences may reflect in part differing assumptions underlying these models. Econometric models produce positive estimates based on historical observed behavior that often show little fluctuations in water prices, while mathematical programming models yield normative estimates based on both historical and synthetic data. The latter can be adapted to represent a wide range of scenarios, and model the responses to water and product prices for which no historical observations need to exist. In case of the studies based on experiment station data, part of the reason for their inelastic estimates is that while they model changes in water applications for each of a few selected crops, they do not permit changes in the crop mix or provide possibilities for substituting other inputs (e.g. labor) or alternative irrigation technologies.
2. <u>Irrigation Water Price</u>. Due to the definition of the elasticity concept in percentage terms, the price elasticity of demand is not necessarily the same everywhere along

the demand curve. In case of a straight-line demand curve, for example, demand is elastic at higher prices and inelastic at lower prices.

3. Time-Frame of Analysis. The distinction between a long-run and a short-run time-frame of analysis relates to the degree of fixity of certain inputs. A long-established a priori expectation is that price elasticity of demand is likely to be more inelastic in the short-run when decisions are constrained by factors such as water use technologies, than in the longer-run when more adjustments are possible (Johnston).
4. Farmers' Adjustment Options. The inclusion of high-value crops is hypothesized to contribute to a less elastic estimate. With regard to other adjustment options available to farmers, one would expect that in the lower price ranges the higher the substitutability of other resources for water, the more elastic the response of farmers would be. In one of the early studies on irrigation water demand Hartman and Whittlesey already noted that the kind of adjustments farmers are allowed to make in the model in response to changes in water supply determines the value of additional water and thus the shape of the demand curve. This was confirmed in a more recent study that focused on the effect of varying on-farm adjustment possibilities to changes in water price (Scheierling, Young and Cardon).
5. Type of Data. Irrigation water demand studies may be based on field plot/farm data or regional data, and use primary or secondary data. There are no a priori expectations the M&I user does not face a quantity restriction.
6. Climate. Levels of precipitation and temperature in a study region may affect elasticity estimates. Although there is no explicit guidance from the literature, one would assume that estimates would be less elastic in a locale with scarcer precipitation and higher temperature.

Table A1 summarizes various elasticity estimates identified by Sheierling, Loomis and Young. These are categorized by method of analysis.

Table A1
Irrigation Water Demand Elasticities

Mathematical Programming Studies		
Author	**Number of Estimates**	**Range of Estimates**
Moore, C.V. and Hedges (1963)	1	-.07
Heady, Madsen, Nicol and Hargrove (1973)	1	-0.15
Shumway (1973)	1	-1.97
Kelso, Martin and Mack (1 973)	8	-.0002 to –1.01
Moore, C.V., Snyder and Sun (1974)	1	-0.42
Hedges (1977)	1	-.04
Gisser, Landford, Gorman, Creel and Evans (1979)	2	-0.10 to –0.12
Howitt, Watson and Adams (1980)	1	-0.97
Bemardo, Whittlesey, Saxton, Bassett (1987)	1	-0.12
Hooker and Alexander (1998)	1	-0.22
Scheierling, Young and Cardon (2003)	3	-0.02 to-0.16
Econometric Studies		
Author	**Number of Estimates**	**Range of Estimates**
Frank and Beattie (1979)	16	-1.01-1.69
Nieswiadomy (1985)	1	-0.80
Ogg and Gollehon (1989)	1	-0.26
Moore, R.M., Gollehon and Carey (1994)	4	-0.03 to -0.10
Field Experiment Studies		
Author	**Number of Estimates**	**Range of Estimates**
Hexem and Heady (1978)	4	-0.06 to -0.10
Ayer and Hoyt (1981)	3	-0.06 to -0.16
Kelley and Ayer (1982)	3	-0.04 to -0.56

The variability in these estimates underscores the uncertainty of basing demand estimates on a single data point and an assumed constant elasticity.

Appendix 2: Sample application of *IRRIGATION DEMAND CALCULATOR*

Question 1: Change in cost of electricity for pumping ground water

The first sample treats the question of changes in pumped volume of water, if the price of electricity were to increase. We assume a 59,900-irrigated-acres region irrigated from ground water. Table A2 lists hypothetical crop acreages and parameters entered into worksheet "6_Crop." Figure A2 shows the aggregated (horizontal summation) of demand from the worksheet, along with the estimated demand equation. We assume that the current marginal cost of electricity for irrigation pumping is $ 0.03/ kilowatt hour and that data indicate about 205,000 acre feet per year of pumping for the region.

Table A2. Crop Acreage and Parameters for Illustration
"Change in Cost of Electricity for Pumping Ground Water"

Crop	**Acres**	I_m	ET_m	Y_m	Y_d	P_c
Alfalfa (low mgt)	10,000	3.80 ft	2.20 ft	4 ton/acre	0.5 ton/acre	$100/ton
Alfalfa (high mgt)	20,000	3.80 ft	3.00 ft	5.5 ton/acre	0.5 ton/acre	$100/ton
Barley (low mgt)	5,000	3.00 ft	1.80 ft	85 bu/acre	13 bu/acre	$7.00/bu
Barley (high mgt)	10,000	2.60 ft	2.10 ft	100 bu/acre	13 bu/acre	$7.00/bu
Potatoes	3,400	2.25 ft	2.00 ft	300 cwt/acre	40 cwt/acre	$5.00/cwt
Wheat	6,500	2.75 ft	2.20 ft	100 bu/acre	12.5 bu/acre	$10.00/bu

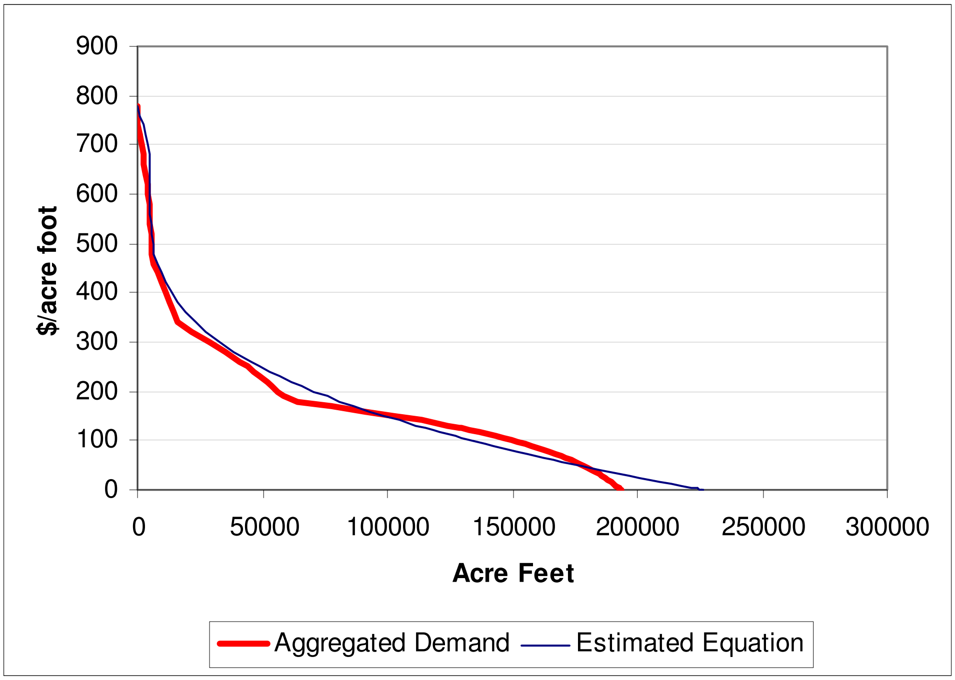

Figure A2. Derived demand for Question 1.

The aggregated demand curve would be very difficult to fit exactly; it has an irregular shape due to the horizontal summation being comprised of only six components. An investigator would need to carefully evaluate the estimated equation and its intended use prior to continuing. If the intended use was for evaluations at very high marginal cost of water, perhaps the approximation could be deemed acceptable. At low marginal costs of water, the investigator would have to carefully weigh whether the concave-to-the-origin shape of the tail of the aggregated curve were reasonable. If indeed the data in Table A2 were representative of all the acreages, prices and irrigation efficiencies of the lower-revenue crops in the region, the shape might be reasonable. However, if there were additional low-revenue, low-application-efficiency uses of water not included in Table A2, or if some of the uses in Table A2 actually experienced lower efficiencies than those implicit in the table, the extended tail and convex-to-the-origin shape of the estimate might actually better reflect reality.

For this illustration, we applied the assumed status-quo marginal price of electricity of $ 0.03/ kilowatt hour to worksheet “GW_Marg_Cost.” With values of 220 feet of lift, 15 feet of column friction, 70 psi discharge pressure and 60% pumping plant efficiency, the marginal cost of water is $20.23 per acre foot. Entering a new electricity price of $ 0.05/ kilowatt hour, we find a marginal cost increases to $33.72 per acre foot.

Figure A3 shows the analysis on a magnified portion of the demand curve. We examined the aggregated-curve prediction of volume at the status-quo marginal cost (about 185,000 acre feet) and the estimated-curve prediction (about 205,000 acre feet). Since the estimated-curve prediction better matches our hypothetical observed regional pumpage volume, we determined that the estimated curve actually better represents actual pumper behavior, probably due to the factors described above. Following the estimated demand curve back up to the hypothetical new supply price of $33.72/kilowatt hour, we predicted that if the price of electricity were to change, pumping would drop by about 10,000 acre feet per year. As a bracket to the uncertainty in our estimates, we also reported that if the lower-elasticity aggregate demand curve better represents actual irrigator behavior, the reduction in pumping could be as little as 5,000 acre feet per year.

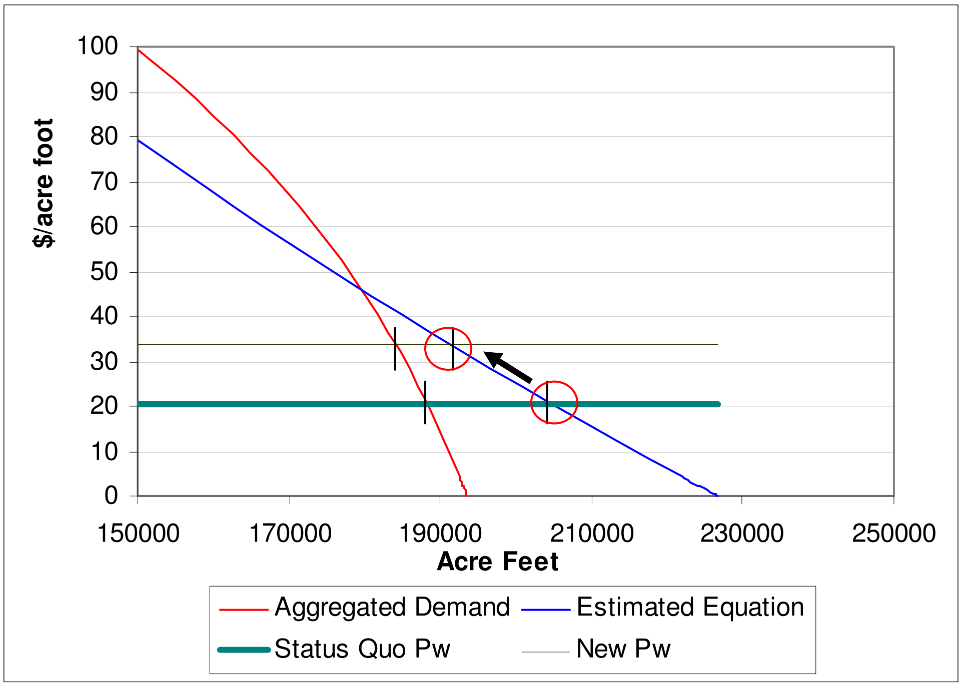

Figure A3. Analysis of Question 1.

Question 2: Change in pricing mechanism for surface-water irrigation

This hypothetical situation involves a canal company of the same acreage and crop mix as Question 1, but irrigated by surface water. Historically the district was developed with lower-efficiency irrigation systems, and not all of these have been upgraded. High-efficiency systems are used primarily on higher-value crops. Water charges are a flat rate per-acre rate for operation and maintenance, with no marginal charge per acre foot of delivery. Deliveries are limited, however, by available supplies. In water short years, users do not have sufficient water and there are calls to take an engineering approach to increase surface-water supply in dry years. Table A3 shows the parameters entered into worksheet “6_Crop” and Figure A4 shows the resulting aggregated and estimated demand.

Table A3. Crop Acreage and Parameters for Illustration
"Change in Pricing Mechanism for Surface-water Irrigation"

Crop	Acres	I_m	ET_m	Y_m	Y_d	P_c
Alfalfa (low mgt)	10,000	3.80 ft	2.20 ft	4 ton/acre	0.5 ton/acre	$100/ton
Alfalfa (high mgt)	20,000	3.80 ft	3.00 ft	5.5 ton/acre	0.5 ton/acre	$100/ton
Barley (low mgt)	5,000	3.00 ft	1.80 ft	85 bu/acre	13 bu/acre	$7.00/bu
Barley (high mgt)	10,000	2.60 ft	2.10 ft	100 bu/acre	13 bu/acre	$7.00/bu
Potatoes	3,400	2.25 ft	2.00 ft	300 cwt/acre	40 cwt/acre	$5.00/cwt
Wheat	6,500	2.75 ft	2.20 ft	100 bu/acre	12.5 bu/acre	$10.00/bu

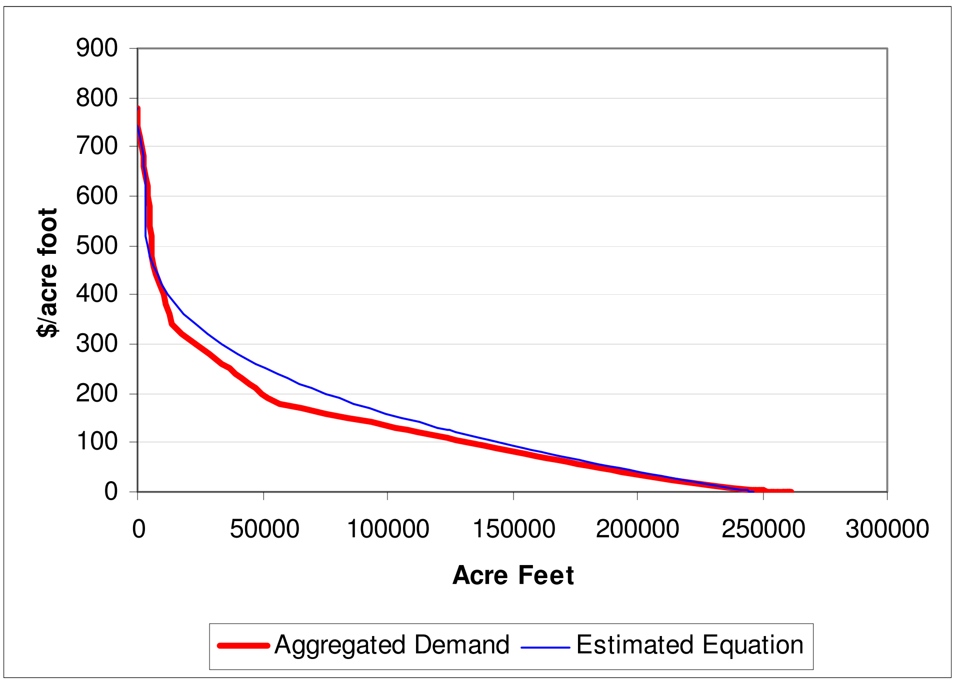

Figure A4. Derived demand for Question 2

In a typical water-short year, deliveries of 215,000 acre feet are possible. Figure A5 shows that at marginal price of zero, there is a perceived shortage of about 35,000 acre feet. As an alternative, if the flat-rate charge were replaced with a marginal charge of about $26 per acre foot, 215,000 acre feet of supply would be adequate; all demands at that price could be satisfied with a supply of 215,000 acre feet. Analysis could then proceed to compare the cost of developing an additional 35,000 acre feet of dry-year supply with the social and income effects of replacing the flat rate charge with the per-acre charge. The company would also need to compare the flat-rate revenues with projected revenues of 215,000 acre feet at the variable rate, to ensure that expenses could still be covered.

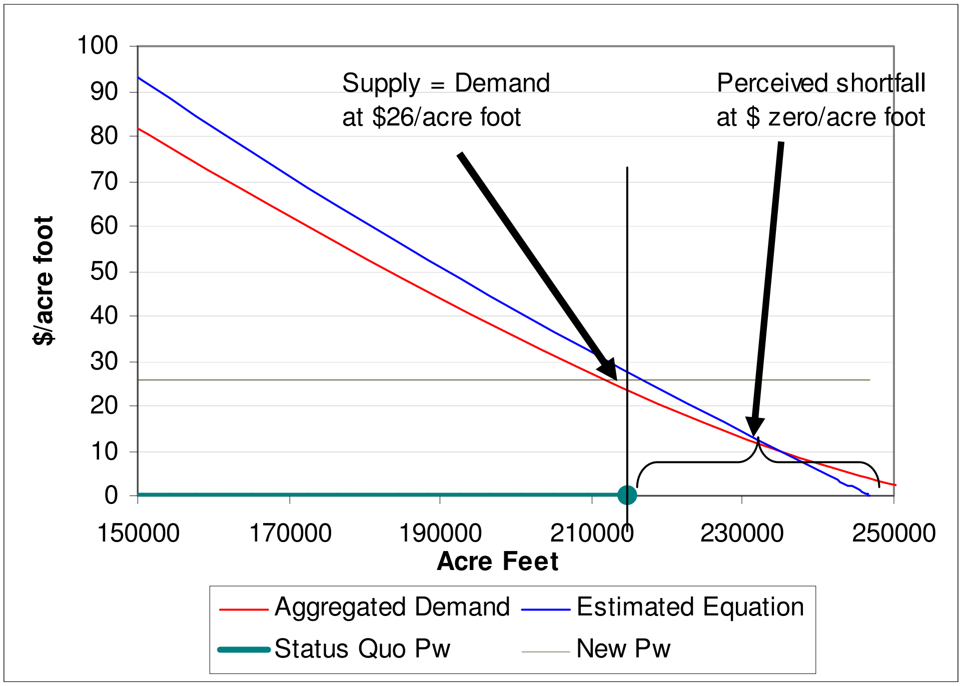

Figure A5. Analysis of Question 2.

Question 3: Effect of current unprecedented high commodity prices

Table A2 and Table A3 include commodity prices that only a few years ago would have been considered absurd. To illustrate the effect that these prices have had on demand for irrigation water, the analysis of the hypothetical canal company is repeated with prices that would have appeared reasonable two years ago, listed in Table A4.

Table A4. Crop Acreage and Parameters for Illustration
"Effect of Current Unprecedented Commodity Prices"

Crop	Acres	I_m	ET_m	Y_m	Y_d	P_c
Alfalfa (low mgt)	10,000	3.80 ft	2.20 ft	4 ton/acre	0.5 ton/acre	$75/ton
Alfalfa (high mgt)	20,000	3.80 ft	3.00 ft	5.5 ton/acre	0.5 ton/acre	$75/ton
Barley	5,000	3.00 ft	1.80 ft	85 bu/acre	13 bu/acre	$2.50/bu

(low mgt)						
Barley (high mgt)	10,000	2.60 ft	2.10 ft	100 bu/acre	13 bu/acre	$2.50/bu
Potatoes	3,400	2.25 ft	2.00 ft	300 cwt/acre	40 cwt/acre	$5.00/cwt
Wheat	6,500	2.75 ft	2.20 ft	100 bu/acre	12.5 bu/acre	$3.25/bu

Figure A6 compares the estimated equations from the low-price and high-price versions of the hypothetical canal company. The zero-quantity price is driven by the commodity price of the highest-revenue crop. In this case, the low price is 80% of the high price and the maximum per-acre-foot willingness to pay for the low-price scenario is 80% of the maximum for the high-price scenario. The zero-price quantity is the same; at zero marginal cost for water, rational irrigators will apply water until the last acre foot applied produces no additional crop production. This point is driven by crop physiology and irrigation efficiency and is independent of commodity price.

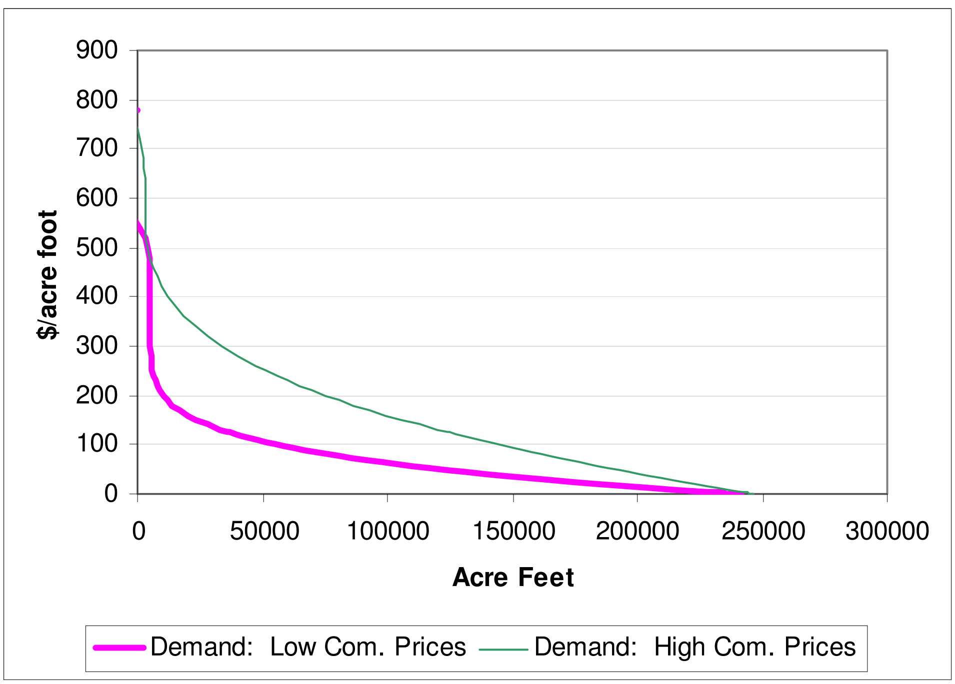

Figure A6. Low-commodity-price and high-commodity-price demand curves.

The "changing pricing mechanism" scenario can be repeated for the lower commodity prices, as shown in Figure A7. At a non-zero marginal cost of water, the marginal production value is a function of commodity price, and as expected, at lower commodity prices a water price of only $9/acre foot would allow a supply of 215,000 acre feet to fully satisfy all irrigation demand. (Note that in Figure A7, both curves would cross the zero-price axis at the same point if the estimated demand equations were perfect.)

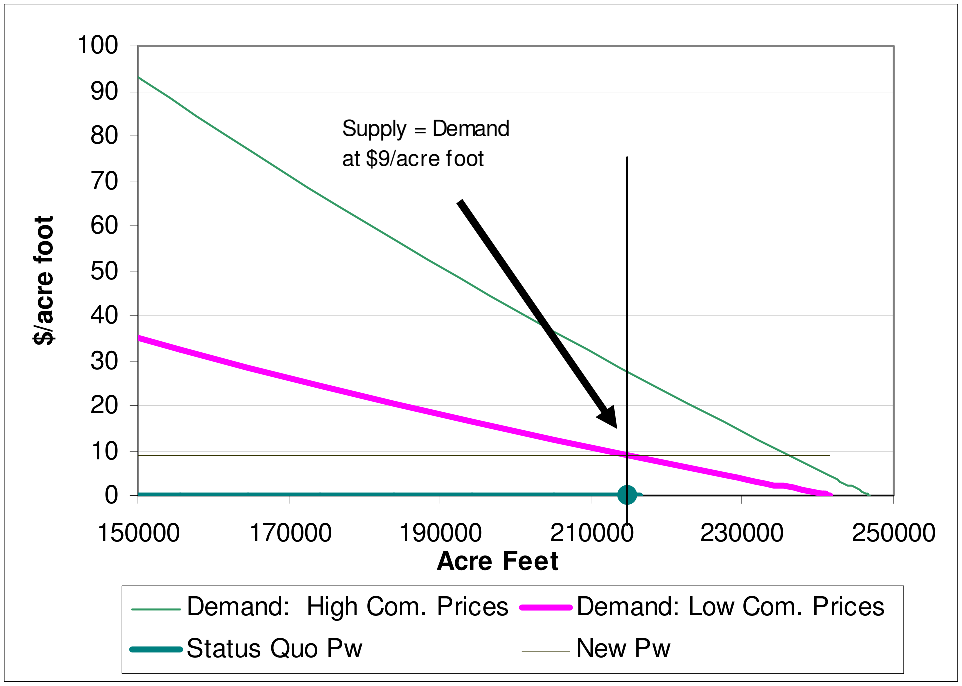

Figure A7. Illustration of effect of commodity price on equilibrium water price.

Question 4: Potential effect of climate change

Most climate-change models predict an increase in temperature, and some predict an increase in precipitation. Starting with the high-price (approximate current-price) simulation of Question 2, we used the "Climate" worksheet to estimate increased demand for irrigation water (and corresponding changes in crop yields) associated with a 10% increase in evapotranspiration and a three percent increase in precipitation, for each crop. The resulting new parameters were then entered in worksheet "6_Crop" and a new estimation equation was parameterized. Parameter values are shown in Table A5, and the resulting estimated demand curve is illustrated in Figure A8 along with the other curves for the hypothetical canal company.

Table A5. Crop Acreage and Parameters for Illustration
"Potential Effect of Climate Change"

Crop	Acres	I_m	ET_m	Y_m	Y_d	P_c
Alfalfa (low mgt)	10,000	5.79 ft	2.31 ft	4.20 ton/acre	0.52 ton/acre	$100/ton
Alfalfa (high mgt)	20,000	5.26 ft	3.15 ft	5.78 ton/acre	0.52 ton/acre	$100/ton
Barley (low mgt)	5,000	4.21 ft	1.89 ft	89.25 bu/acre	13.4 bu/acre	$7.00/bu
Barley (high mgt)	10,000	3.69 ft	2.205 ft	105 bu/acre	13.4 bu/acre	$7.00/bu
Potatoes	3,400	2.37 ft	2.10 ft	315 cwt/acre	41.2 cwt/acre	$5.00/cwt
Wheat	6,500	3.69 ft	2.31 ft	105 bu/acre	12.88 bu/acre	$10.00/bu

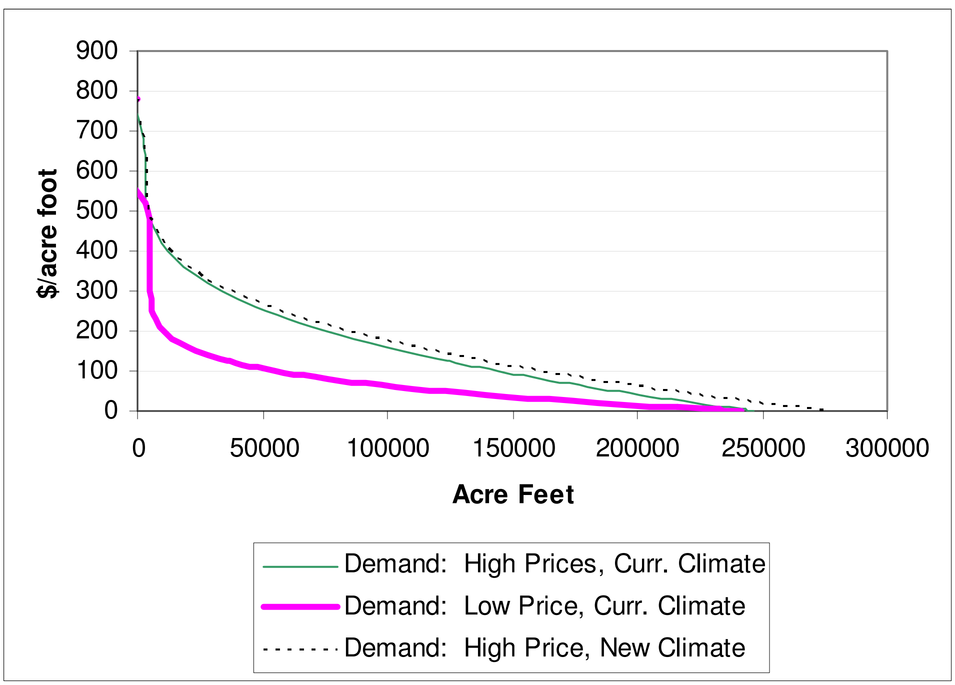

Figure A8. Demand curves for Question 4.

A more rigorous exercise would include careful population of the crop acreages and parameters from data describing irrigation in an actual region of interest, and careful calculation of a range of changes in ET and precipitation implied by various climate models. This sample illustrates the method, and suggests the possibility that the water-demand impact of potential climate change may be significantly less than the water-demand impact of recent commodity-price changes.

Appendix 3: Derivation of equations in *IRRIGATION DEMAND CALCULATOR*

Equations in the report and appendices rely on the following notation:

Y = Crop yield (yield units/L^3)
Y_m = Yield at full irrigation and crop production (yield units/L^3)
Y_d = Yield at rain fed (dryland) production (yield units/L^3)
I = Irrigation depth (L)
I_m = Irrigation depth at full irrigation (L)
ET_m = Evapotranspiration at full irrigation and crop production (L)
ET_d = Evapotranspiration at rain fed (dryland) production (L)
B = $(ET_m - ET_d)/I_m$ (unitless)
a = 1/B (unitless)
R = Effective precipitation (L)
A = Irrigated area (L^2)
P_c = Crop price (currency units/yield unit)
P_{wd} = Price of water, depth basis (currency units/L)
P_{wv} = Price of water, volume basis (currency units/L^3)
b_i = Empirical parameter for estimation equation
K_1 = Crop-production coefficient (yield units/L)
$E_{V,ETm}$ = Elasticity of demanded volume, with respect to full-yield ET
$E_{V,R}$ = Elasticity of demanded volume, with respect to effective rainfall
$E_{V,Pwv}$ = Elasticity of demanded volume, with respect to water price
$E_{V,Pc}$ = Elasticity of demanded volume, with respect to crop price
$E_{Pwv,Pc}$ = Cross-elasticity of demand price, with respect to crop price
$E_{Pwv,K1}$ = Cross-elasticity of demand price, with respect to crop-production coefficient
$E_{Pwv,ETm}$ = Cross-elasticity of demand price, with respect to full-yield ET
$E_{Pwv,a}$ = Cross-elasticity of demand price, with respect to parameter (a), which is closely related to [/(irrigation efficiency)]

Some of these are defined with equation (1) above, others are defined below.

Derivation of the equations begins with substituting (a) for (1/B) in equation (1), giving:

$$Y = Y_m - (Y_m - Y_d)\,(1 - I/I_m)^a \qquad (2)$$

Multiplying yield by irrigated area (A) and commodity price[13] (P_c) gives the gross revenue (R):

$$R = A\,P_c\,Y_m - A\,P_c\,(Y_m - Y_d)\,(1 - I/I_m)^a \qquad (3)$$

The derivative of revenue with respect to irrigation depth (I) is:

$$\partial R/\partial I = (1/I_m)\,a\,A\,P_c\,(Ym-Yd)\,(1 - I/I_m)^{(a-1)} \qquad (4)$$

The derivative ($\partial R/\partial I$) is the marginal production value of water[14] and may be considered the willingness to pay for irrigation water, or the water-depth demand price (P_{wd}). Solving equation (4) for irrigation depth (and substituting (B) for (1/a) where convenient), the depth of irrigation water demanded as a function of price is:

$$I = I_m - I_m\left(\frac{I_m\,B\,P_{wd}}{A\,P_c\,(Y_m - Y_d)}\right)^{(1/(a-1))} \qquad (5)$$

Equation (5) gives a relationship between *depth* of irrigation demanded and *price per depth* of irrigation. Price of Water Volume (P_{wv},, currency/length^3) equals Price of Water Depth (P_{wd}, currency/length) divided by area (A, length^2), so ($P_{wd} = P_{wv} * A$). Substituting ($P_{wv} * A$) for (P_{wd}) and multiplying both sides of equation (5) times area (A, length^2) to obtain volume (V, length^3) gives equation (6), the *volume* of irrigation demanded as a function of the *price per volume:*

$$V = A\,I_m - A\,I_m\left(\frac{I_m\,B\,P_{wv}}{P_c\,(Y_m - Y_d)}\right)^{(1/(a-1))} \qquad (6)$$

This equation will give a nonsensical result of negative volumes of water at high prices; therefore, the spreadsheet uses equation (7), which includes a conditional test:

$$V = \mathrm{Max}\left(0,\ A\,I_m - A\,I_m\left(\frac{I_m\,B\,P_{wv}}{P_c\,(Y_m - Y_d)}\right)^{(1/(a-1))}\right) \qquad (7)$$

---

[13](P_c) is the net price after deducting per-unit harvest costs such as hay twine or drying.
[14]This derivative depends on the important assumptions that commodity prices are perfectly competitive (i.e. independent of local production quantity) and that allocation of crop acres is fully constrained by considerations besides water supply.

If the contemplated use of the composite demand function can accommodate multiple conditional tests, then the composite demand for the farm or region in question is simply the horizontal summation of all individual crop demands:

$$V = \sum \text{Max}\left(0, A_i I_{mi} - A_i I_{mi} \left(\frac{I_{mi} B_i P_{wv}}{P_{ci}(Y_{mi} - Y_{di})}\right)^{(1/(a-1))}\right) \qquad (8)$$

Where subscript (i) denotes an individual crop, with its unique acreage and other parameters.

For uses where the contemplated use of the demand function cannot accommodate conditional statements for each component of the summation, the spreadsheet tool offers an opportunity to manually calibrate two approximations of the composite demand function:

$$V = b_0 + b_1/(P_{wv} - b_3) + b_2(P_{wv} - b_3) \qquad (9)$$

$$V = b_4 (P_{wv} + b_5)^{b_6} + b_7 \qquad (10)$$

Where

b_i = empirical parameter

Both these approximations will give nonsensical results beyond the price-axis and quantity-axis intercepts. Therefore, if either equation is to be used in further computer processing, steps must be taken to limit calculations to an appropriate reasonable range of values.

Appendix 4: Derivation of independent-variable equations for "Climate" worksheet

The underlying production function equation and the derivations described above are defined using readily-available input data, but these data are not independent. Therefore, marginal analyses using partial derivatives of equation (6), or iterative exploration performed by varying one input at a time in the "1_Crop" worksheet, will not be valid. For instance, a climate-change analysis that was performed by adjusting only ET_m (evapotranspiration at full yield) would not be valid because a climate change that impacted ET_m would simultaneously impact full-irrigation yield and irrigation requirement.

Worksheet "Climate" uses the input data to define underlying independent variables, and uses these with additional user input to analyze climate conditions different from the base condition. The derivation of the independent exogenous variables relies on the following assumptions and simplifications:

1) Calculation of demand in the spreadsheet uses the following relationships and derived parameters:

$$ET_d = (Y_d/Y_m)\,ET_m \quad (11)$$
$$B = (ET_m - ET_d) / I_m \quad (12)$$
$$a = 1/B \quad (13)$$
$$K_1 = Y_m/ET_m \quad (14)$$

2) Because of the linear yield/ET relationship implicit in equation (1)[15], parameter K_1 (crop-specific yield factor) can be defined which will be true in both equation (15) and equation (16):

$$Y_m = K_1\,ET_m \quad (15)$$
$$Y_d = K_1\,ET_d \quad (16)$$

3) We define effective precipitation (R) as the fraction of precipitation that satisfies crop ET. This leads to two additional relationships:

$$ET_d = R \quad (17)$$
$$Y_d = K_1\,R \quad (18)$$

4) The relationship that defines B is a function of irrigation system, crop agronomy and management. It will be essentially unaffected by the range of climate differences for which this simplified analysis is appropriate. This leads to:

$$I_m = a(ET_m - R) \quad (19)$$

Note that if effective precipitation exceeds ET_m, I_m will be negative. This is simply an indication that irrigation is not required; the magnitude of I_m is the depth by which effective rainfall could decrease without affecting yield (assuming appropriate temporal distribution of rainfall).

Appendix 5: Elasticities

Partial derivatives are absolute rates of change and can be used as a basis for calculating elasticities. Substituting the simplifications in equation (11) through equation (19) into equation (6) gives equation (20):

$$V = A\,a\,(ET_m - R) - A\,a\,(ET_m - R)\,(P_{wv}/P_cK_1)^{(1/(a-1))} \quad (20)$$

Implicit in these simplifications is an assumption that K_1 (yield factor) and parameter (a) (closely related to 1/efficiency) are independent of climate. If one further assumes that

[15]The linear relationship is implicit in many other equations, including equation (11).

(P_c) (price of commodity) is independent of (P_{wv}) (price of water volume) and climate, the following rates of change can be derived from equation (20):

$$\partial V/\partial ET_m = A\ a - A\ a\ (P_{wv}/P_c K_1)^{(1/(a-1))} \quad (21)$$
$$\partial V/\partial R_e = -\ A\ a + A\ a\ (P_{wv}/P_c K_1)^{(1/(a-1))} \quad (22)$$
$$\partial V/\partial P_{wv} = -(1/(a-1))\ A\ a\ (ET_m - R)\ (1/P_c K_1)^{(1/(a-1))}\ P_{wv}^{(1/(a-1)-1)} \quad (23)$$
$$\partial V/\partial P_c = (1/(a-1))\ A\ a\ (ET_m - R)\ (P_{wv}/K_1)^{(1/(a-1))}\ P_c^{(-1/(a-1)-1)} \quad (24)$$

Note that the derivatives with respect to effective rainfall (R) and evapotranspiration (ET_m) are identical except for sign. Similarly, water price (P_{wv}) and commodity price (P_c) derivatives are similar in form but opposite in sign, with ($1/P_{wv}$) taking the place of P_c.

Because elasticities are relative rates of change, elasticity can be obtained from a derivative by cross-multiplication.

$$E_{x,y} = (\partial x/\partial y)\ (y/x) \quad (25)$$

Where

$E_{x,y}$ = the y elasticity of x or elasticity of x with respect to y
x, y = variables of interest

Applying equation (25) to the derivatives in equations (21) through (24) and simplifying gives various elasticities of demanded volume of water:

$$E_{V,ETm} = ET_m / (ET_m - R) \quad (26)$$
$$E_{V,R} = -\ R / (ET_m - R) \quad (27)$$
$$E_{V,R} = -\ E_{V,ETm} + 1 \quad (28)$$
$$E_{V,Pwv} = [(-1/(a-1)]\ [(P_{wv}/P_c K_1)^{(1/(a-1))}] / [1 - (P_{wv}/P_c K_1)^{(1/(a-1))}] \quad (29)$$
$$E_{V,Pc} = [(1/(a-1)]\ [(P_{wv}/P_c K_1)^{(1/(a-1))}] / [1 - (P_{wv}/P_c K_1)^{(1/(a-1))}] \quad (30)$$

Prices do not appear in the ET/rainfall elasticities, and rainfall and ET do not appear in the price elasticities. The expression for water-price elasticity includes price of water, which is inconsistent with a constant-elasticity assumption. This means that a constant-elasticity demand curve would require some kind of departure from the assumptions that this analysis or the underlying production function rely upon.

Cross-elasticities may also be of interest. For instance, "What is the change in willingness-to-pay for water (P_{wv}) when commodity prices change?" These can be explored mathematically using derivatives based on equation (20), along with equation (25). They can also be explored conceptually, at least in a qualitative sense. For instance, one can reason that an increase in commodity prices will increase the marginal value product of inputs into commodity production, including water, and therefore increase willingness-to-pay for water.

Equation (20) can be rearranged to begin the process of exploring cross elasticities. In the cross-elasticities and derivations below, we have considered (A) (area), (a) (related to irrigation efficiency), (ET_m) (evapotranspiration at full yield), (R) (effective precipitation), crop price (P_c) and (K_1) (crop yield coefficient) as exogenous variables. Note that (A) (area) has been divided out of equation (31), since it appears on both sides of equation (20), being implicitly included in (V).

$$P_{wv} = P_c K_1 \left[(a(ET_m - R) - I)/(a(ET_m - R))\right]^{(a-1)} \qquad (31)$$

Derivatives can be taken of equation (31) to consider cross-elasticities with price of water:

$$\partial P_{wv}/\partial P_c = K_1[(a(ET_m-R) - I)/(a(ET_m-R))]^{(a-1)} \qquad (33)$$
$$\partial P_{wv}/\partial K_1 = P_c[(a(ET_m-R) - I)/(a(ET_m-R))]^{(a-1)} \qquad (34)$$
$$\partial P_{wv}/\partial ET_m = a\,(a-1)\,P_c K_1[(a(ET_m - R) - I)/(a(ET_m - R))]^{(a-2)}[I/(a(ET_m-R))^2] \qquad (35)$$
$$\partial P_{wv}/\partial R = -\,a\,(a-1)\,P_c K_1[(a(ET_m - R) - I)/(a(ET_m - R))]^{(a-2)}[I/(a(ET_m-R))^2] \qquad (36)$$
$$\partial P_{wv}/\partial a = P_c K_1(a-1)\,[(a(ET_m-R)-I)/(a(ET_m-R))]^{(a-2)}[I/(a^2(ETm-R))] + P_c K_1[(a(ET_m-R)-I)/(a(ET_m-R))]^{(a-1)} \ln[(a(ET_m-R)-I)/(a(ET_m-R))] \qquad (37)$$

Equation (25) can be applied to the results of equation (33) through (37), substituting equation (31) for P_{wv}, to obtain water-price cross-elasticities:

$$E_{Pwv,Pc} = 1 \qquad (38)$$
$$E_{Pwv,K1} = 1 \qquad (39)$$
$$E_{Pwv,ETm} = [\,a\,(a-1)\,ET_m\,I] \,/\, [(a(ET_m - R) - I)(a(ET_m - R)] \qquad (40)$$
$$E_{Pwv,R} = [\,-\,a\,(a-1)\,R\,I] \,/\, [(a(ET_m - R) - I)(a(ET_m - R)] \qquad (41)$$
$$E_{Pwv,a} = (a-1)\,I\,/(a(ET_m-R)-I) + a\ln[(a(ET_m-R)-I)/(a(ET_m-R))] \qquad (42)$$

In discussion of elasticities it is helpful to consider the numerical relationships between (B) (equation (12)), (a) (equation (13)), and various combinations of parameter (a) that appear in the elasticity equations. These are shown in Table A6:

Table A6
Relationship Between Parameter(B) and
Combinations of Parameter (a)

B	**a**	**(a-1)**	**(a-2)**	**a(a-1)**	**a/(a-1)**	**1/(a-1)**
0.3	3.33	2.33	1.33	7.78	1.43	0.43
0.4	2.50	1.50	0.50	3.75	1.67	0.67
0.5	2.00	1.00	0.00	2.00	2.00	1.00
0.6	1.67	0.67	-0.33	1.11	2.50	1.50
0.7	1.43	0.43	-0.57	0.61	3.33	2.33
0.8	1.25	0.25	-0.75	0.31	5.00	4.00
0.9	1.11	0.11	-0.89	0.12	10.00	9.00

With this basis, each of the elasticities and cross-elasticities can be considered conceptually:

$E_{V,ETm} = ET_m / (ET_m - R)$ (equation (26))

The sign of this elasticity is positive; an increase in ET requirement produces an increase in demanded irrigation volume. Conceptually, all existing ET is met by some mix of precipitation and irrigation. However, any *change* in ET must be accommodated by a change in irrigation, since precipitation is assumed to be beyond the control of the irrigator. When the fraction of ET currently met by irrigation is small, any change in required irrigation is large on a percentage basis; we expect elasticity to be high (elastic). However, if nearly all the ET is satisfied by irrigation, a change in ET is small, percentage-wise, and we expect lower elasticity.

The equation meets these expectations. The denominator (ET_m - R) represents the depth that must be met from irrigation, so [(ET_m - R)/ET_m] is the fraction of depth that irrigation must supply. The elasticity is the inverse of this fraction. If rainfall is high, the fraction met by irrigation is small and elasticity is high. As rainfall decreases, elasticity decreases and approaches unitary elasticity (1.0).

$E_{V,R} = - R_e / (ET_m - R)$ (equation ((27), alternately expressed as equation (28))

The elasticity of demanded irrigation volume with respect to rainfall, as expected, is negative. The relationship is similar to the ET relationship; any change in rainfall affects the portion of ET that must be satisfied by irrigation. Consequently, the absolute value of elasticity with respect to rainfall is very high when rainfall supplies most of the ET and drops to 1.0 when the base condition is for all ET to be supplied by irrigation.

$E_{V,Pwv} = [(-1/(a-1)] [(P_{wv}/P_c K_1)^{(1/(a-1))}] / [1 - (P_{wv}/P_c K_1)^{(1/(a-1))}]$ (equation((29))

The elasticity of demanded volume with respect to water price is a function of the ratio of water price to crop value (term ($P_{wv}/P_c K_1$)) and a function of irrigation efficiency (term (1/(a-1)). Term ($P_{wv}/P_c K_1$) can be considered an indexed price-of-water, and will be less than or equal to one in the rational production region. With fixed-rate pricing (or subsidized pricing), (P_{wv}/P_cK_1) will be very low.

Term (1/(a-1)) is a function of parameter (B), which is closely related to irrigation efficiency. It is less than one when (B) is less than 0.50 and is greater than 9.0 when (B) exceeds 0.9. Table A7 shows how elasticity varies with (1/(a-1)) and (P_{wv}/P_cK_1), and includes values of (B) for reference. It indicates that elasticity increases as marginal cost of water increases, and decreases as irrigation efficiency increases. Elasticities are negative, indicating that an increase in the price of water prompts a reduction in water use.

Table A7
Price Elasticity of Demand for Irrigation Water (Volume Basis)
for Combinations of (1/(a-1)) and (P_{wv}/P_cK_1).

1/(a-1)	**B**	**P_{wv}/P_cK_1 = 0.25**	**P_{wv}/P_cK_1 = 0.50**	**P_{wv}/P_cK_1 = 0.75**	**P_{wv}/P_cK_1 = 0.99**
0.43	**0.30**	*-0.5*	*-1.2*	*-3.3*	*-99.3*
0.67	**0.40**	*-0.4*	*-1.1*	*-3.2*	*-99.2*
1.00	**0.50**	*-0.3*	*-1.0*	*-3.0*	*-99.0*
1.50	**0.60**	*-0.2*	*-0.8*	*-2.8*	*-98.8*
2.33	**0.70**	*-0.1*	*-0.6*	*-2.4*	*-98.3*
4.00	**0.80**	*-1.6E-02*	*-0.3*	*-1.9*	*-97.5*
9.00	**0.90**	*-3.4E-05*	*-1.8E-02*	*-0.7*	*-95.1*

$E_{V,Pc} = [(1/(a-1)] [(P_{wv}/P_cK_1)^{(1/(a-1))}] / [1 - (P_{wv}/P_cK_1)^{(1/(a-1))}]$ (equation(30))

The elasticity of demanded volume with respect to crop price is identical in absolute value but opposite in sign to the elasticity with respect to water price. This makes sense; the demand is derived from the marginal production value of water, which is a function of the amount of water applied and the price of the crop. As with water price, crop-price responses are elastic at high water-application efficiencies and low water price/crop price ratios. An increase in crop price, like a decrease in water price, increases elasticity. Positive elasticities indicate that demand for irrigation water increases when crop prices rise.

$E_{Pwv,Pc} = 1$ (equation(38))

The cross elasticity between water demand price and crop price is unity (1.0), indicating that other things being equal, willingness-to-pay for water moves proportionally and in the same direction as crop prices.

$E_{Pwv,K1} = 1$ (equation(39))

Parameter K_1 is an ET production coefficient for the specific crop. Its cross elasticity with water demand price is also unity, indicating that some technological or genetic breakthrough that increased crop production (relative to ET) would induce a proportional increase in willingness-to-pay for irrigation water.

$E_{Pwv,ETm} = [\ a\ (a\text{-}1)\ ET_m\ I] / [(a(ET_m - R) - I)(a(ET_m - R)]$ (equation((40))

The water-price/full-yield ET cross-elasticity indicates the change in willingness-to-pay for a given volume of irrigation water if evapotranspiration (driven by climate) were to change. To consider this cross-elasticity conceptually, it is helpful to use equation (19), repeated below, to substitute into elasticity equations:

$$I_m = a(ET_m - R) \qquad (19)$$

With this substitution, equation (40) (price elasticity with respect to full-yield ET) may be expressed as:

$$E_{Pwv,ETm} = a\ (a\text{-}1)\ [ET_m/(I_m\text{-}I)]\ [I/I_m] \qquad (43)$$

Table A6 shows that a(a-1) is large when efficiency is low. Term $[ET_m/(I_m\text{-}I)]$ is dependent on (a) and (R), as well as adequacy of irrigation. It is small under deficit irrigation and approaches infinity at full irrigation. Term $[I/I_m]$ may be considered an index of adequacy of irrigation. It is zero at zero irrigation and increases to 1.0 at full irrigation. Table A8 shows how this cross-elasticity varies across various levels of efficiency and irrigation adequacy.

Table A8
Cross-elasticity of Willingness-to-pay with Respect to
ET at Full Irrigation (ET_m)

SECTION ONE: R = ½ ETm							
B	**a**	**$E_{Pwv,ETm}$ for I/I_m = 0.25**	**$E_{Pwv,ETm}$ for I/I_m = 0.33**	**$E_{Pwv,ETm}$ for I/I_m = 0.5**	**$E_{Pwv,ETm}$ for I/I_m = 0.67**	**$E_{Pwv,ETm}$ for I/I_m = 0.71**	**$E_{Pwv,ETm}$ for I/I_m = 0.95**
0.3	**3.33**	*1.6*	*2.3*	*4.7*	*9.3*	*11.7*	*93*
0.4	**2.50**	*1.0*	*1.5*	*3.0*	*6.0*	*7.5*	*60*
0.5	**2.00**	*0.7*	*1.0*	*2.0*	*4.0*	*5.0*	*40*
0.6	**1.67**	*0.4*	*0.7*	*1.3*	*2.7*	*3.3*	*27*
0.7	**1.43**	*0.3*	*0.4*	*0.9*	*1.7*	*2.1*	*17*
0.8	**1.25**	*0.2*	*0.3*	*0.5*	*1.0*	*1.3*	*10*
0.9	**1.11**	*0.1*	*0.1*	*0.2*	*0.4*	*0.6*	*4.4*
SECTION TWO: R = 1/10 ETm							
B	**a**	**$E_{Pwv,ETm}$**	**$E_{Pwv,ETm}$**	**$E_{Pwv,ETm}$**	**$E_{Pwv,ETm}$**	**$E_{Pwv,ETm}$**	**$E_{Pwv,ETm}$**

		for I/I_m = 0.25	for I/I_m = 0.33	for I/I_m = 0.5	for I/I_m = 0.67	for I/I_m = 0.71	for I/I_m = 0.95
0.3	**3.33**	*0.9*	*1.3*	*2.6*	*5.2*	*6.5*	*52*
0.4	**2.50**	*0.6*	*0.8*	*1.7*	*3.3*	*4.2*	*33*
0.5	**2.00**	*0.4*	*0.6*	*1.1*	*2.2*	*2.8*	*22*
0.6	**1.67**	*0.2*	*0.4*	*0.7*	*1.5*	*1.9*	*15*
0.7	**1.43**	*0.2*	*0.2*	*0.5*	*1.0*	*1.2*	*10*
0.8	**1.25**	*0.1*	*0.1*	*0.3*	*0.6*	*0.7*	*6*
0.9	**1.11**	*0.0*	*0.1*	*0.1*	*0.2*	*0.3*	*2.5*

$E_{Pwv,R} = [- a (a-1) R I] / [(a(ET_m - R) - I)(a(ET_m – R)]$ (equation((41))

The cross-elasticity between water price and effective rainfall is very similar to the water price/ET relationship. The sign is opposite (an increase in rainfall induces lower willingness-to-pay) and rainfall takes the place of ET in the elasticity.

$E_{Pwv,a} = (a-1) I /(a(ET_m-R)-I) +a \ln[(a(ET_m-R)-I)/(a(ET_m-R))]$ (equation(42))

The mathematical complexity of this cross-elasticity arises from the fact that parameter (a) appears in both the base and the exponent of the demand equation. Consequently, parameter (a) appears in both the numerator and denominator of the first term of the elasticity, and both the numerator and denominator of the logarithmic expression. Therefore, it is difficult from simple inspection of the equation to anticipate or describe the effect upon cross-elasticity of various components. Experimentation shows that the cross-elasticity is independent of levels of ET_m and R (as long as (I/I_m) is held constant), but sensitive to (a) and irrigation adequacy (I/I_m). Table A9 is presented to map the effect of varying levels of irrigation efficiency and deficit irrigation upon this cross-elasticity.

Table A9
Cross-elasticity of Willingness-to-pay with Respect to
Parameter (a) (related to irrigation efficiency)

B	a	$E_{Pwv,a}$ for I/I_m = 0.25	$E_{Pwv,a}$ for I/I_m = 0.33	$E_{Pwv,a}$ for I/I_m = 0.50	$E_{Pwv,a}$ for I/I_m = 0.67	$E_{Pwv,a}$ for I/I_m = 0.71	$E_{Pwv,a}$ for I/I_m = 0.95
0.3	**3.33**	*4.4*	*3.5*	*2.3*	*2.36*	*2.78*	*36.7*
0.4	**2.50**	*3.2*	*2.5*	*1.5*	*1.27*	*1.46*	*22.5*
0.5	**2.00**	*2.5*	*1.9*	*1.0*	*0.61*	*0.67*	*14.0*
0.6	**1.67**	*2.1*	*1.5*	*0.7*	*0.18*	*0.14*	*8.3*
0.7	**1.43**	*1.7*	*1.2*	*0.4*	*-0.13*	*-0.24*	*4.3*
0.8	**1.25**	*1.5*	*1.0*	*0.3*	*-0.37*	*-0.52*	*1.3*
0.9	**1.11**	*1.3*	*0.8*	*0.1*	*-0.55*	*-0.74*	*-1.1*

This elasticity indicates the percentage change expected in willingness-to-pay, given a one-percent change in irrigation efficiency. Positive elasticities, such as occur with low irrigation efficiencies and low adequacy of irrigation, indicate that an increase in efficiency will produce an increased ability and willingness to pay for irrigation water. For a given efficiency, elasticity is higher at low or very high irrigation adequacies. For a given adequacy, elasticity decreases at higher efficiencies. In fact, at higher adequacy and efficiency, the cross-elasticity is negative, indicating that an increase in efficiency results in a reduced willingness-to-pay. This likely indicates that at high adequacy and efficiency, an increase in efficiency "frees up" water and offsets some demand.