EVALUATION OF POTENTIAL ERRORS RESULTING FROM IMPOSING LINEARITY IN DEVELOPMENT OF CAPTURE RESPONSE FUNCTIONS FOR THE SPOKANE VALLEY – RATHDRUM PRAIRIE AQUIFER

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Introduction

For many decades ground water hydrologists have taken advantage of the linearity of the confined ground water flow or diffusion equation to simplify hydrogeologic analyses. The scalability and additive responses of linear systems have given rise to many applications of superimposing effects of different stresses (Reilly et al., 1987; Cosgrove, 2001). These applications include image well analysis, analytical expressions for stream depletion (Jenkins, 1968; Glover,. 1968), the development of capture response functions (Maddock, 1972; Cosgrove and Johnson, 2004, 2005; Leake et al., 2008), and others.

In the Spokane Valley-Rathdrum Prairie (SVRP) aquifer, Taylor et al. (2008) and Johnson et al. (2008) developed response function maps and a spreadsheet tool for assisting water interests with assessing surface water impacts resulting from changes or proposed changes in aquifer recharge or discharge. The response functions were calculated using the SVRP aquifer model developed by Hsieh et al. (2007). Those response function products are valid so long as surface and ground water interaction respond in a near linear fashion. The purpose of this report is to describe the potential error in SVRP response functions that results from linear approximations to non-linear conditions represented in the SVRP aquifer model developed by Hsieh et al. (2007).

Linearity and Response Function Concepts

Capture response functions describe how surface water gains and losses vary depending on aquifer recharge and discharge events (aquifer stresses). Response functions may be used to describe the effects of aquifer pumping or the effect of a recharge event on a river, lake, or stream. Response function application allows users to add the effects (change in surface water gains or losses) of individual aquifer stresses together to determine the net effect of multiple stresses, or scale the effects in proportion to the magnitude of the stress. For example, if a pumping rate is doubled, the depletion effects on a surface water body will also be doubled. Consequently, effects are considered additive and scalable. This allows the development of simple generic tools that can be applied to understand how surface water gains and losses change in response to changes in aquifer stress, such as the maps and spreadsheet tool for the SVRP developed by Johnson et al. (2008) and Taylor et al. (2008).

Capture response functions may be developed from an existing numerical ground water flow model (for example Modflow), or from more simple analytical techniques such as equations developed by Jenkins (1968) and Glover (1968). The use of analytical techniques requires more extensive assumptions related to aquifer and river geometry and homogeneity of aquifer properties and therefore may have a greater level of uncertainty. The response functions developed for the SVRP (Taylor et al., 2008; Johnson et al., 2008) were determined from simulations of the numerical Modflow model of the SVRP developed by Hsieh et al. (2007) and therefore are based on the same concepts and properties used in that model.

The valid development and use of response functions require that the aquifer and surface water system can be represented by linear, or nearly linear, equations. The SVRP Modflow model developed by Hsieh et al. (2007) included two components that were not represented by strictly linear relationships: 1) the unconfined ground water flow, and 2) some surface water boundaries that were simulated as piecewise linear. Unconfined conditions can create non-linear responses because aquifer thickness changes in response to temporal changes in recharge and discharge. Changes in aquifer thickness translate to changes in aquifer transmissivity. Since the response of surface water systems to aquifer stress is partially a function of aquifer transmissivity, then response (and response functions) varies over time depending on the distribution of aquifer recharge and discharge. Under these conditions, responses to multiple stresses are not purely additive, nor scalable, and response function application is invalid or compromised. Piecewise linear representation of model boundaries representing surface water features occurs when a water body transitions between being perched above the aquifer to hydraulically interconnected with the aquifer, or vice-versa. In the SVRP model, Lake Coeur d'Alene and a portion of the Spokane River were represented as transitioning in this piecewise fashion.

Because the SVRP model developed by Hsieh et al. (2007) included potentially non-linear features, some errors are introduced when response functions are used to represent model results. This report describes some of those errors, but cannot describe all errors involved with using response functions under all possible conditions. It should also be noted that the errors described in this report are only errors occurring as a result of simplifying the numerical model to a response function approach. They do not include errors in the original model resulting from conceptual uncertainty, model parameterization, numerical or discretization error.

Description of the Spokane Valley-Rathdrum Prairie Aquifer Model

The SVRP aquifer is a trans-boundary resource located in northern Idaho and eastern Washington State (Figure 1). The aquifer underlies a rapidly developing area of about 330 square miles. The largest cities overlying the aquifer are Spokane WA and Coeur d'Alene ID. The aquifer is recharged primarily by runoff from mountains on the north, west, and east sides of the Rathdrum Prairie in Idaho. The aquifer gains water from nine lakes bordering the aquifer and exchanges water with the Spokane and Little Spokane rivers. The model developed by Hsieh et al. (2007) represented Lake Pend Oreille, Lake Coeur d'Alene, the Spokane, and Little Spokane rivers as interconnected with the aquifer. The ground water flow model represents the aquifer as a single layer, unconfined system except for an area beneath part of the City of Spokane where a clay confining layer is believed to divide the aquifer into shallow and deep units. There are few wells drilled into the deeper unit and a high degree of uncertainty exists on the characteristics and properties of the deep system. Consequently, response functions developed by Taylor et al. (2008) and Johnson et al. (2008) do not include response to stresses on the deeper aquifer.

The model employed a square model grid of 1320 feet on a side. This resulted in 5268 active model grid cells. The Spokane River was represented using Modflow's Streamflow Routing Package. The eastern portion of the Spokane River, above approximately Greenacres, is continuously perched above the aquifer. To the west of Greenacres, the river is modeled as interconnected with the aquifer except for 19 cells that transition between interconnected and perched (Figure 1). The Little Spokane River and Lake Pend Oreille are continuously interconnected with the aquifer and were modeled with the Modflow River Package. Lake Coeur d'Alene was modeled as intermittently connected with the aquifer also using the River Package. Long Lake, a reservoir on the Spokane River, was modeled as interconnected with the deeper model layer beneath the City of Spokane. Other model boundaries were either no-flow or a fixed flow boundary where peripheral lakes and streams seep into the aquifer.

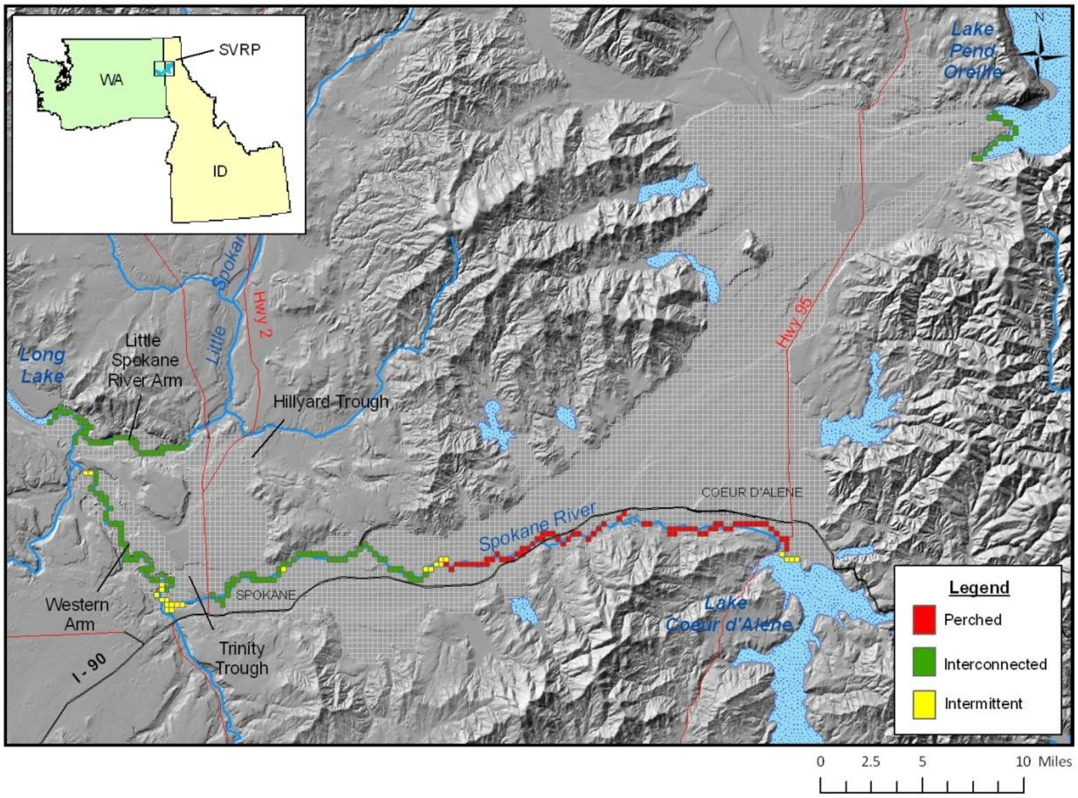

Figure 1. Perched, hydraulically interconnected, and intermittently perched conditions simulated in the Modflow model by Hsieh et al. (2007).

The model was calibrated in monthly stress periods to aquifer water levels and river and lake gain and loss estimates for the October 1995 through September 2005 period. Aquifer hydraulic conductivity and specific yield estimates were calibrated for a network of zones described in Hsieh et al. (2007). Conductance was calibrated for 11 reaches of the Spokane River and one reach of the Little Spokane River based on measured river gains and losses at gaging stations along the rivers. Conductance was also estimated to limit seepage from Coeur d'Alene and Pend Oreille lakes, however, there is little known about the seepage from these lakes resulting in additional uncertainty in these estimates.

One of the principal reasons the model was constructed was to enhance understanding of surface and ground water interactions in the SVRP. To further develop this understanding, capture response function maps (Johnson et al., 2008) and a spreadsheet tool (Taylor et al., 2008) were developed from the model. The capture response functions were identified for stresses located at each model cell and the resulting effect on each of six surface water reaches shown in Figure 2: 1) the Spokane River above the Spokane gage; 2) the Spokane River between Avista Dam and Deep Creek; 3) the Spokane River below Deep Creek combined with Little Spokane below Painted Rocks gage and Long Lake; 4) Little

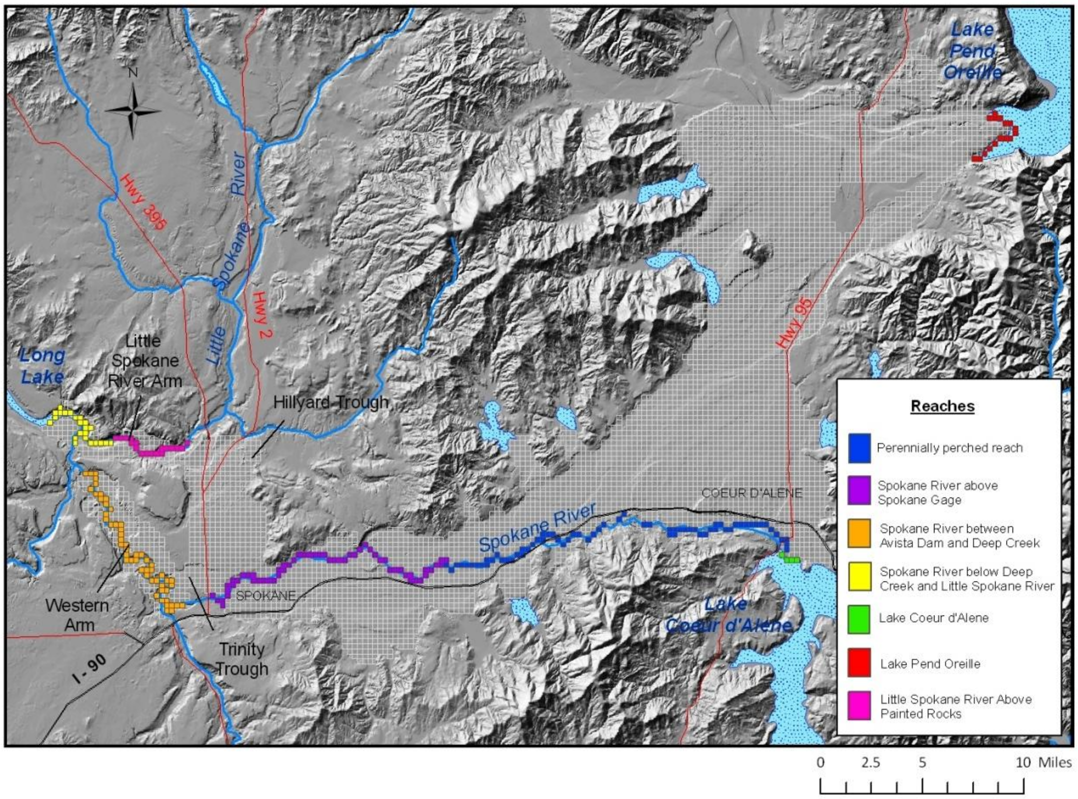

Figure 2. Reaches used in the capture response function maps of Johnson et al. (2008) and spreadsheet of Taylor et al. (2008).

Spokane River above Painted Rocks gage; 5) Lake Coeur d'Alene; and 6) Lake Pend Oreille. This report will discuss response function errors associated with these same reaches.

Description of Approach

The purpose of this report is to assess the error associated with two basic types of linearity conditions imposed by response function application in the SVRP: 1) assuming constant aquifer thickness in an unconfined aquifer, and 2) assuming linear boundary response in boundaries otherwise simulated as piecewise linear. This analysis determines errors as the differences in simulated response functions between the original, nonlinear model and two model versions representing a constant thickness aquifer and a fully linear model with constant aquifer thickness and linear boundary representation.

Response functions were determined for each surface water reach and each possible stress location (5268 cells) in each model version. This required a total of 15,804 simulations that required multiple weeks on multiple personal computers to complete. Errors associated with linearity assumptions were determined as the absolute value of differences between response functions for the nonlinear and linearized model versions. Simulations were executed that represent the following conditions:

1) The original model developed by Hsieh et al. (2007) that includes non-linear representation of both aquifer thickness and some surface water boundaries (referred to as the Nonlinear Version).

2) A constant thickness model developed by modifying the Nonlinear Version to confined and representing aquifer thickness as the average thickness simulated (using the Nonlinear Version) during the October 1995 through September 2005 period. Specific storage in this version was simulated as the specific yield values of the Nonlinear Version divided by the calculated aquifer thickness. This model version is subsequently referred to as the Confined Version.

3) A model version with constant thickness as described immediately above for the Confined Version and linear boundary representation. Linear boundaries were developed by identifying boundary cells in the Nonlinear Version that transitioned between perched and interconnected with the aquifer and representing those cells as continuously interconnected. A factor (hydraulic conductance) controlling exchange of water between the aquifer and surface water in these cells was reduced in proportion to the amount of time that each cell was hydraulically interconnected in the October 1995 through September 2005 period in the Nonlinear Version of the model. This was expected to create a linear model version that would generate nearly correct long term results (that is, matching the Nonlinear Version) but may fail to match specific months when the Nonlinear version experiences boundary cells transitioning to or from perched conditions. This version is referred to as the Fully Linear Version.

Differences between surface water responses among the above model versions represent the effects of assumptions of linearity. Differences in simulated surface water responses to an aquifer stress between the Nonlinear and Confined versions indicate errors introduced through the assumption of constant aquifer thickness. These results are described in the Interpretation of Results section on *Confined Conditions* below. Differences between the Nonlinear and Fully Linear Versions indicate the combined effect of assumed constant thickness and linearized boundaries and are discussed in the section titled *Fully Linear Representation* .

Since the magnitude of induced errors from nonlinear effects varies with both time and stress location, thousands of simulations were needed with each model version to evaluate linearity assumption effects. Temporal variation was evaluated by examining the difference in surface water responses at each stress period during the October 1995 through September 2005 simulation period. In each simulation, an added stress of 10,000 cubic feet per day (0.116 cfs) was represented in a specific cell. Surface water response to that stress was determined by differencing simulated surface water gains and losses between simulations with and without the stress. The stress was shifted among each of the 5268 active model cells to describe how effects vary based on stress location. Presentation of the combination of temporally and spatially distributed errors is accomplished by mapping the maximum error (difference between model versions) at any stress period during the 10 year period and by mapping the error averaged among all 120 stress periods within the 10 year simulation period.

Results

Response function maps are created by simulating the change in gains and losses of a surface water reach resulting from an aquifer stress at a specific location or grid cell. The stress location is shifted among all active grid cells to create a map where response changes depending on location. Maps showing time-averaged response functions (averaged for the 120 stress periods) determined from the three model versions are presented in Appendix A for each of the six response function reaches identified in Johnson et al. (2008). The response functions percentages are the ratio of the sum of changes in surface water response for all cells within a reach divided by the magnitude of the stress and multiplied by 100. The maps in Appendix A differ slightly from those of Taylor (2008) because the maps in Appendix A are determined as average values over a 10 year period. The maps presented in Taylor (2008) represent responses after a specific period of continuous stress. Average values were required for this analysis because the Nonlinear Model Version results in erratic temporal variations in responses in surface water bodies that are intermittently perched.

Although many of the response function maps for a given reach appear similar among model versions, subtle differences are evident in some of the maps in Appendix A. Differences are more apparent when the response functions of one model version are differenced from another version. Appendix B presents the differences for the time-averaged response functions between the Nonlinear and the Confined versions of the model (effects of assuming constant thickness) and also between the Nonlinear and Fully Linear versions (combined effects of constant thickness and linear boundaries). Differences are also presented for the maximum difference that occurred in any of the 120 stress periods representing the 1995 through 2005 period.

Interpretation of Results

Confined Conditions - Time-averaged Differences

Time-averaged differences in response functions present a picture of the degree to which imposing linear conditions on nonlinear model elements affects results (values of the response functions) under long-term or average conditions. Since response functions are intended to assess general impacts, often associated with future development, the average conditions probably provide the most meaningful interpretation of potential errors in application of the response function maps of Johnson et al. (2008) and the spreadsheet analysis tool presented by Taylor et al. (2008).

The greatest differences in time-averaged responses (interpreted as errors) occur in the reaches of the Spokane River above Spokane Gage, and in the Little Spokane River as a result of stress in the northern portion of the Hillyard Trough (Figures B1 and B11). In this area, subtle changes in aquifer thickness (either increase or decrease) magnify responses in the Little Spokane River above the Painted Rocks gage and correspondingly diminish response in the Spokane River above the Spokane Gage. This unexpected result appears to be due to a gradual thinning of the aquifer (upper layer in this area) progressively northward from the Spokane River toward the Little Spokane River. This phenomenon occurs despite the fact that the aquifer is near or greater than 100 feet thick throughout this area (Figure 3). Even subtle changes in aquifer thickness have a proportionately greater effect in the direction of the Little Spokane River (relative to a variable thickness simulation). An increase in aquifer thickness will steer a greater proportion of aquifer flow toward the Little Spokane River. Since total ground water flow greatly exceeds the magnitude of the simulated response function stress, subtle changes in this total flow appear large relative to the magnitude of the added stress. This results in large apparent errors in the response function values. Response functions to Little Spokane increase regardless of whether added stress is recharge or discharge, but seem to be insensitive to magnitude of the added stress. For this reason, response functions of Johnson et al. (2008) and Taylor et al. (2008) were developed with an unconfined model, reducing the errors for stresses in Hillyard Trough.

To a lesser degree, stress in the Trinity Trough area creates changes in aquifer thickness that redirect impacts of aquifer stress between the reach identified as the Spokane River between Avista Dam and Deep Creek and the reach above the Spokane Gage (Figures B1 and B3). Although response functions for this area are sensitive to changes in aquifer thickness, the area is very small and regardless of the errors, the effects of stress are felt on the Spokane River, it is just an issue of which reach is impacted.

Time-averaged response functions also show slight sensitivity to aquifer thickness in the vicinity of Lake Coeur d'Alene (Figure B7). These errors are small and may be a result of compounding effects of intermittent perching occurring in Lake Coeur d'Alene.

Aquifer thickness appears to have little effect on simulated responses of Lake Pend Oreille (Figure B9). A slight sensitivity appears along the northern edge of the aquifer and in the Chilco Channel where the aquifer is relatively thin (Figure 3).

Confined Conditions - Maximum Errors

Maximum errors express the magnitude of maximum error in response functions that can be expected at any given time. The maximum errors were determined for the 120 monthly stress periods simulated by the model for October 1995 through September 2005, consequently, dryer or wetter extremes than experienced during this period will likely cause greater errors in response functions than shown here. It should be noted, however, that response functions in SVRP are intended to represent system responses under long-term average conditions, not one specific month.

Generally, the maximum simulated errors during the 120 monthly stress periods were similar to those for time averaged effects, but slightly larger in magnitude (e.g. Figure B13 and B19). Effects were more often apparent near the aquifer perimeter where the aquifer is modeled as thinning. Greater effects are also apparent near Lake Coeur d'Alene where the intermittent nature of hydraulic connection causes dramatic changes in response between stress periods (Figure B19). This is most apparent in the Fully Linear model version (relative to the Nonlinear Version shown in Figure B19) since boundaries in the Fully Linear Version do not transition between perched and interconnected.

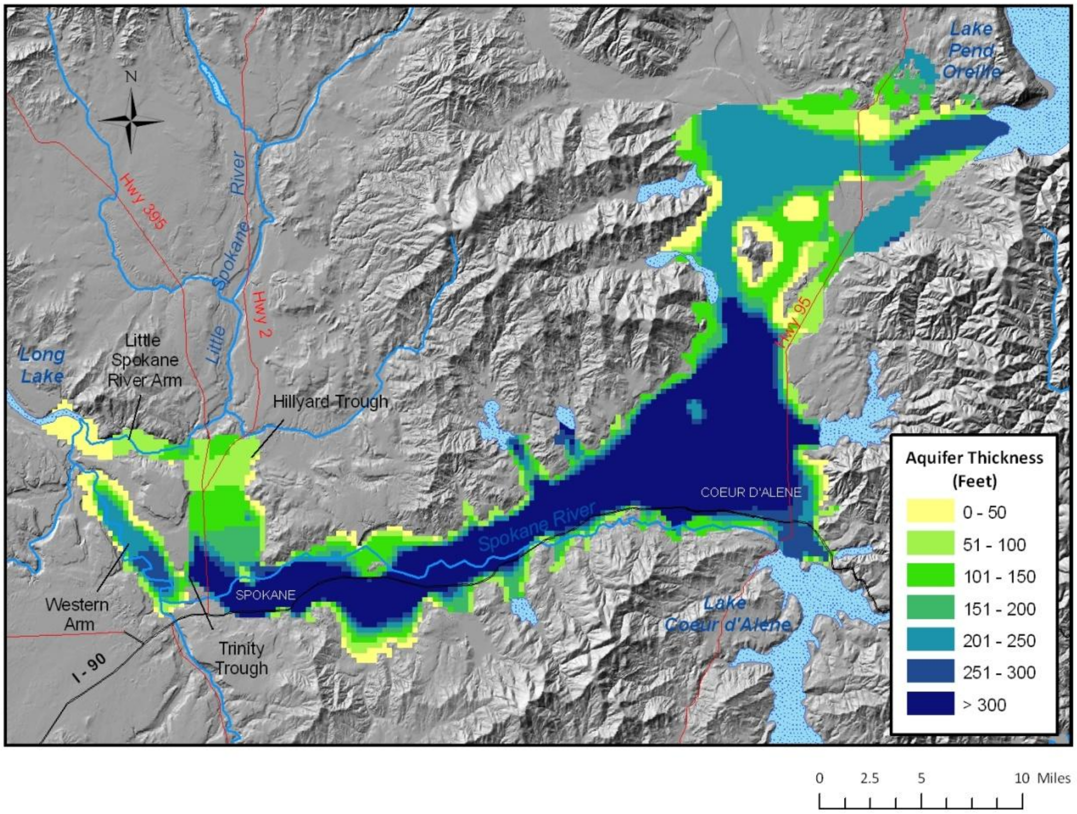

Figure 3. Average SVRP aquifer thickness for the period October 1995 through September 2005 for model layer 1.

Fully Linear Representation – Time Averaged Errors

The Fully Linear Model Version includes assumptions of constant aquifer thickness (as in the Confined Version) and also includes a linear representation of the aquifer boundaries. This may be consider the "entire error" resulting from imposing linear relationships on the aquifer model.

In general, the errors created by the Fully Linear Model Version are similar to, but sometimes slightly greater than the errors created by use of the Confined Model Version. This is evident from a comparison of figures B1 and B2. Since this version includes approximations that include both constant aquifer thickness and linear boundaries, the errors are expected to be slightly greater than the Confined Version. The similarity of errors from these two versions, however, suggests that in the SVRP most of the error in the time-averaged response functions is associated with the assumption of constant aquifer thickness rather than the linearization of the boundaries.

Fully Linear Representation – Maximum Errors

The maximum errors encountered during the 120 stress period simulation were similar to the time-averaged errors, with the greatest differences apparent in the response of Lake Coeur d'Alene. This is likely the result of proximity to the intermittently perched cells representing the boundary with Lake Coeur d'Alene.

Conclusions

During the development of response functions for the SVRP aquifer, it was determined that an unexpected sensitivity to nonlinearity induced by aquifer thickness changes occurred in the Hillyard Trough area. This sensitivity was managed by developing the response functions from a time-variant thickness model (Johnson et al., 2008b), and by cautioning users of the response functions (Taylor et al., 2008). The extreme sensitivity, however, also justified further evaluation resulting in this report.

The results of this evaluation are consistent with the linearity conclusions of the earlier reports by Johnson et al. (2008) and Taylor et al. (2008). Response functions developed for the SVRP are valid to represent the effects of moderate aquifer stresses under recharge and discharge conditions that are similar to the average conditions during the 1995 through 2005 period. The possible exception is for aquifer stress at the north end of Hillyard Trough, near Lake Coeur d'Alene, and near the margins of the SVRP where the aquifer is thin. The maximum errors presented in Appendix B indicate response functions for stresses throughout most of the basin are also valid under a wide range of recharge and discharge conditions.

In the SVRP aquifer model, Lake Coeur d' Alene is represented with intermittent perching conditions. This situation appears to create errors for response functions at specific points in time when the lake is either perched or fully connected with the aquifer. The time-averaged response function errors, those considered most appropriate in assessing validity for use, are not substantial. The cause of the errors appears to be associated with the nonlinearity of the boundary conditions.

In the Hillyard Trough area, even minor changes in aquifer thickness can create significant differences in the responses simulated in the Little Spokane River and in the Spokane River above the Spokane Gage. This sensitivity creates concern in use of response functions for stresses in this area for two reasons: 1) the nonlinear model response is not consistent with the assumptions of response functions, and 2) the extreme sensitivity may mean that even small errors in model development and parameterization may have a significant impact on response estimates. It is hypothesized that the thickness sensitivity in the Hillyard Trough results from a trend in aquifer thickness from thinner near Little Spokane River to thicker near the Spokane River. This hypothesis will be evaluated in greater detail in a subsequent publication.

Response function errors described in the report are the result of the imposition of linear relationships on a moderately nonlinear simulation model. The errors presented do not include errors inherent in the original SVRP aquifer model developed by Hsieh et al. (2007) that result from flawed conceptual understanding, discretization error, parameterization error, and numerical error.

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Appendix A

Response Function Maps for

Time-Averaged Responses for the Nonlinear, Confined, and Fully Linear Model Versions

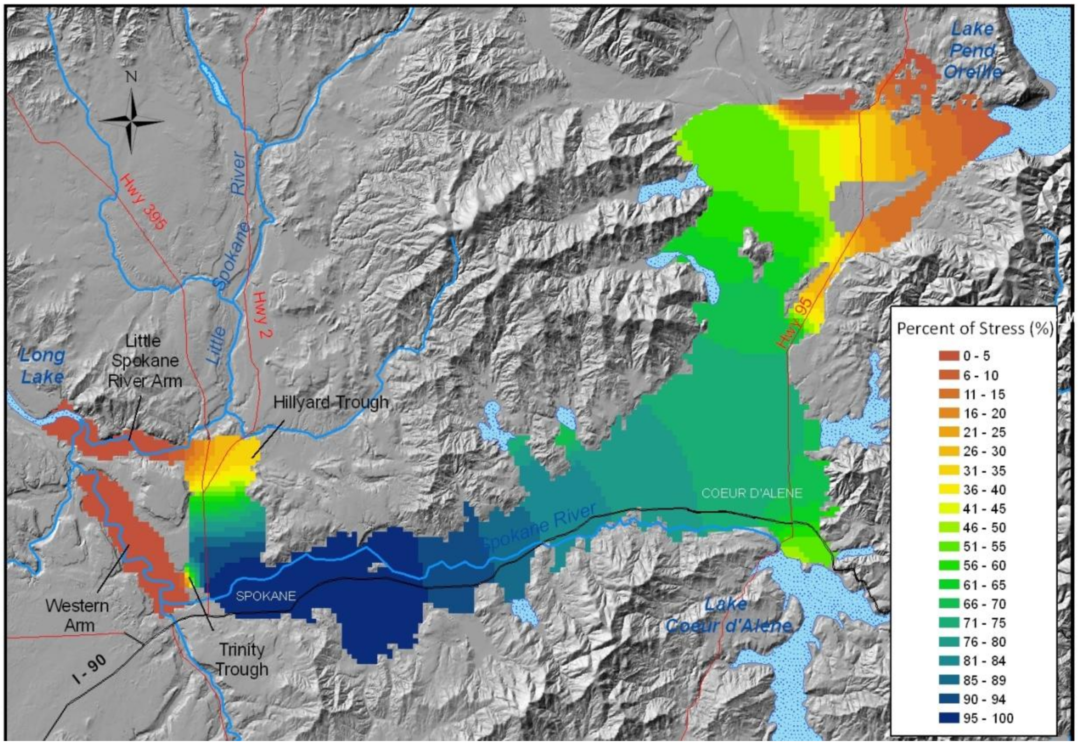

Figure A1. Responses of Spokane River above the Spokane Gage using the Non-linear model

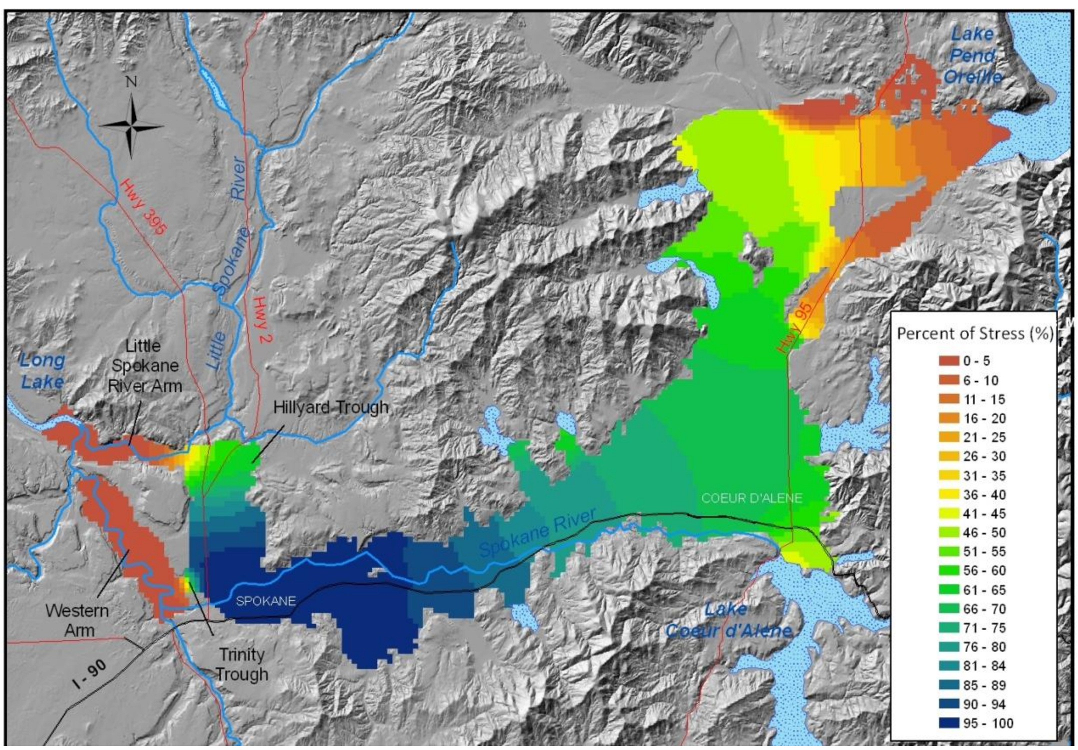

Figure A2. Responses of Spokane River above the Spokane Gage using the Confined model

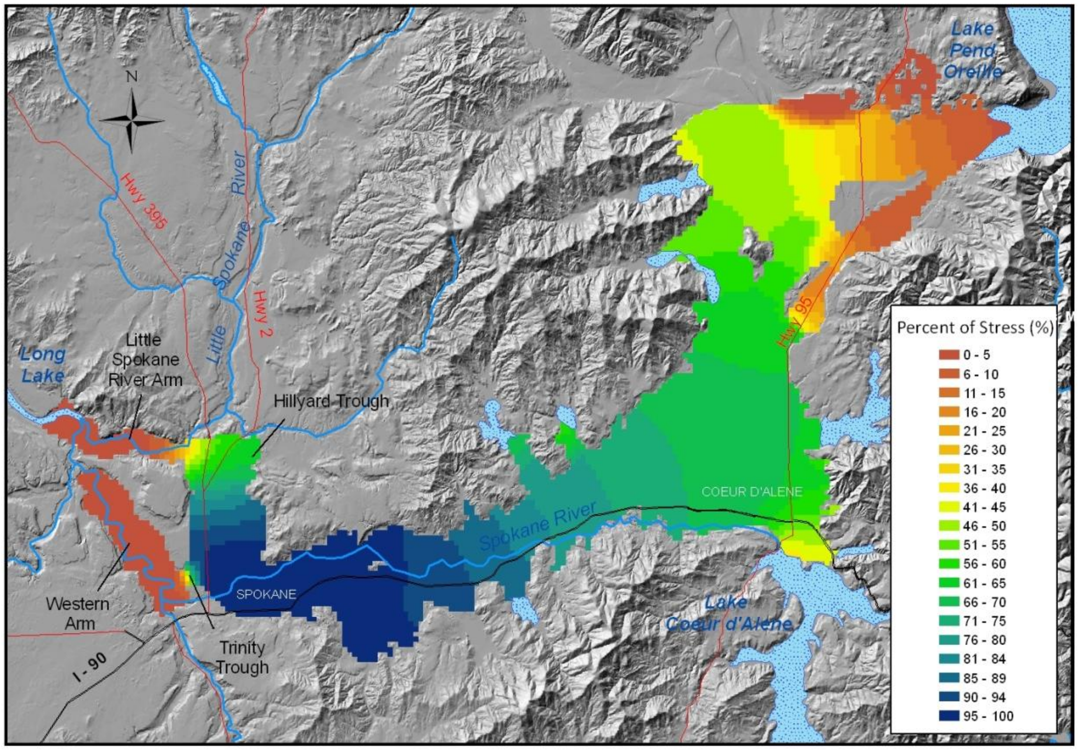

Figure A3. Responses of Spokane River above the Spokane Gage using the Fully Linear model

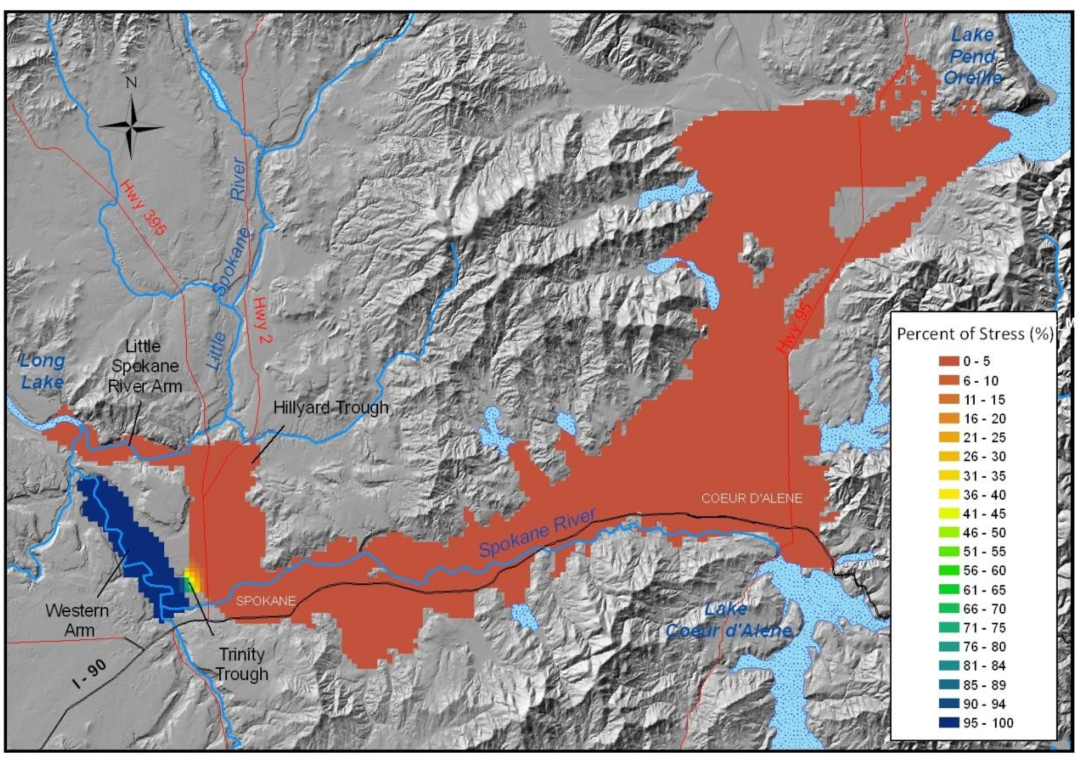

Figure A4. Responses of Spokane River between Avista Dam and Deep Creek using the Nonlineary model

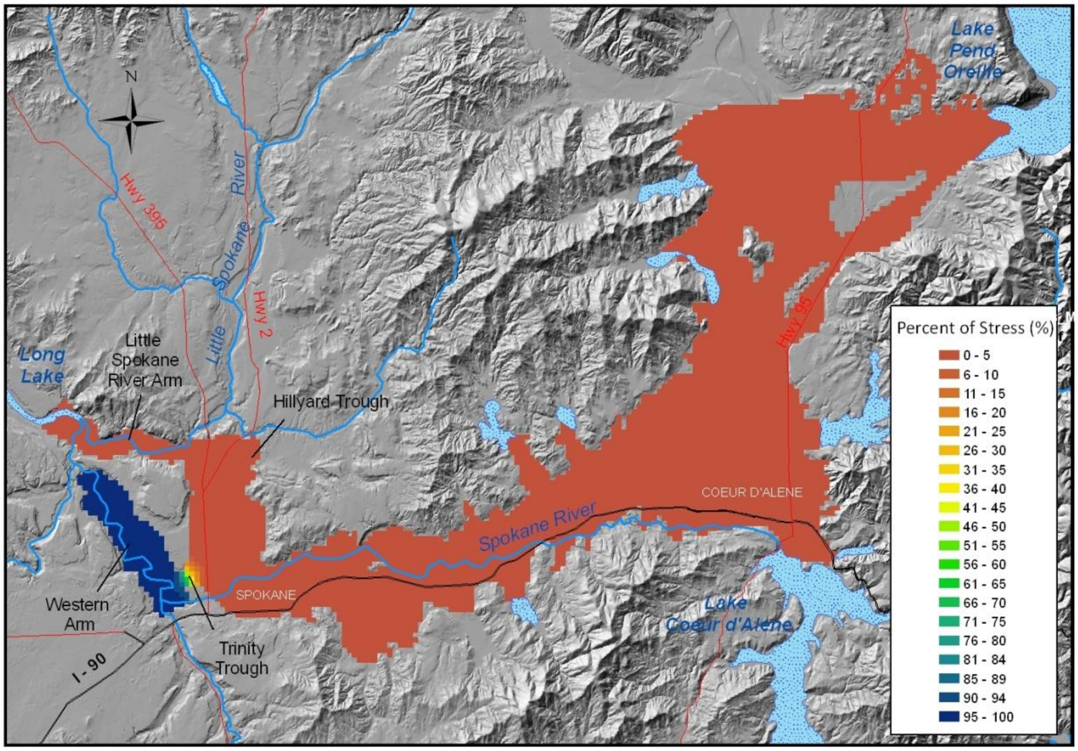

Figure A5. Responses of Spokane River between Avista Dam and Deep Creek using the Confined model

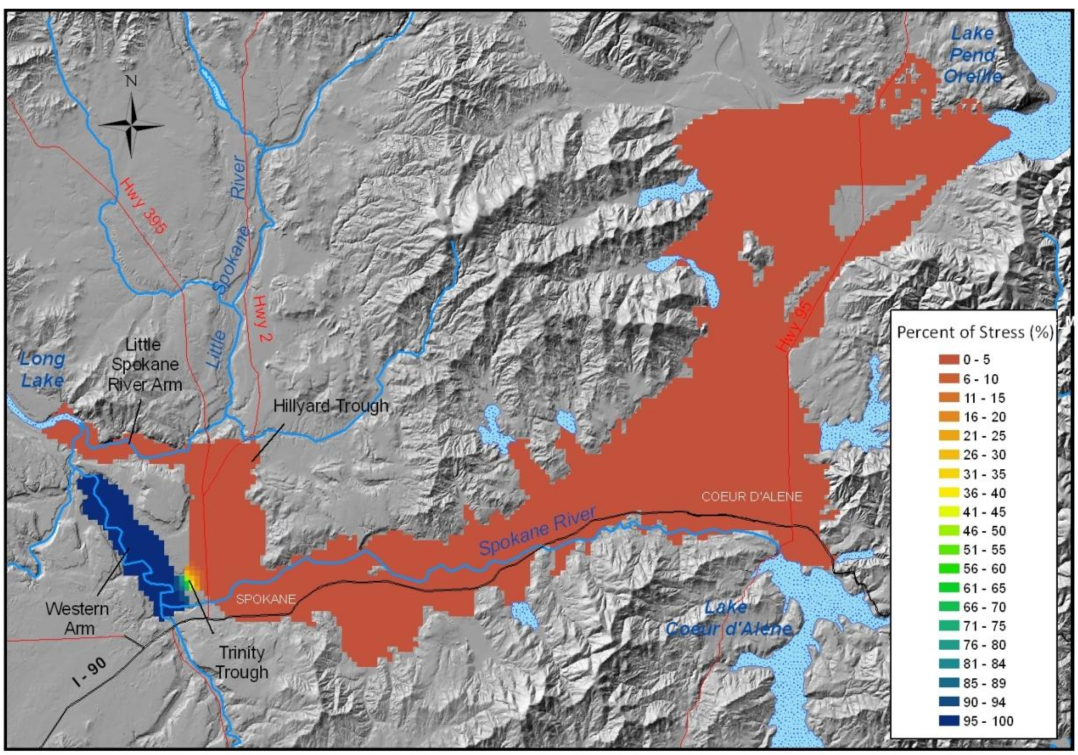

Figure A6. Responses of Spokane River between Avista Dam and Deep Creek using the Fully Linear model

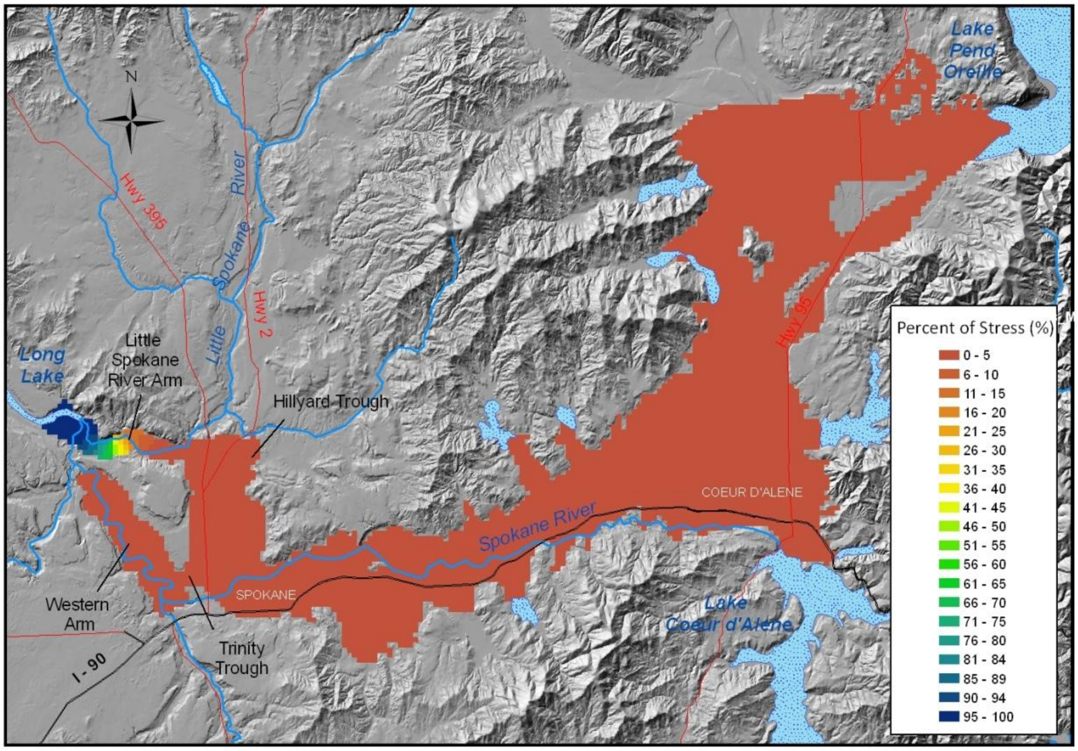

Figure A7. Responses of Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Nonlinear model

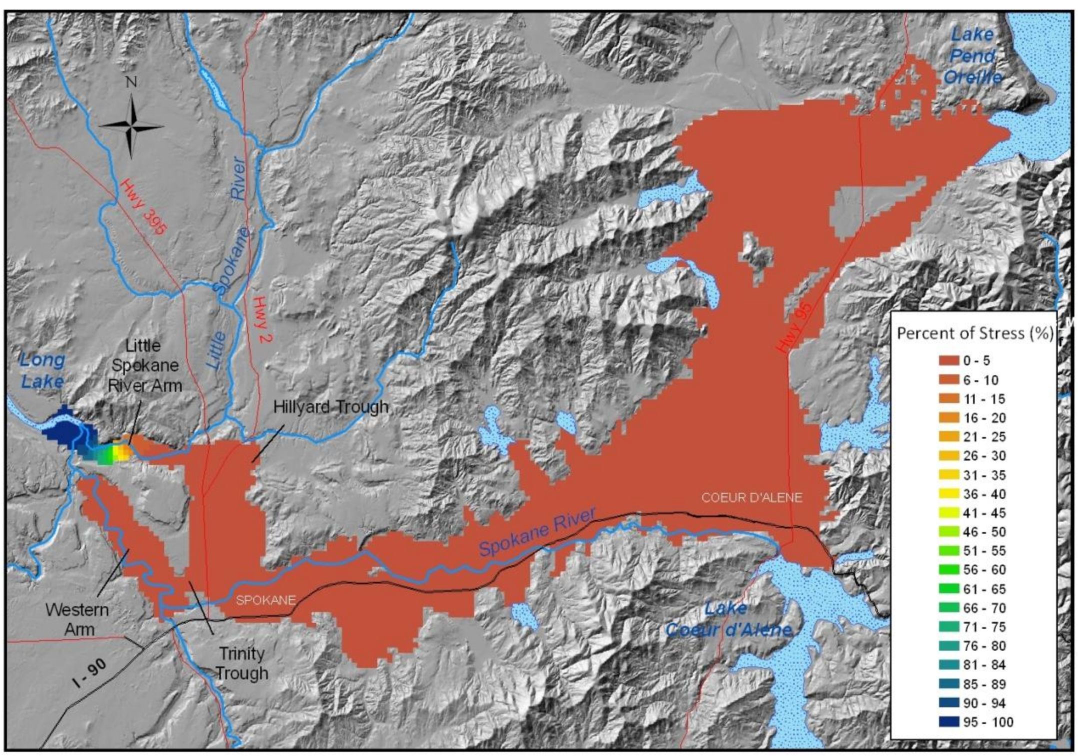

Figure A8. Responses of Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Confined model

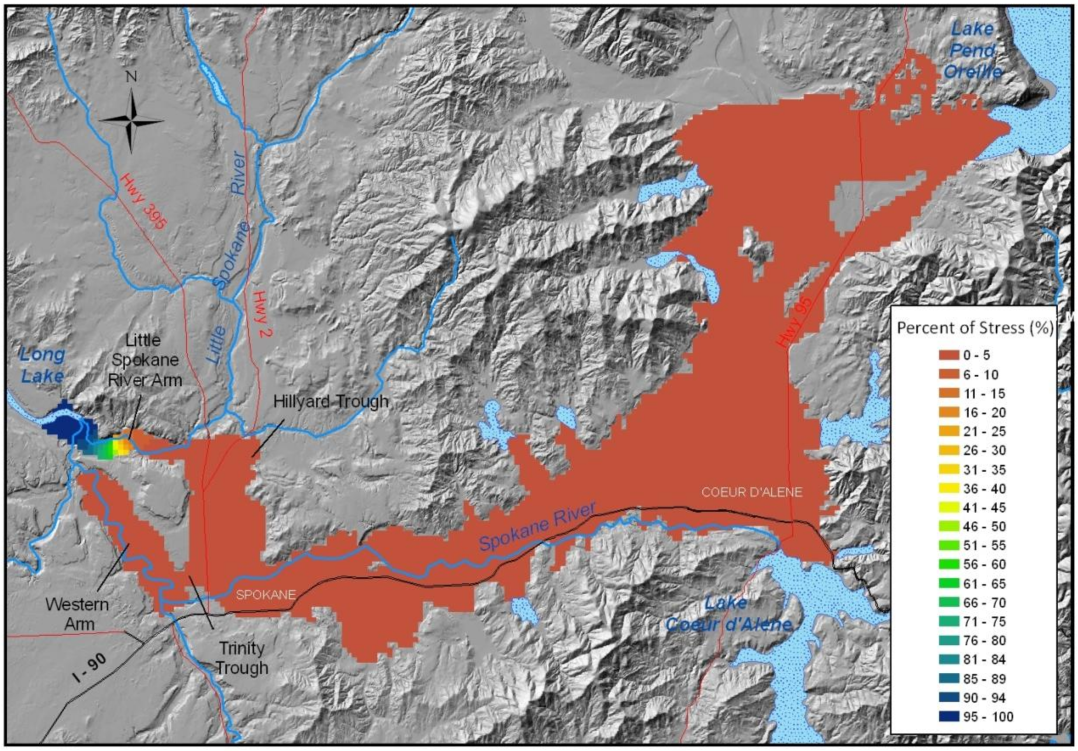

Figure A9. Responses of Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Fully Linear model

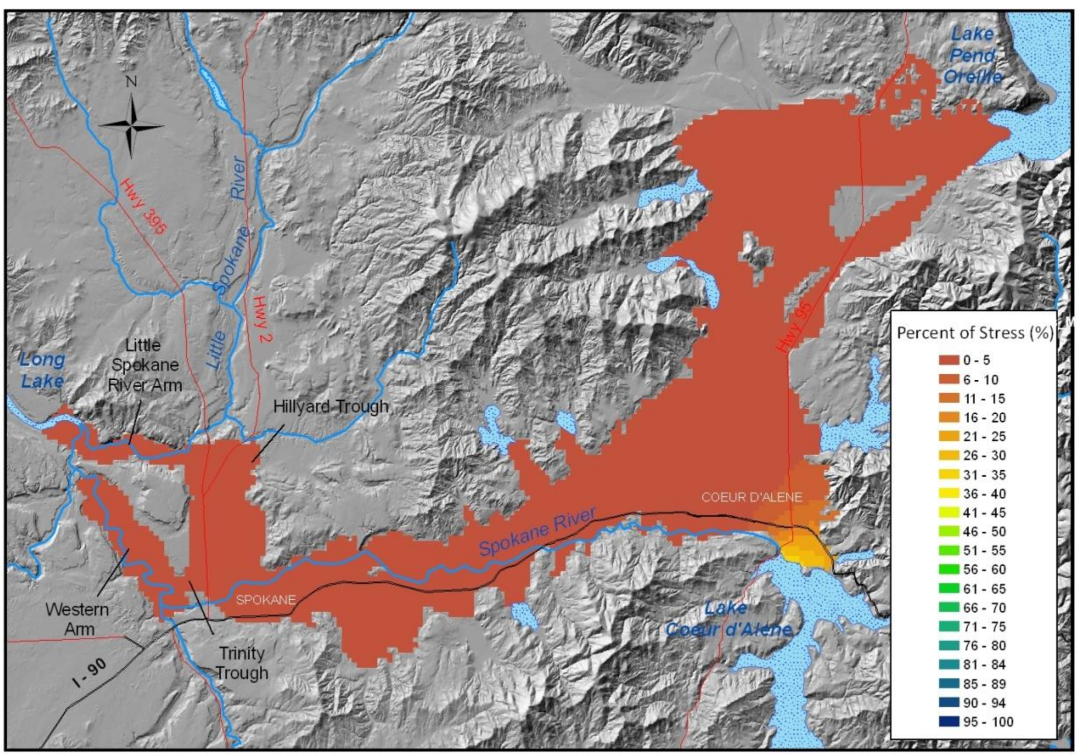

Figure A10. Responses of Lake Coeur d'Alene using the Nonlinear model

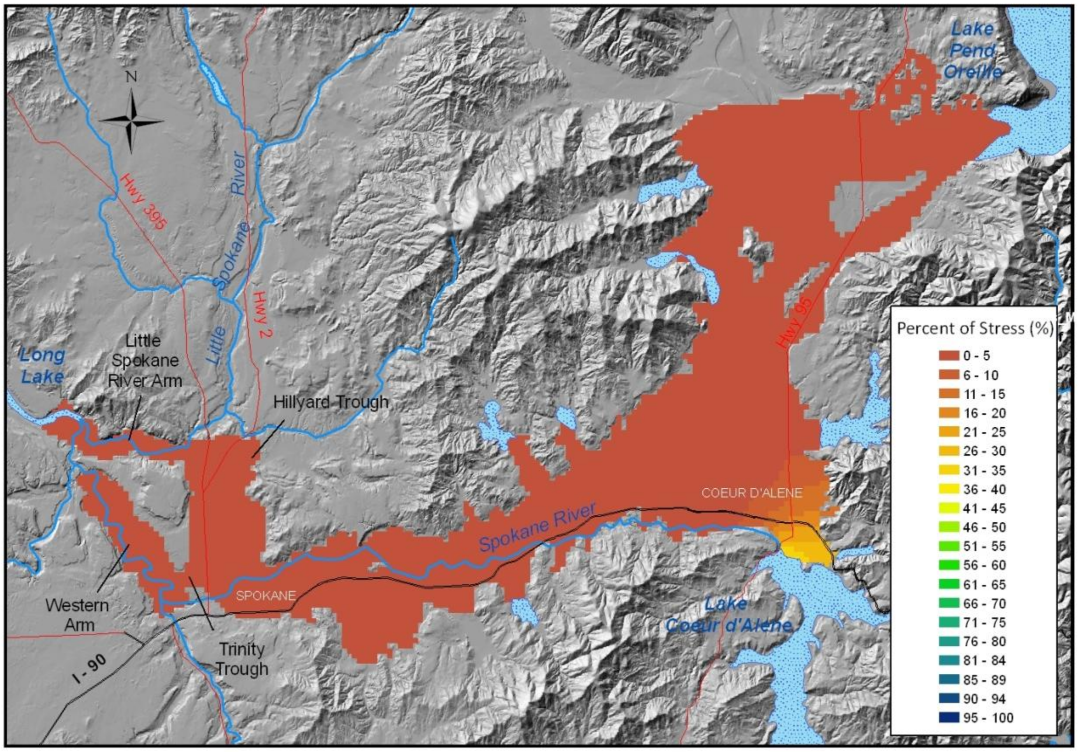

Figure A11. Responses of Lake Coeur d'Alene using the Confined model

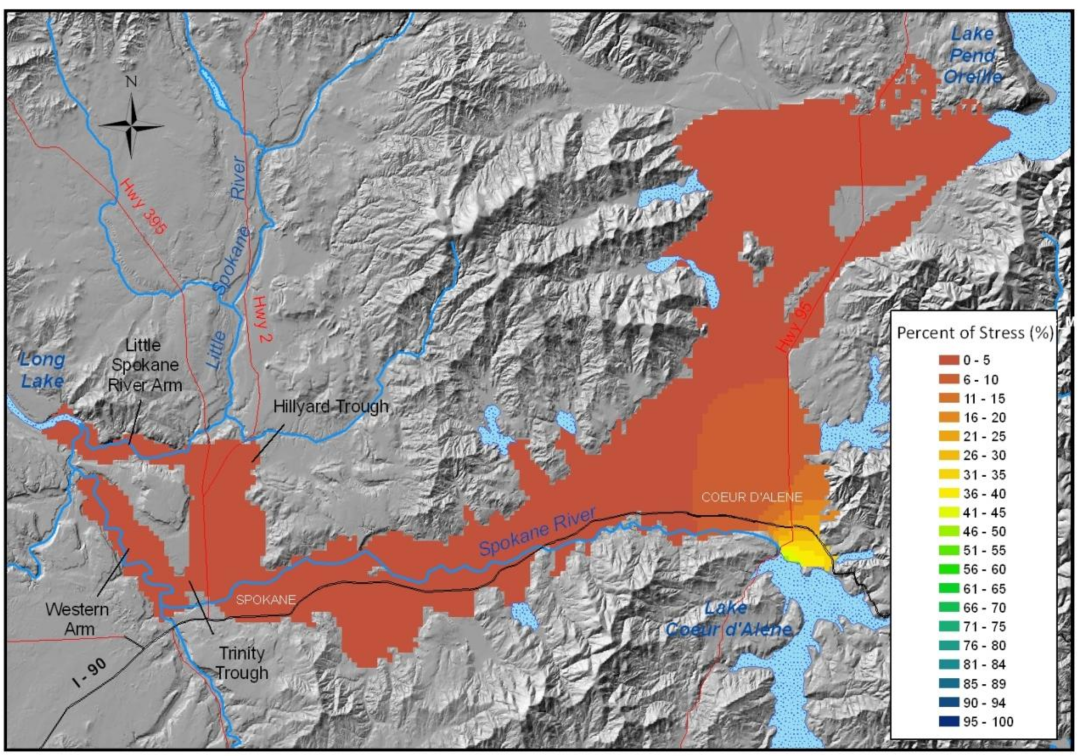

Figure A12. Responses of Lake Coeur d'Alene using the Fully Linear model

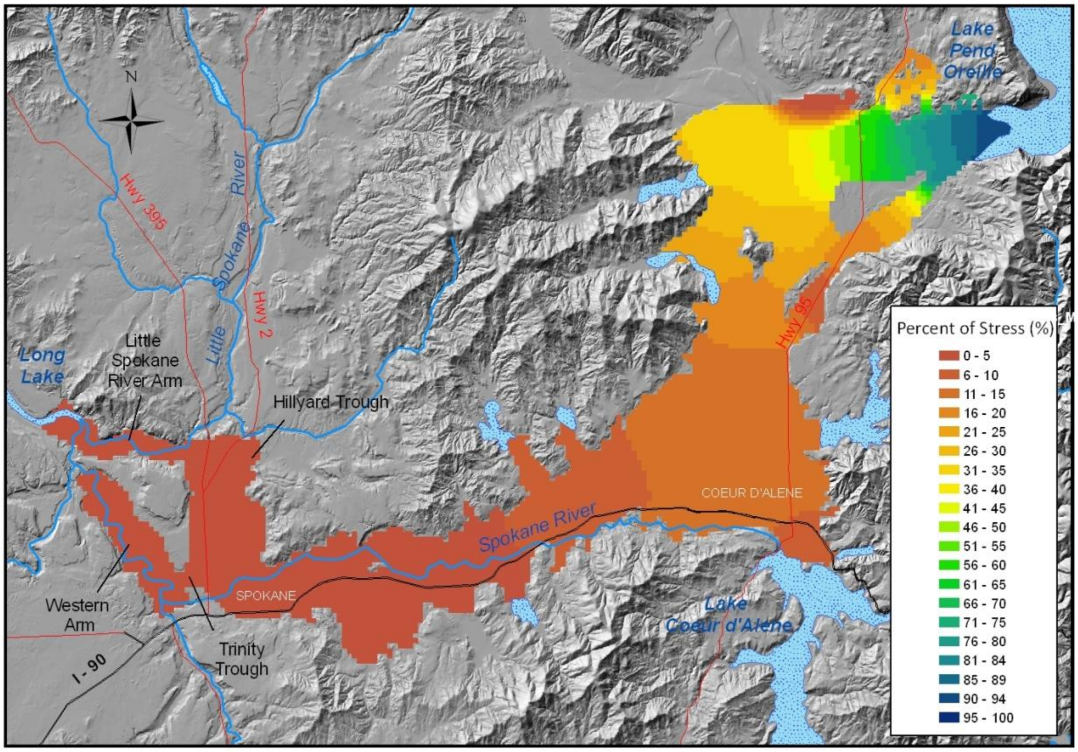

Figure A13. Responses of Lake Pend Oreille using the Nonlinear model

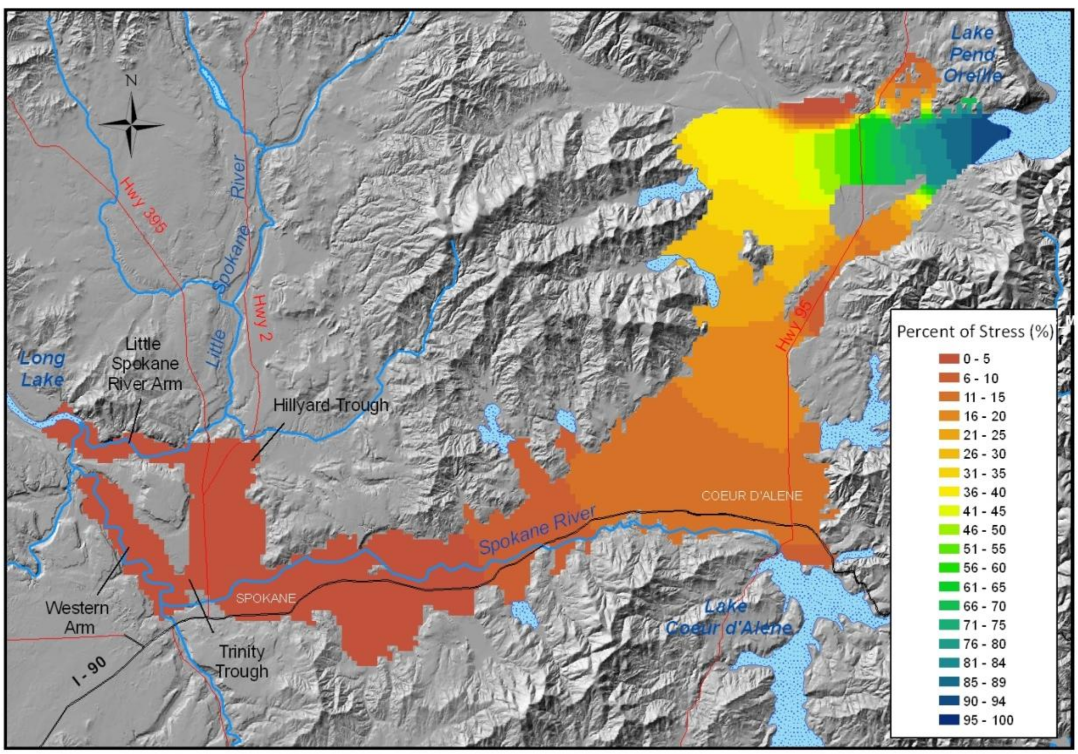

Figure A14. Responses of Lake Pend Oreille using the Confined model

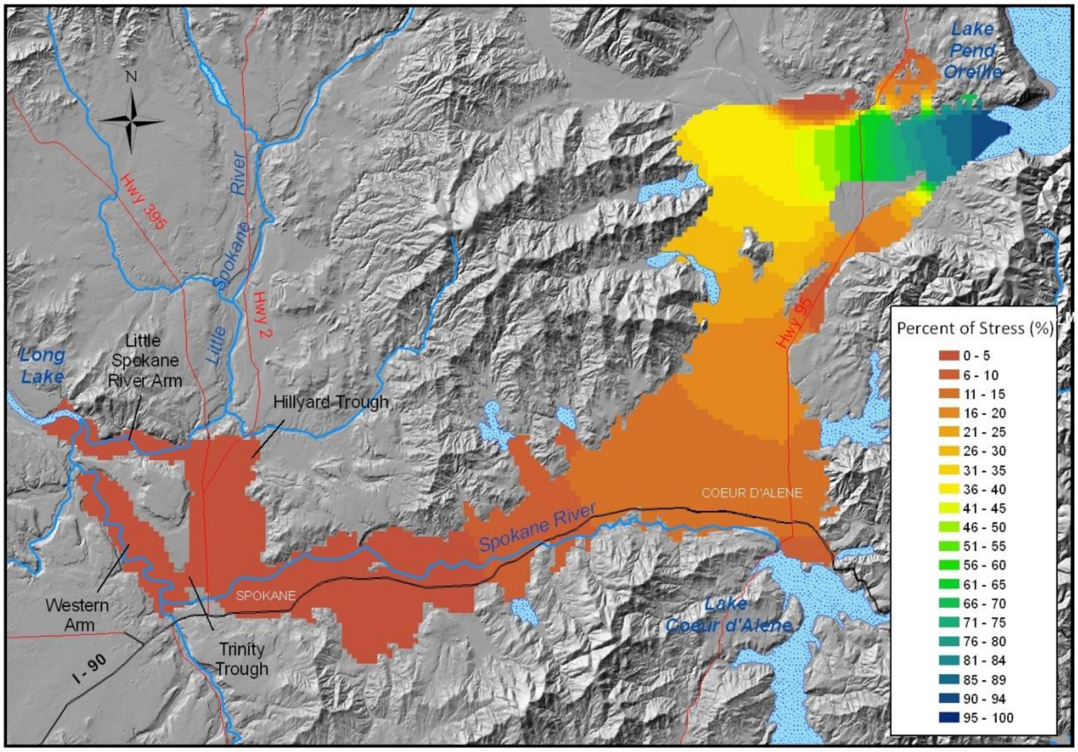

Figure A15. Responses of Lake Pend Oreille using the Fully Linear model

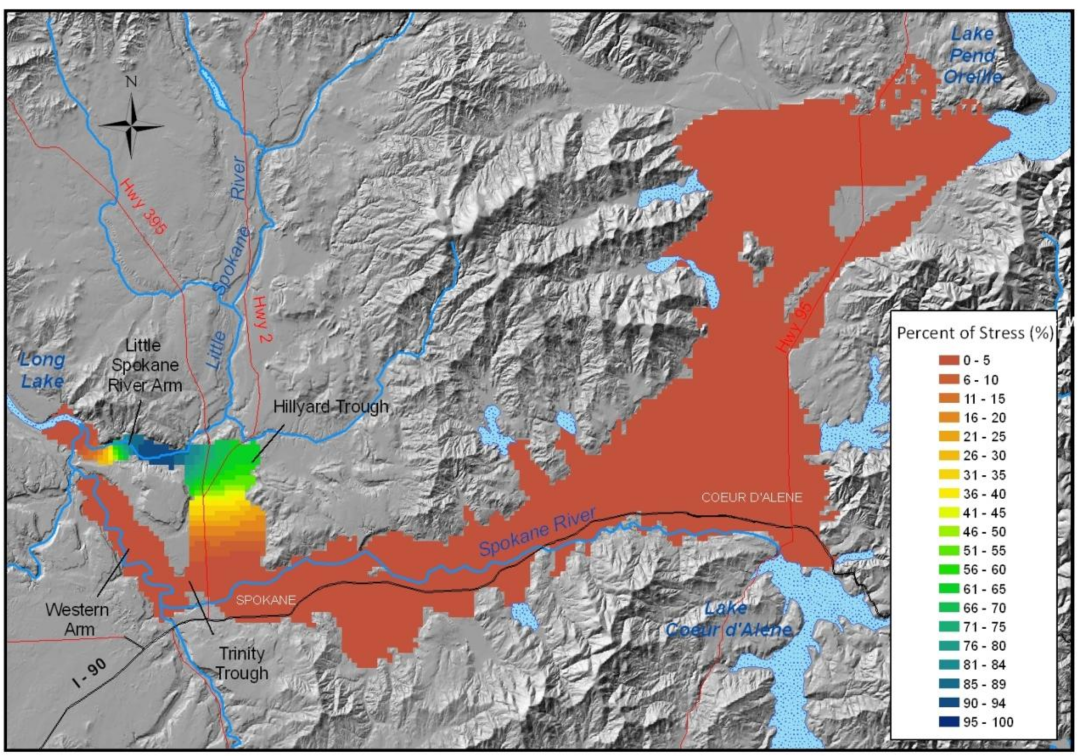

Figure A16. Responses of Little Spokane River above Painted Rocks gage using the Nonlinear model

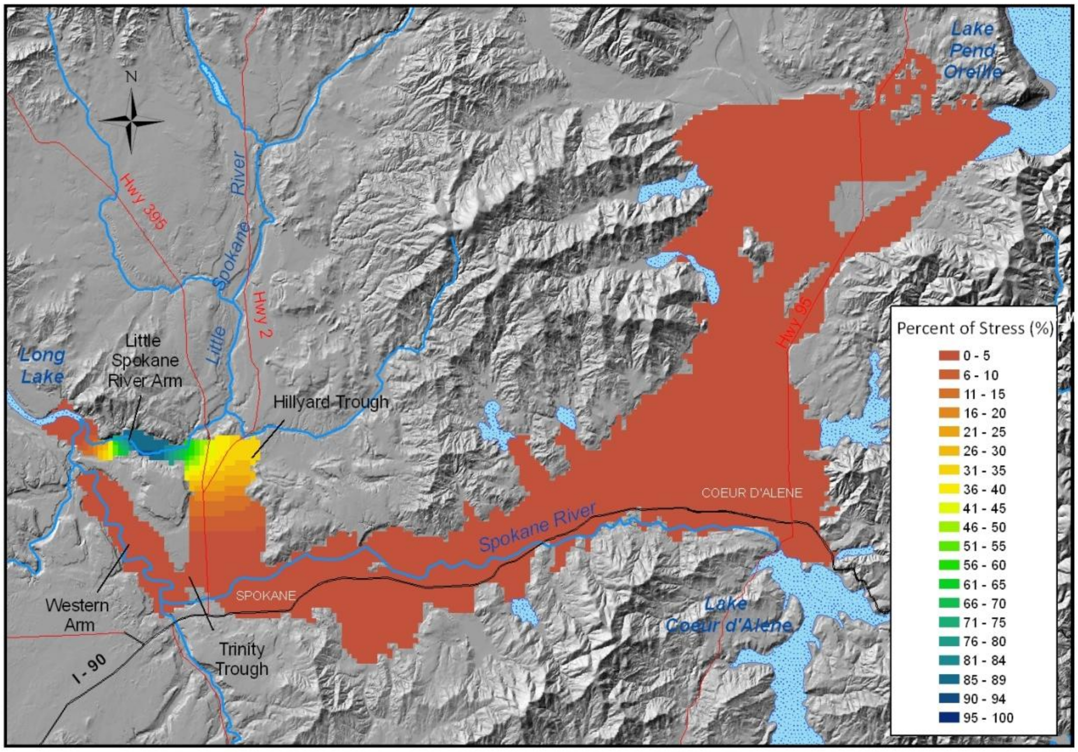

Figure A17. Responses of Little Spokane River above Painted Rocks gage using the Confined model

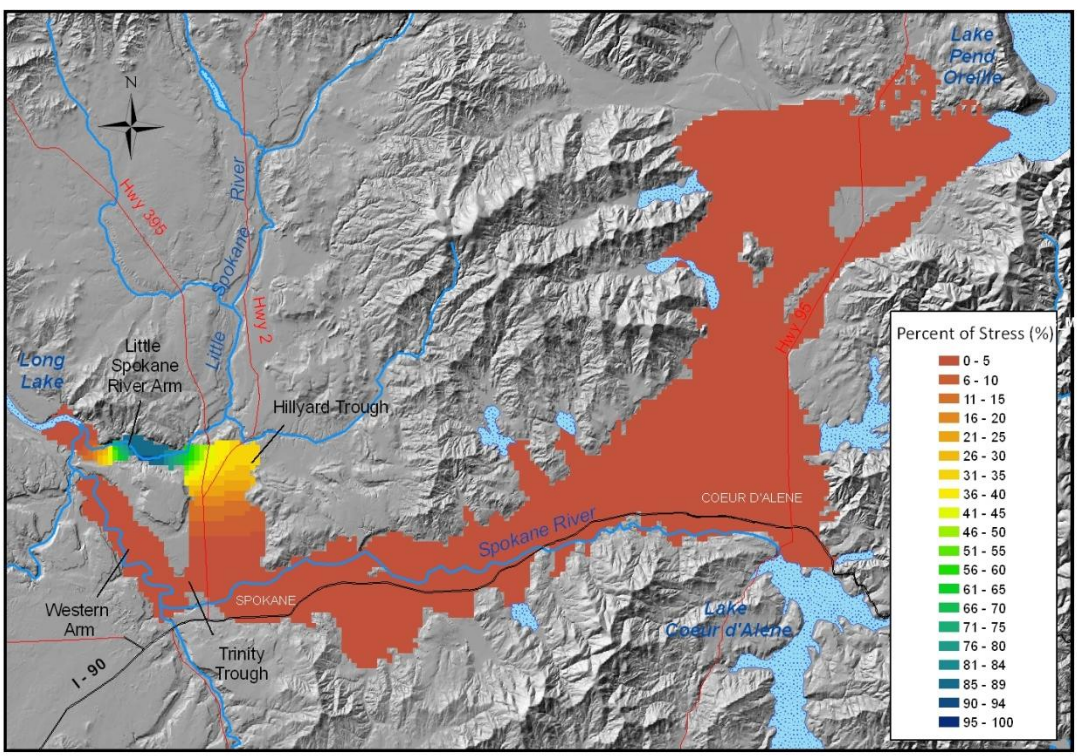

Figure A18. Responses of Little Spokane River above Painted Rocks gage using the Fully Linear model

Appendix B

Time-Averaged Errors With Linearized Model Versions

And

Maximum Errors with Linearized Model Version

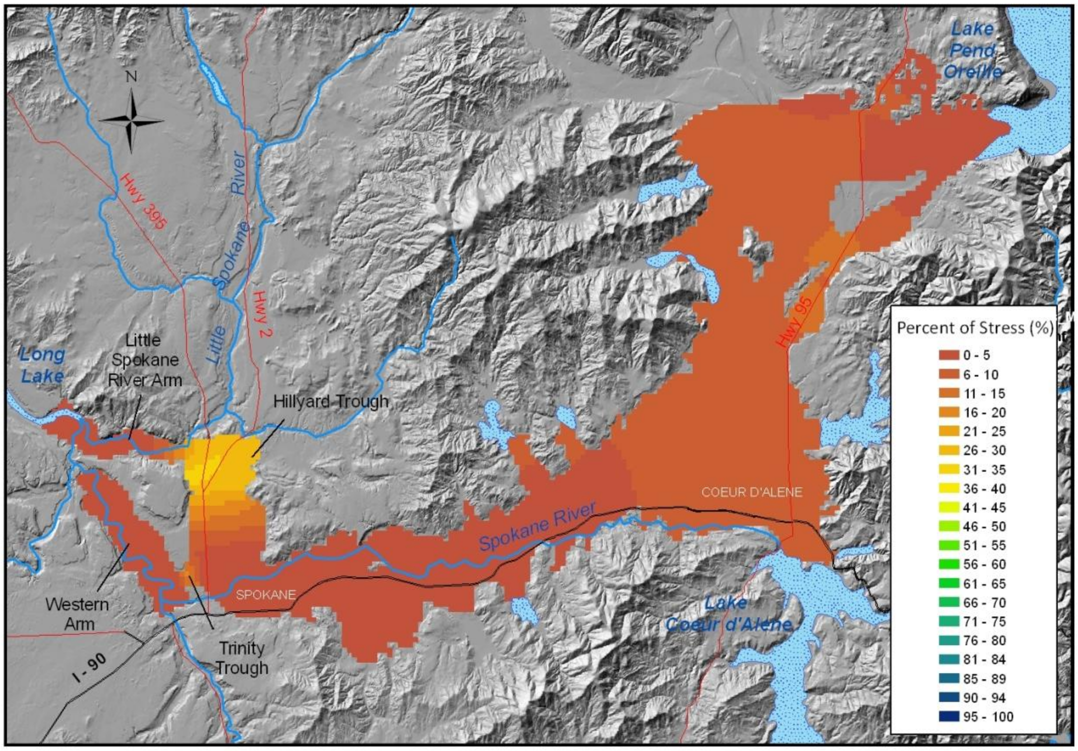

Figure B1. Response errors in Spokane River above the Spokane Gage using the Confined model

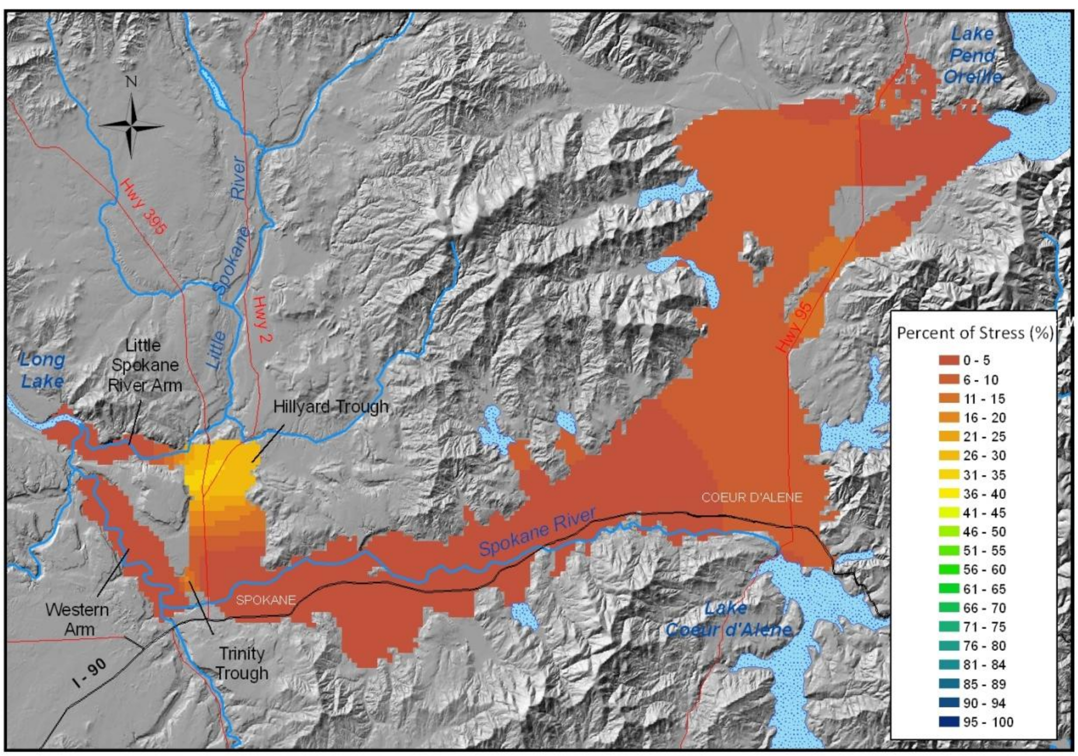

Figure B2. Response errors in Spokane River above the Spokane Gage using the Fully Linear model

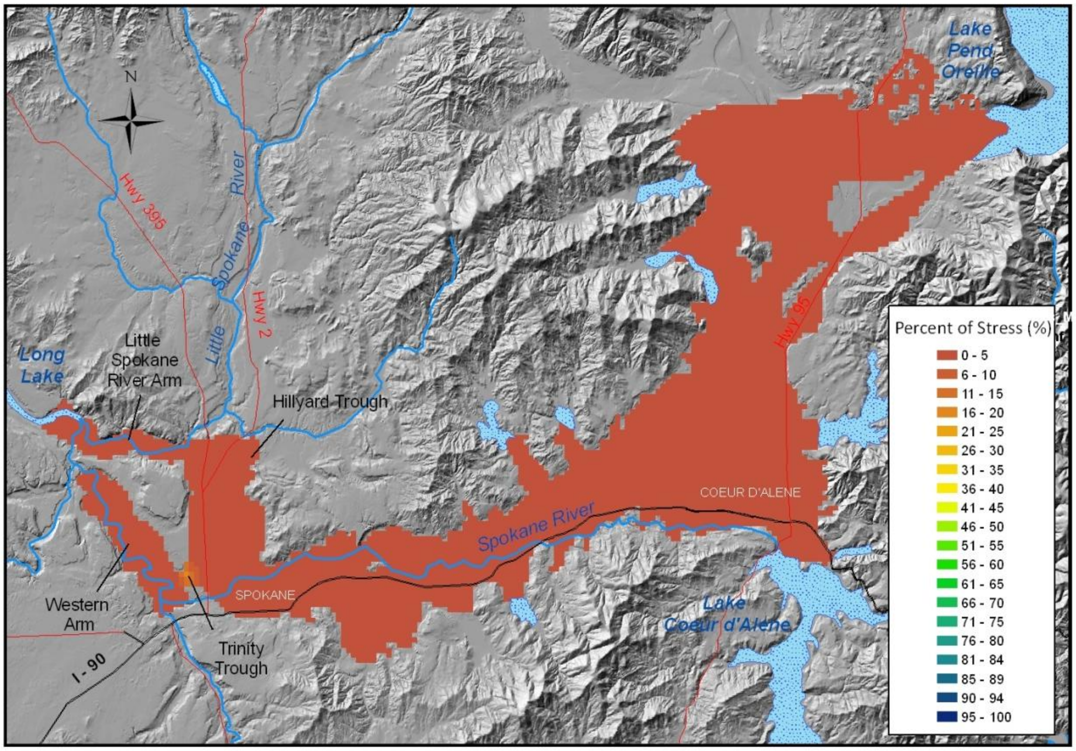

Figure B3. Response errors in Spokane River between Avista Dam and Deep Creek using the Confined model

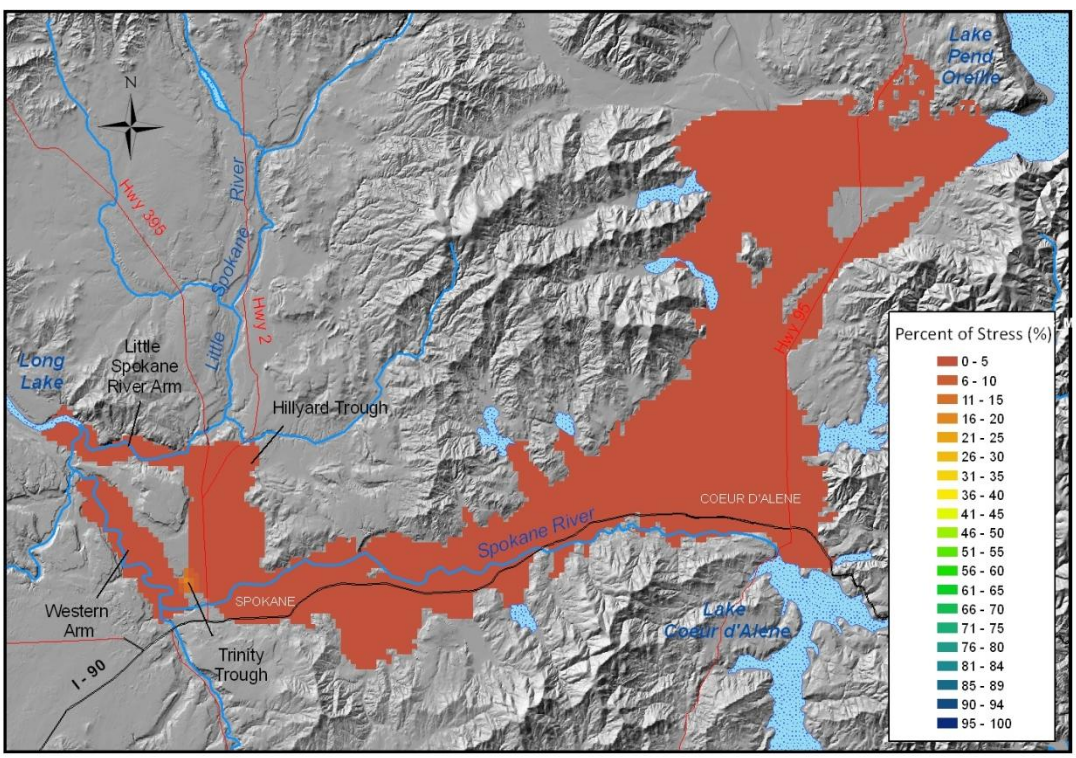

Figure B4. Response errors in Spokane River between Avista Dam and Deep Creek using the Fully Linear model

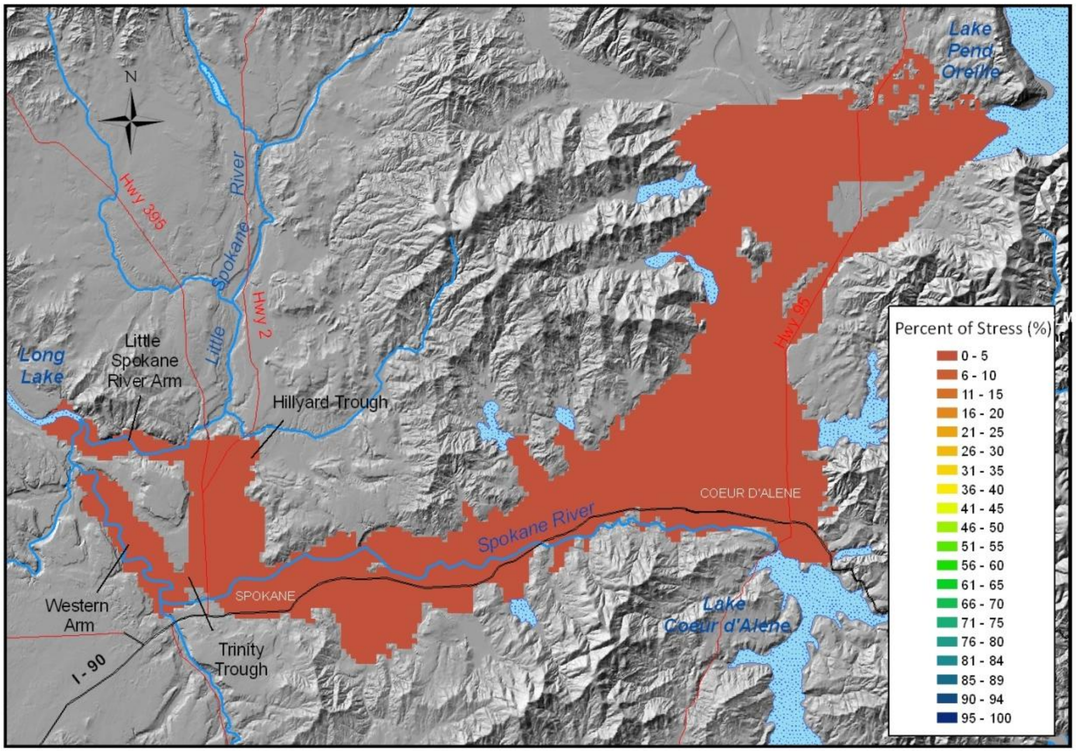

Figure B5. Response errors in Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Confined model

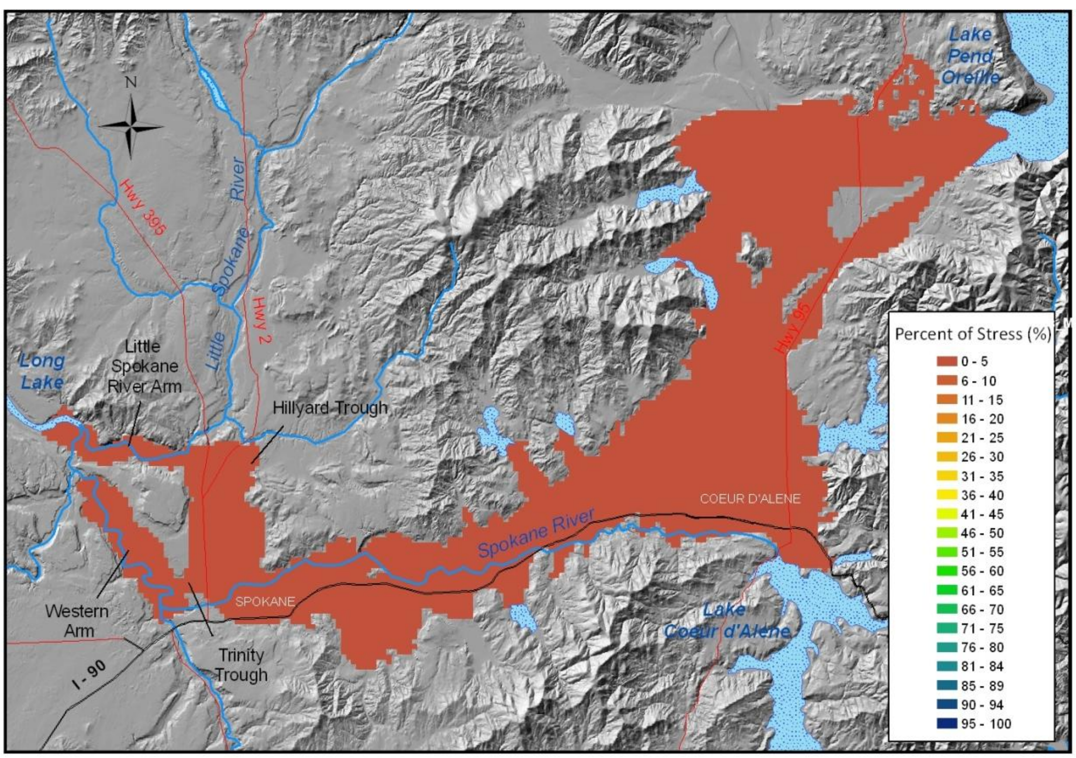

Figure B6. Response errors in Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Fully Linear model

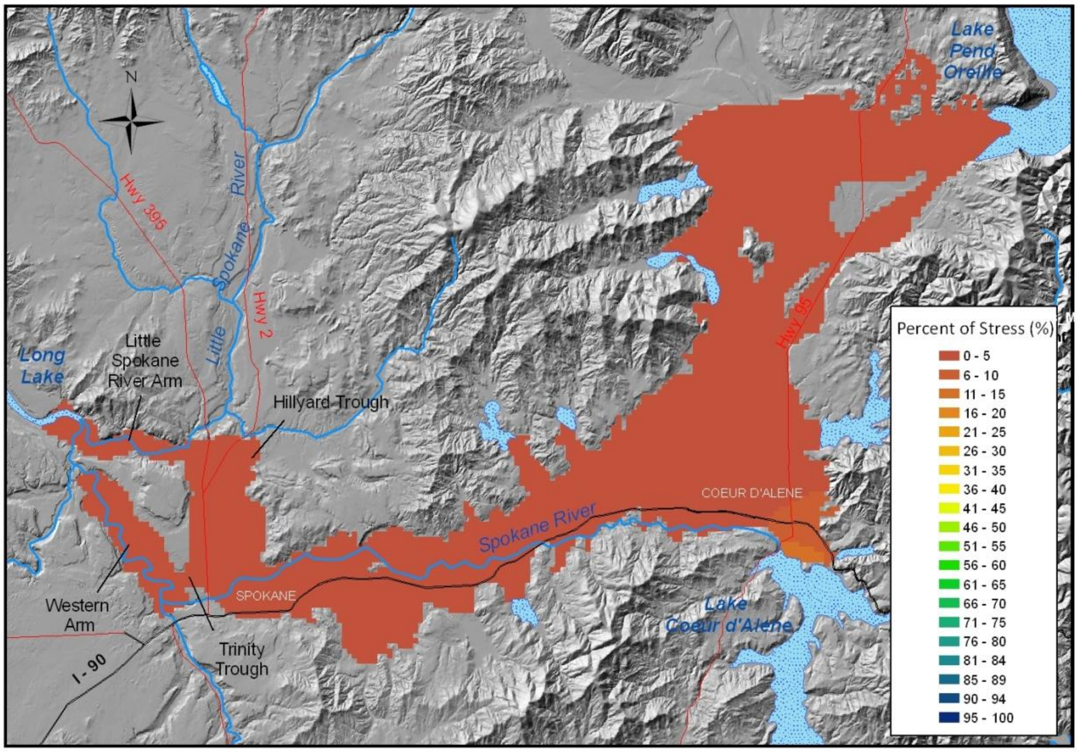

Figure B7. Response errors in Lake Coeur d'Alene using the Confined model

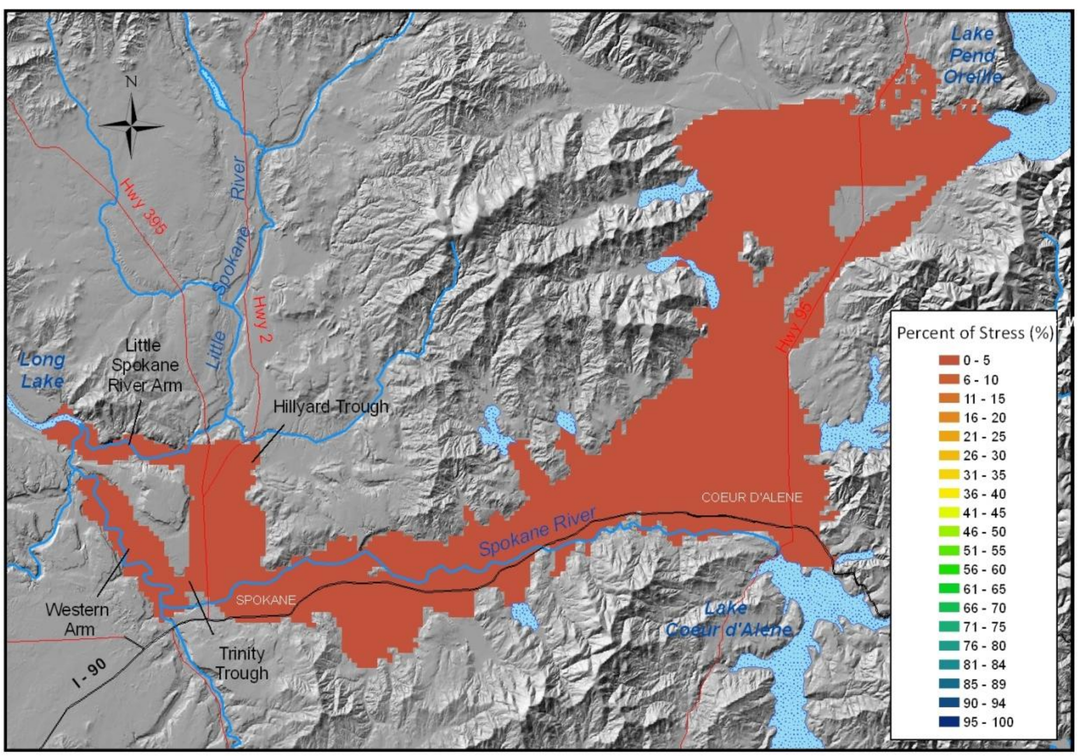

Figure B8. Response errors in Lake Coeur d'Alene using the Fully Linear model

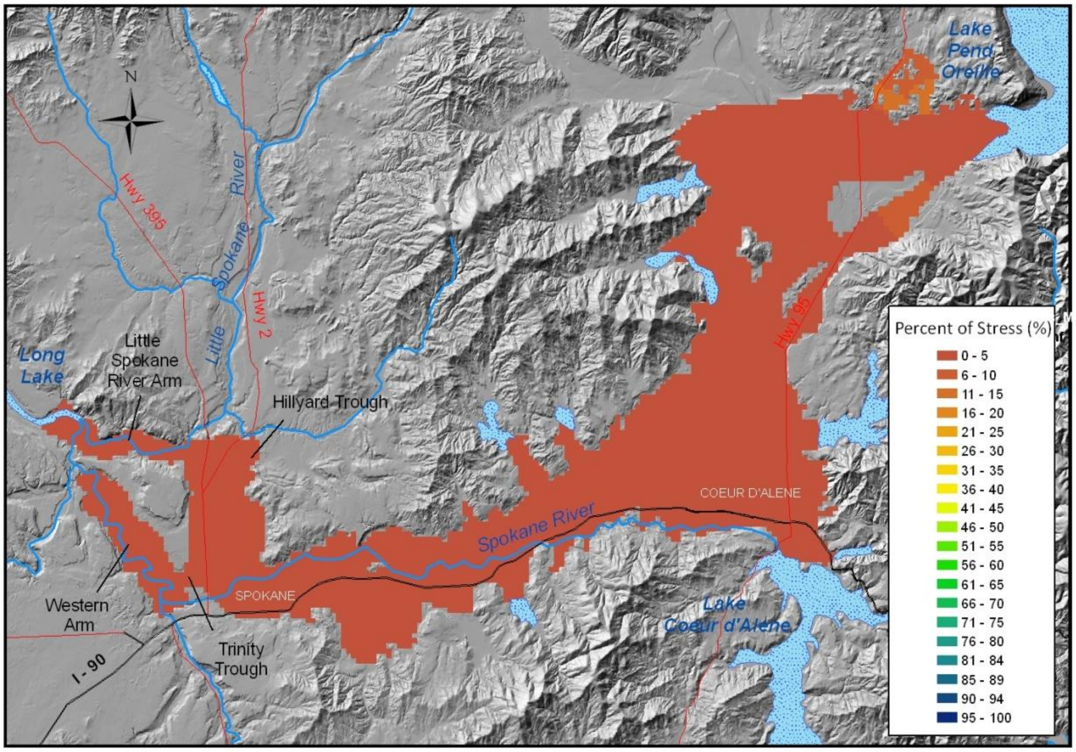

Figure B9. Response errors in Lake Pend Oreille using the Confined model

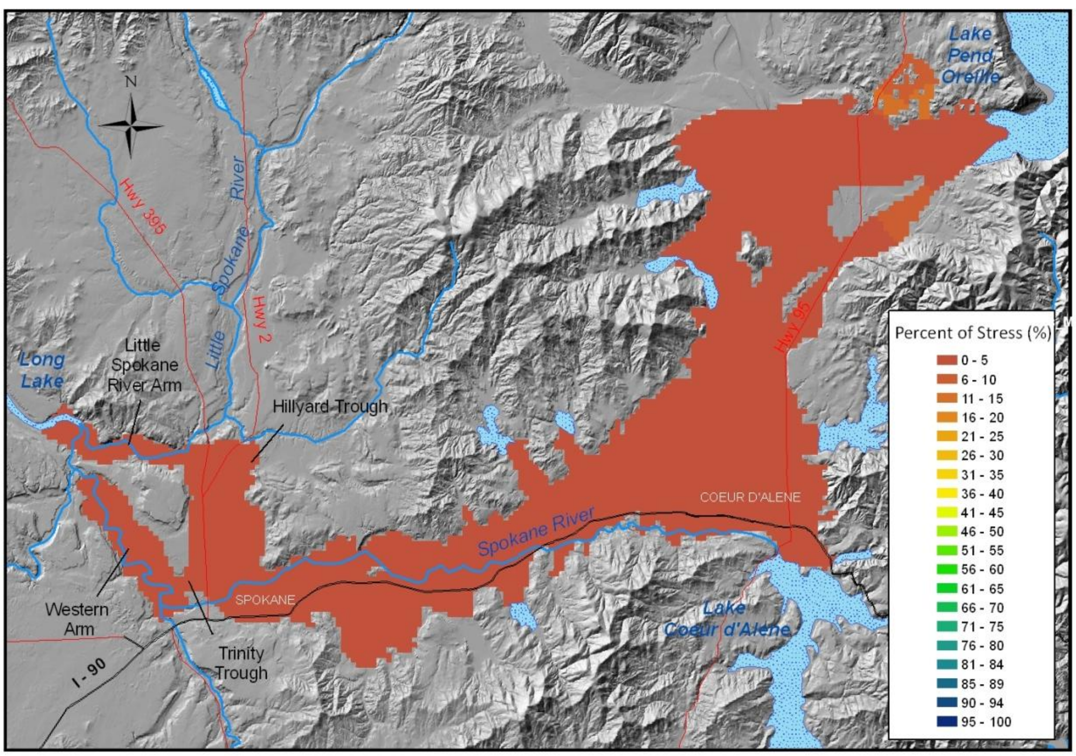

Figure B10. Response errors in Lake Pend Oreille using the Fully Linear model

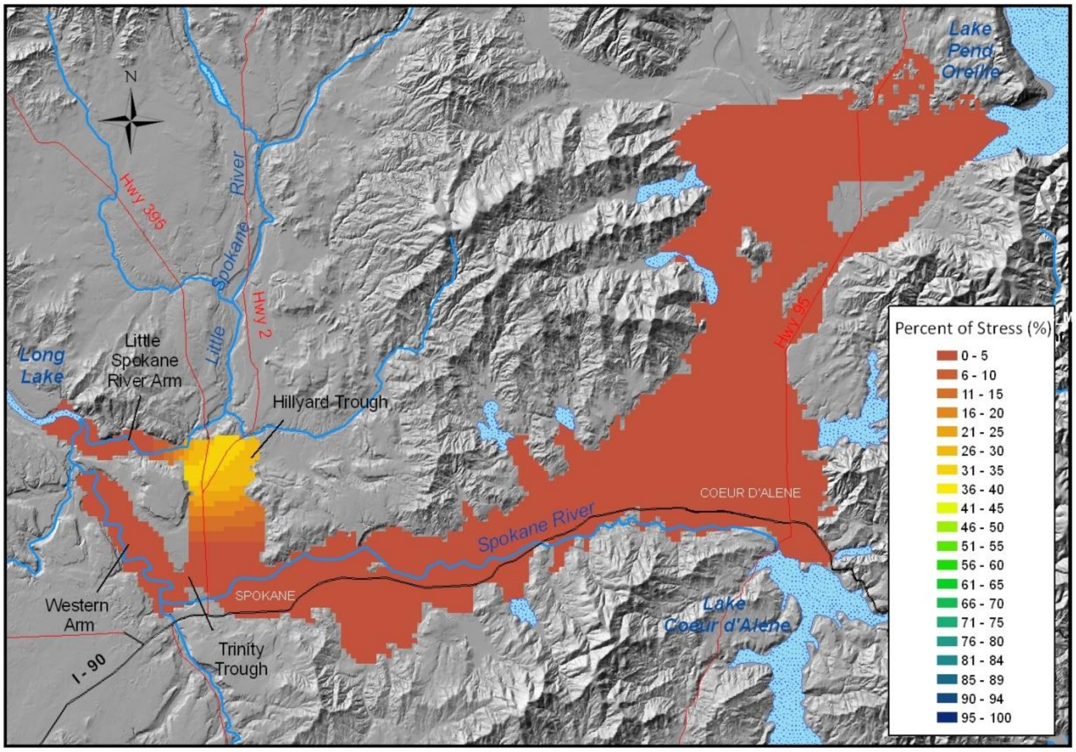

Figure B11. Response errors in Little Spokane River above Painted Rocks gage using the Confined model

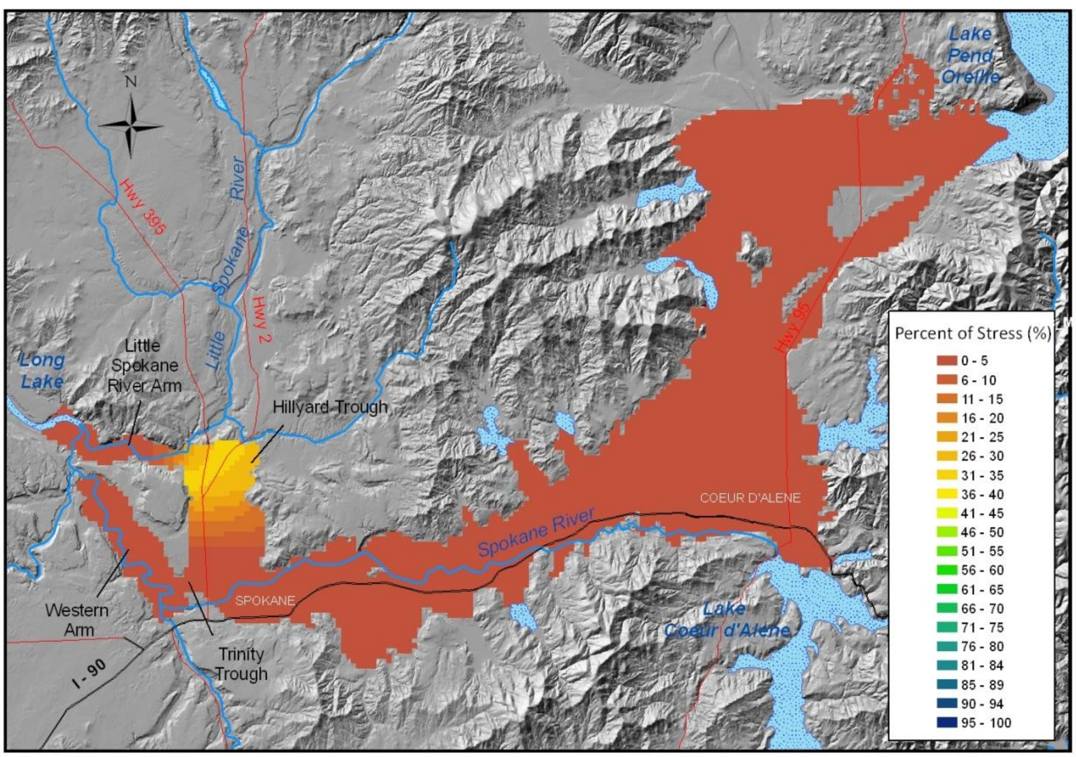

Figure B12. Response errors in Little Spokane River above Painted Rocks gage using the Fully Linear model

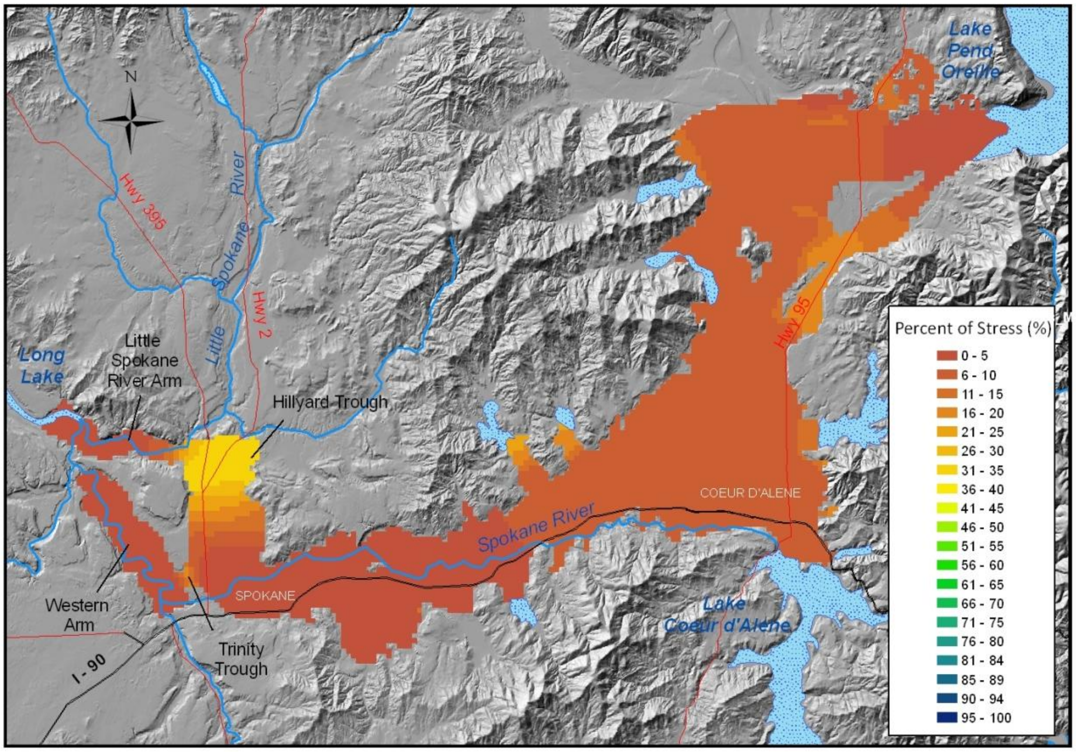

Figure B13. Maximum response errors in Spokane River above the Spokane Gage using the Confined model

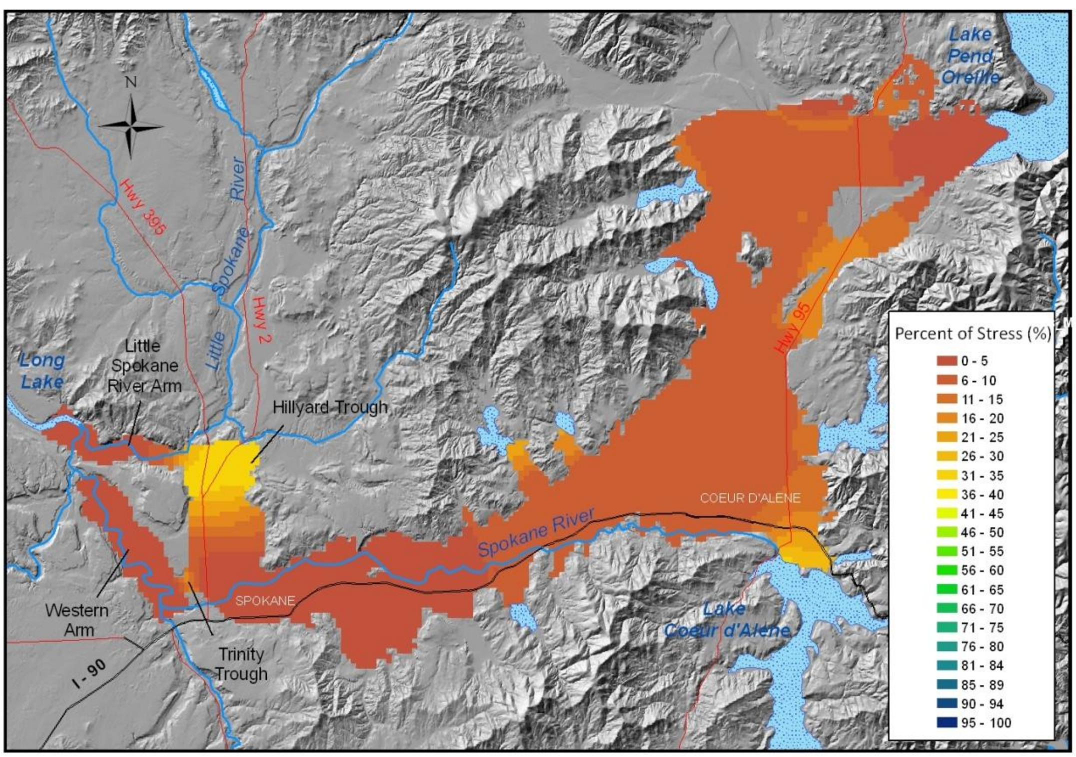

Figure B14. Maximum response errors in Spokane River above the Spokane Gage using the Fully Linear model

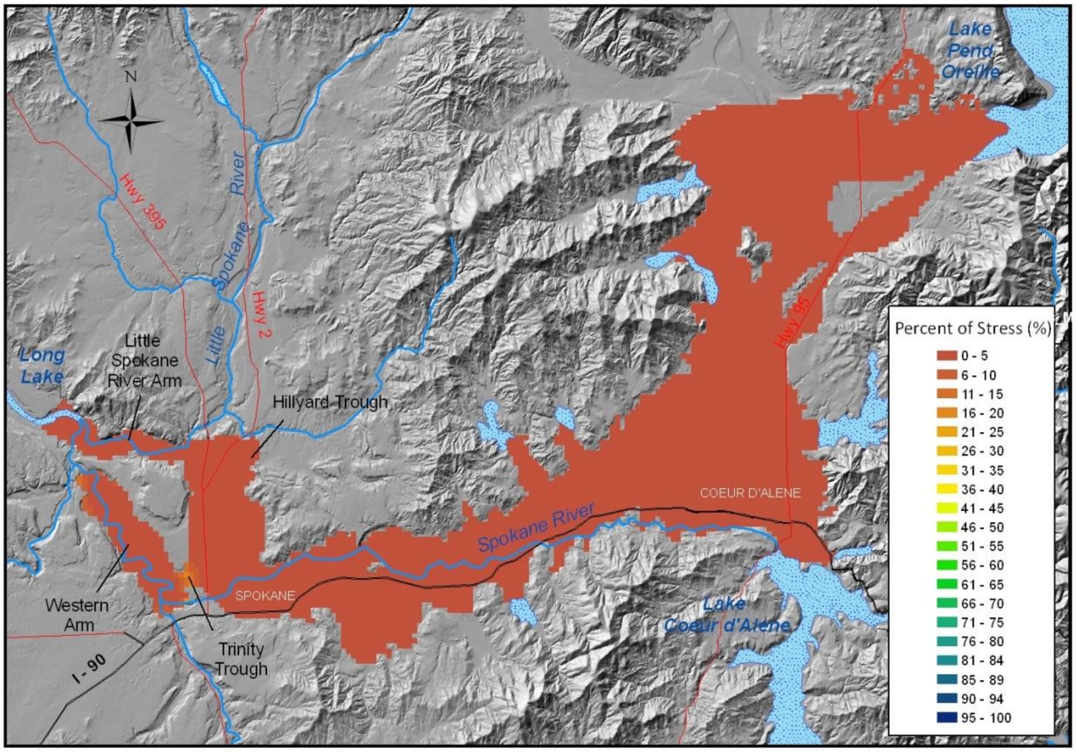

Figure B15. Maximum response errors in Spokane River between Avista Dam and Deep Creek using the Confined model

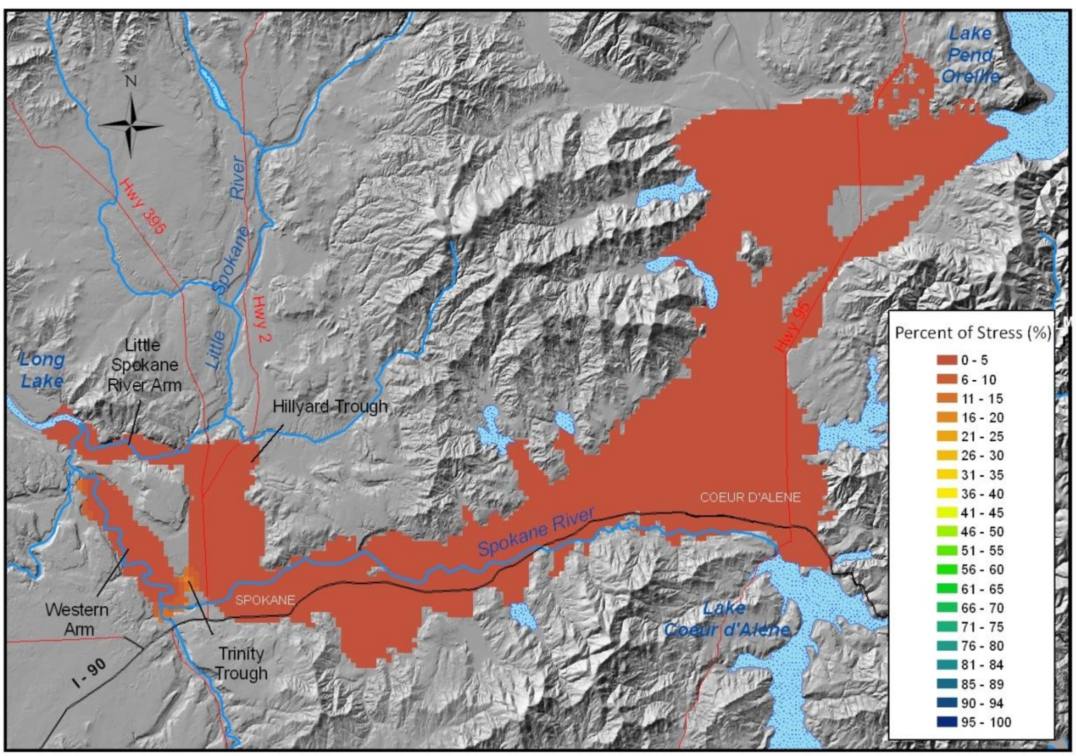

Figure B16. Maximum response errors in Spokane River between Avista Dam and Deep Creek using the Fully Linear model

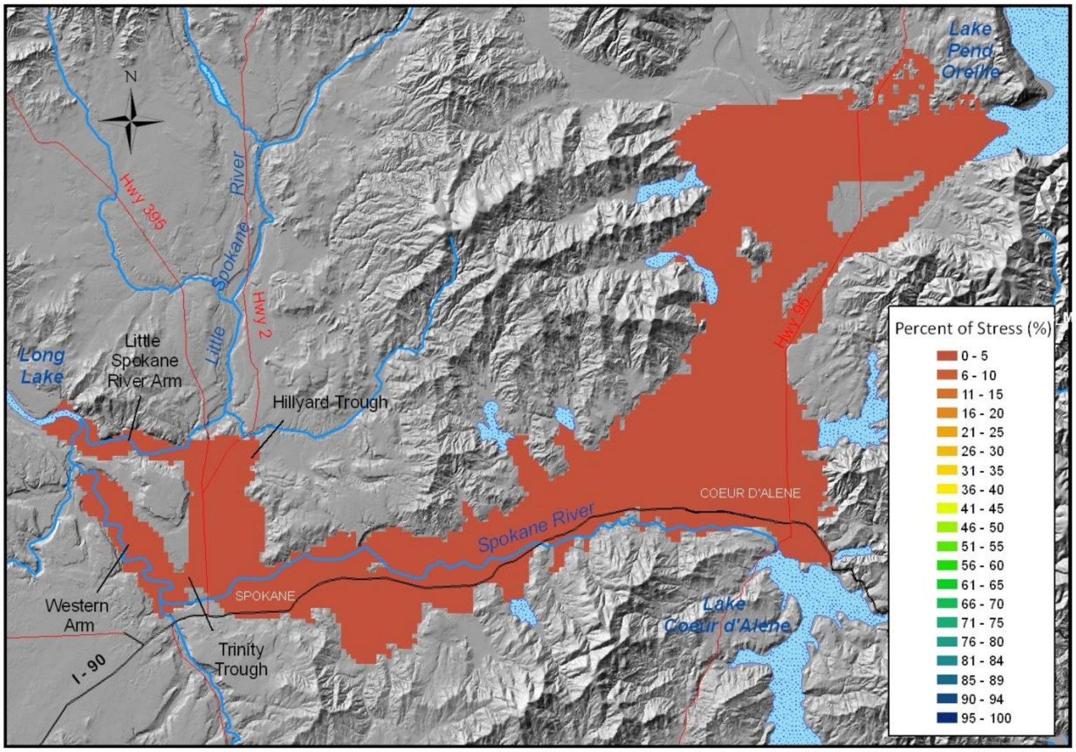

Figure B17. Maximum response errors in Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Confined model

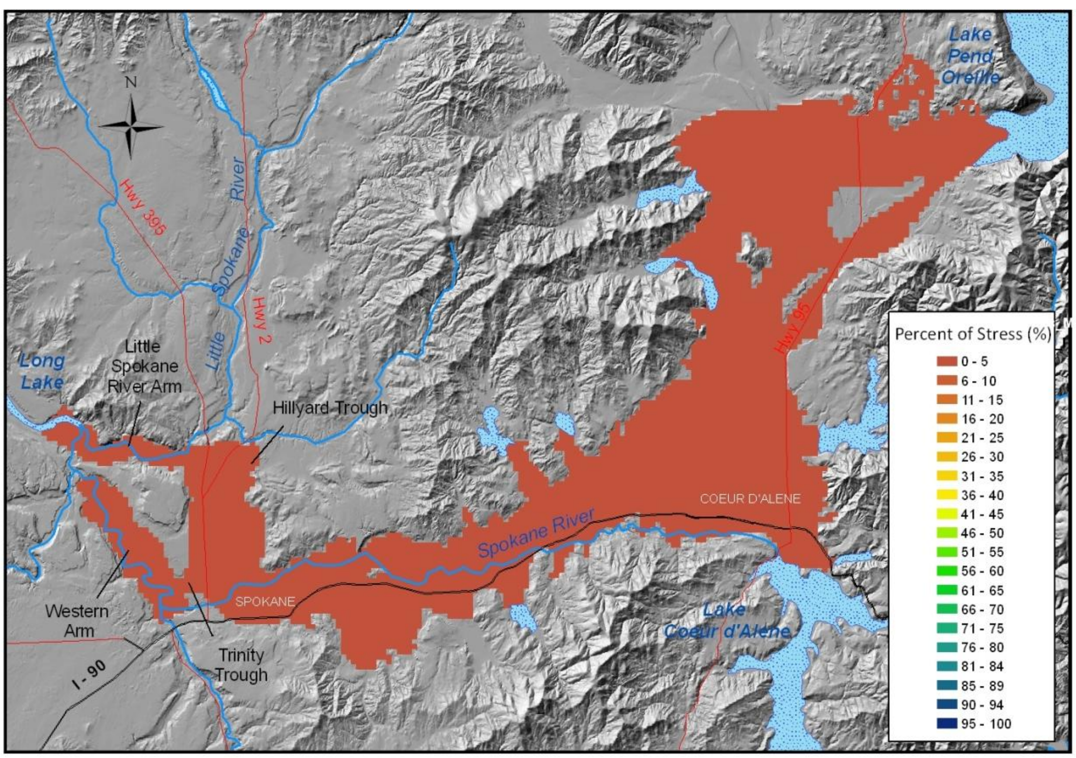

Figure B18. Maximum response errors in Spokane River below Deep Creek and Little Spokane River below Painted Rocks gage and Long Lake using the Fully Linear model

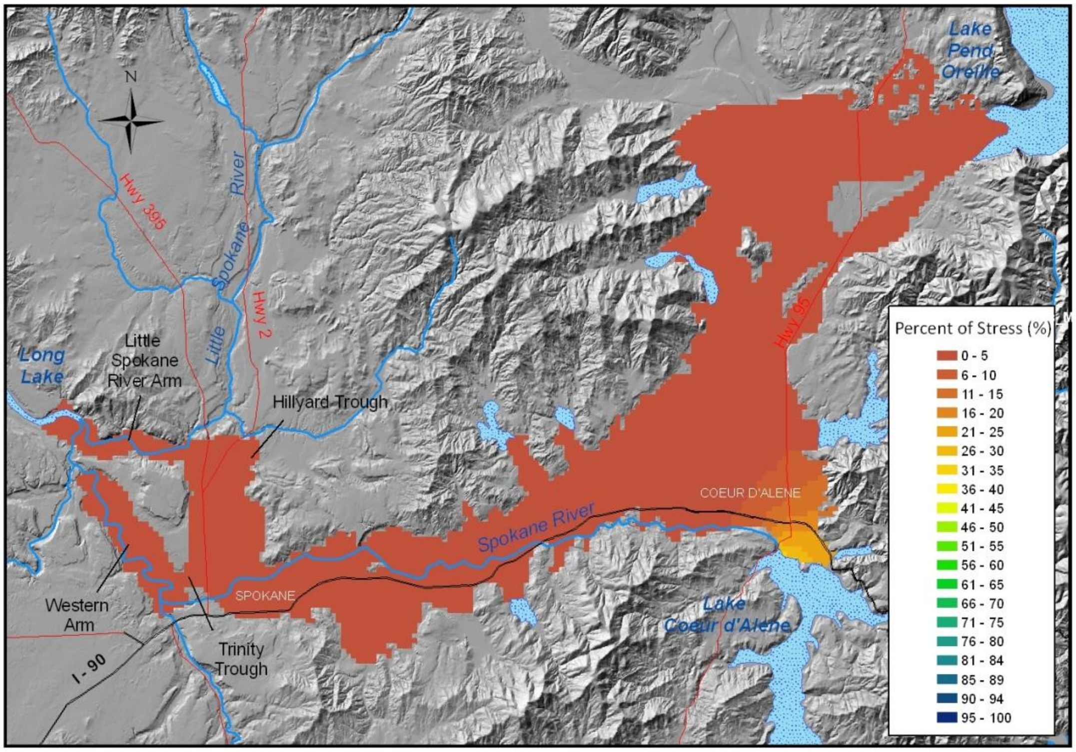

Figure B19. Maximum response errors in Lake Coeur d'Alene using the Confined model

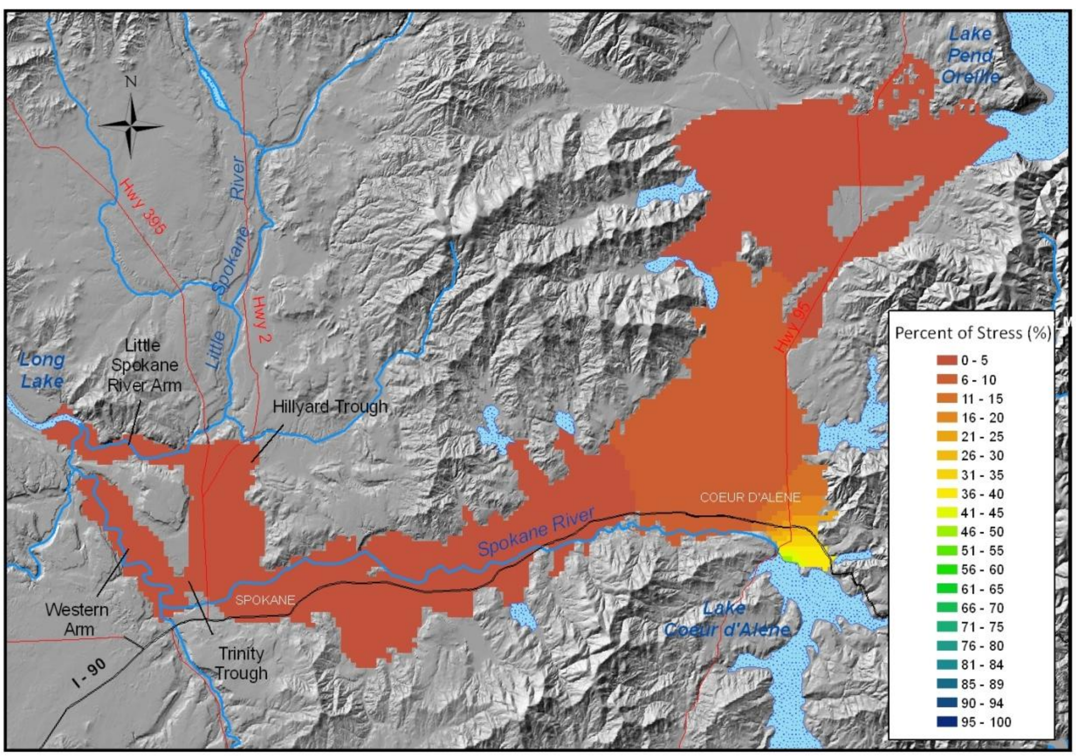

Figure B20. Maximum response errors in Lake Coeur d'Alene using the Fully Linear model

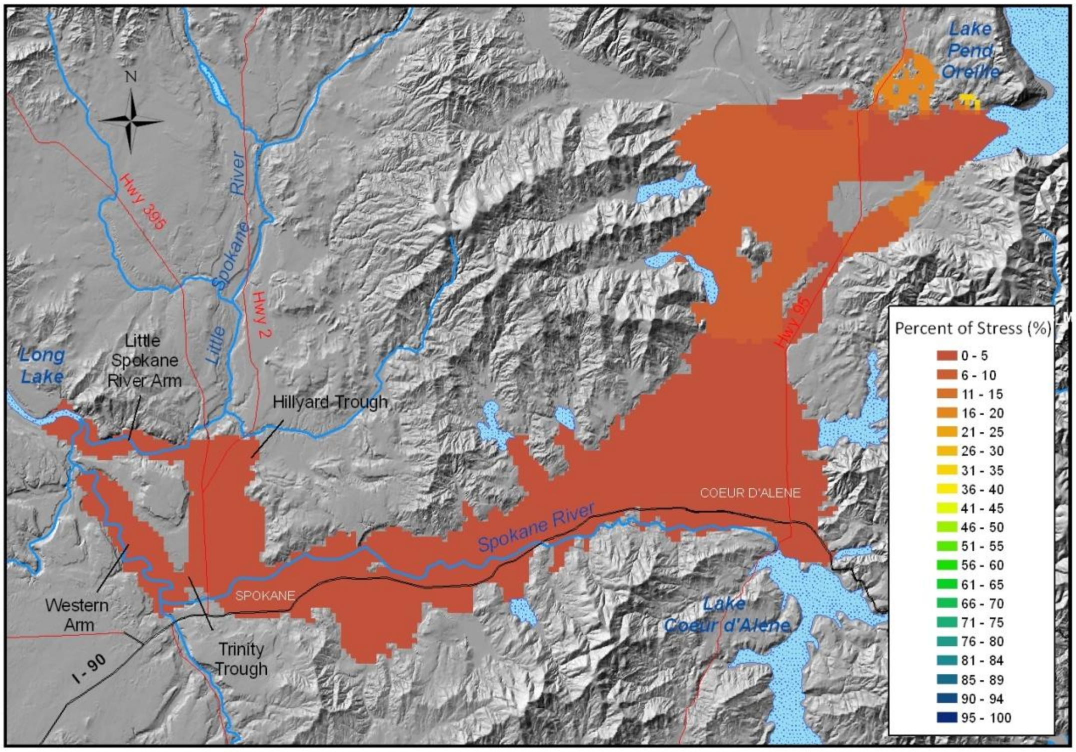

Figure B21. Maximum response errors in Lake Pend Oreille using the Confined model

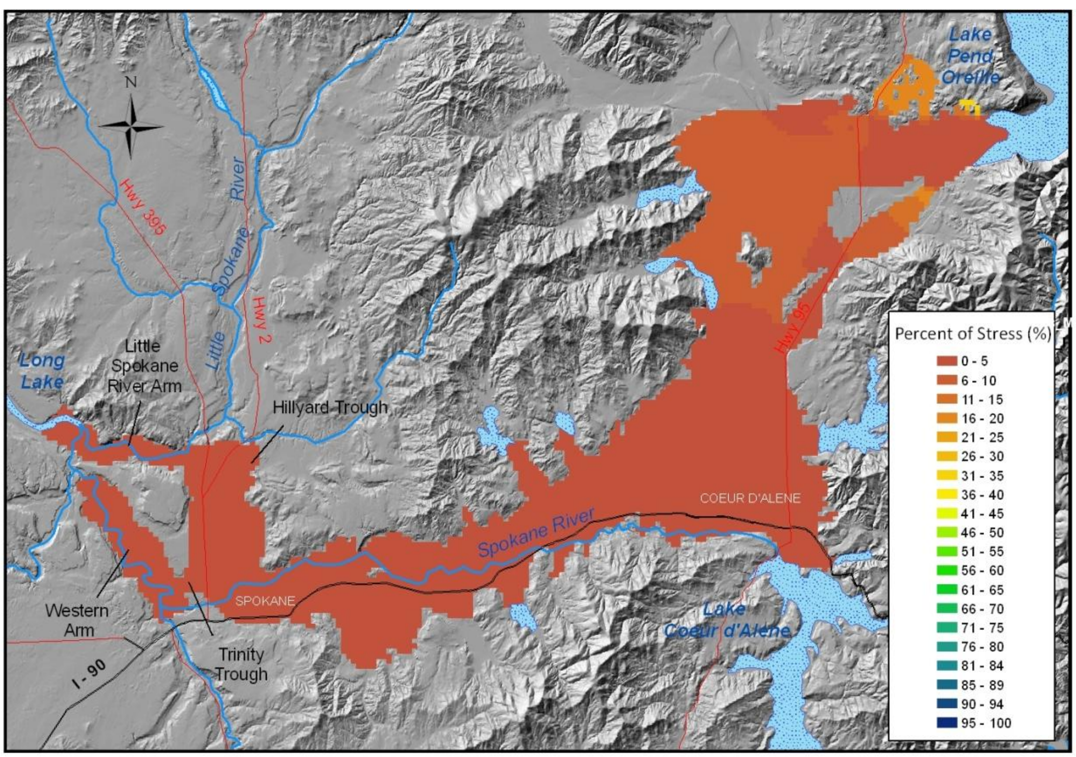

Figure B22. Maximum response errors in Lake Pend Oreille using the Fully Linear model

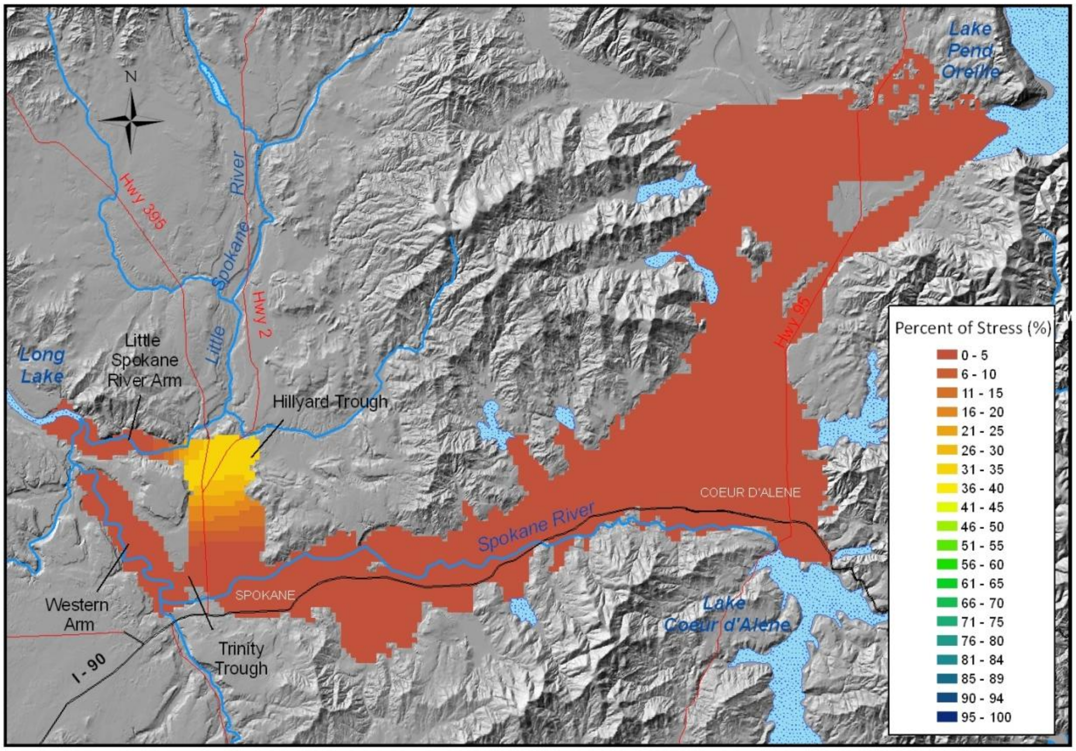

Figure B23. Maximum response errors in Little Spokane River above Painted Rocks gage using the Confined model

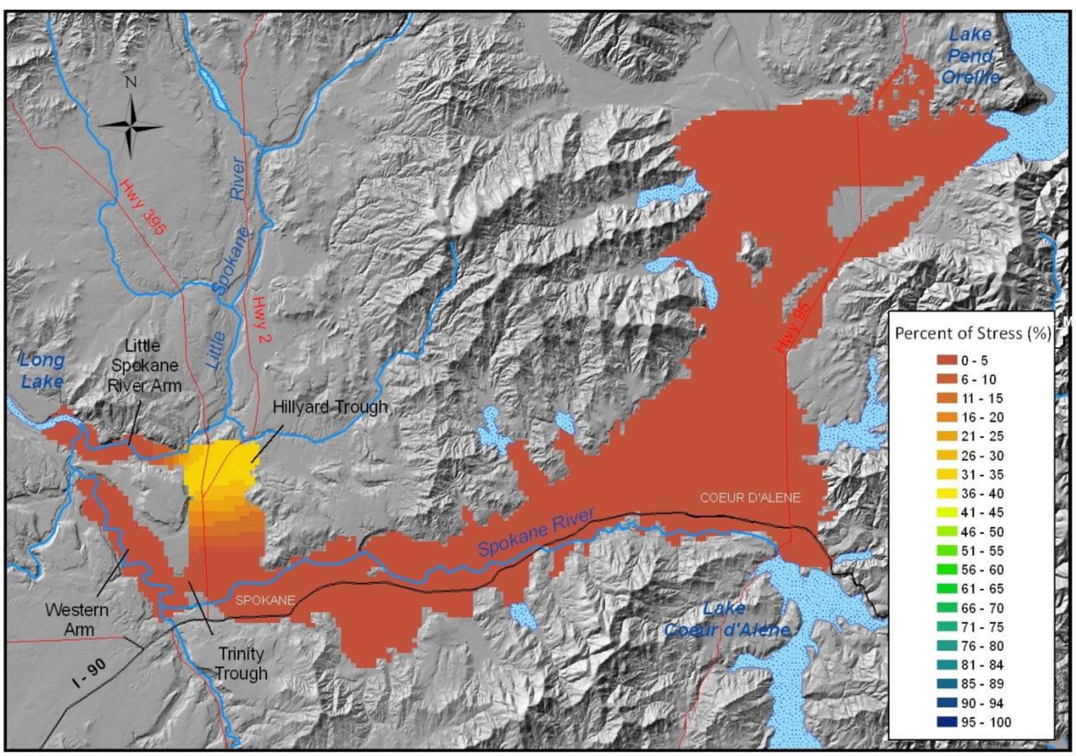

Figure B24. Maximum response errors in Little Spokane River above Painted Rocks gage using the Fully Linear model