

SNAKE RIVER PLAIN AQUIFER MODEL
SCENARIO UPDATE:

*HYDROLOGIC EFFECTS OF CONTINUED
1980-2002 WATER SUPPLY AND USE
CONDITIONS USING SNAKE RIVER PLAIN
AQUIFER MODEL VERSION 1.1*

“Base Case Scenario”

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INTRODUCTION

An evaluation of the scenario, *Hydrologic Effects of Continued 1980-2002 Water Supply and Use Conditions* (also known as the Base Case Scenario), was originally performed using version 1.0 of the Snake River Plain aquifer model [Idaho Water Resources Research Institute (IWRRI), 2004]. The Eastern Snake River Plain aquifer (ESPA) model has been updated to version 1.1, and the results of re-running the Base Case Scenario with the updated model are presented here. The Base Case Scenario is one of many simulations using the ESPA model to provide information and assist in resolution of conflicts among water right holders and guide future water management such as implementation of managed recharge. Water management should be guided by a collective perspective, using many of the scenario evaluations rather than a single document.

The present version of the Snake River Plain aquifer model was developed with funding provided by the State of Idaho, Idaho Power Company, the U.S. Geological Survey, and the U.S. Bureau of Reclamation. The model was designed with the intent of evaluating the effects of land and water use on the exchange of water between the Eastern Snake River Plain aquifer and the Snake River. This evaluation is part of the application of the model towards this purpose.

The model was developed by the IWRRI under the guidance, and with the participation of, the Eastern Snake Hydrologic Modeling Committee (ESHMC). The effort was led by the Idaho Department of Water Resources (IDWR) and active participants in the Committee included Idaho Power Company, the U.S. Geological Survey, the U.S. Bureau of Reclamation, and IWRRI. The ESHMC has also served to guide and review the scenario evaluation process. Documentation of the model and related activities are available from the Idaho Department of Water Resources and the Idaho Water Resources Research Institute at the University of Idaho.

This “Base Case Scenario” consists of repeatedly re-running the 22-year model. This implies that at the end of April 2002, irrigation practices, weather and crop mix immediately revert back to 1980 conditions. Nothing resembling this happened between 2002 and 2005 or can be expected in the future years. Thus, this analysis cannot be used to forecast future conditions, however it will provide an assessment of how close to equilibrium the calibration period was.

The purpose of this scenario evaluation is to determine and describe how spring discharges and river gains and losses are affected by continued future water use in a manner similar to the 1980-2002 period, assuming that weather conditions for that period are also representative of the future. The objectives of this evaluation are to:

- 1) Evaluate and describe the degree to which aquifer inflow and outflow (recharge and discharge) have been out of balance during the 1980-2002 period. Determining the magnitude of the imbalance provides an estimate of further depletion (or accretion) of springs and Snake River flows that can be expected. If

- practices and average weather conditions of that period persist, eventually aquifer outflow (pumping, spring discharge and river gains) will balance inflow.
- 2) Provide a description of estimates of the uncertainty in the water budget. Uncertainty in estimates of the water budget are directly translated into uncertainty in estimates of future expected depletion or accretion of springs and flow of the Snake River.
 - 3) Describe how well the selected period of 1980 through 2002 represents longer term average water supply conditions. If this was an unusually wet or dry period, then the assessment of longer-term impacts will be biased.
 - 4) Estimate and describe how long it may take for a new equilibrium condition to be established. That is, if aquifer inflows are out of balance with outflows, how rapidly will aquifer recharge and discharge change to achieve a new balance.
 - 5) Illustrate results in a fashion that shows expected changes within the context of normal seasonal and inter-year variation in spring discharge and river gains and losses.
 - 6) Provide a base data set for estimating future spring discharge and river gains upon which results from different management schemes (other scenarios) may be superimposed.

The 22-year period on which this evaluation is based extends from May 1 of 1980 through April 30 of 2002. The 1980 to 2002 period was selected because a) intense hydrologic data collection and processing has already been performed for that period for model calibration, b) the period is a reasonable approximation of present water and land use practices, and c) the period is sufficiently long to include multiple wet and dry periods. Results will be presented in terms of model years that extend from May through April. For example, the 1980 model year extends from May 1, 1980 through April 30, 1981. This convention is used to match model stress periods to irrigation and non-irrigation seasons.

The Base Case is not intended as a forecast of future conditions. The recharge and discharge of the 1980 to 2002 period were applied to 2002 through 2024, from 2024 through 2046, and again from 2046 through 2068. This was done to show a degree of natural variability in spring discharge and evidence of longer term trends at the rates of recharge and discharge estimated for the 22-year period of 1980 through 2002. The approximation of future recharge and discharge at levels equivalent to the 1980 through 2002 period is not adequate for forecasting actual river gains and losses.

This evaluation discusses aquifer conditions relative to steady-state. Steady-state conditions are achieved when aquifer recharge and discharge are maintained at a constant level for long periods of time resulting in stable aquifer water levels and river gains and losses; that is, aquifer outflows balance inflows. When seasonal and year-to-year variations are superimposed, but there is no long term trend in recharge and discharge, a dynamic equilibrium is eventually reached where aquifer water levels and river gains and losses fluctuate about the steady state level. Steady state conditions do not necessarily imply that the balance has occurred at a desirable level.

ESPAM Version 1.1

In May 2005, errors were discovered in the calibrated ESPAM v1.0, necessitating recalibration of the model. Correction of these errors and recalibration resulted in ESPAM v1.1, as documented in the ESPAM project final report. Most notable among the errors were a) ESPAM v1.0 was inadvertently calibrated using the old irrigation return flow lag factors, and b) there was a discretization error in the Shelley to Near Blackfoot and Near Blackfoot to Neeley reaches. Each of these errors would cause a re-distribution of recharge and discharge among reaches of the Snake River and affect the final model parameters (transmissivity, storativity and riverbed conductance). The ESHMC decided that the Base Case Scenario should be re-run using ESPAM v1.1. Because both the water budget and the model parameters were impacted by these corrections, the following water budget analysis and figures were modified as part of this revision as well as the model results.

WATER BUDGET ANALYSIS

The ESPA water budget from 1980 through 2002 provides an indication of the degree to which either the aquifer is being depleted or accreted over the long term. If more water has discharged from the aquifer during this time than has recharged, aquifer water levels will decrease, and aquifer storage will have been depleted. The associated decline in aquifer water levels causes a corresponding decline in spring discharge to the Snake River. If these conditions persist, spring discharge would ultimately decline to the point where discharge would equal recharge. If recharge exceeds discharge, the reverse is true, and the volume of water stored in the aquifer will increase, causing water levels to rise and spring discharge to increase. Consequently, determination of the water budget imbalance for the 1980-2002 period may help us understand the degree to which spring discharge may change in the future, if the water supply and use conditions persist.

Evaluation of the Imbalance in Aquifer Recharge and Discharge (Objective 1)

There are multiple components of recharge and discharge to the Snake River Plain aquifer. Recharge components include: precipitation directly on the Plain (both irrigated and non-irrigated portions); seepage from rivers, streams, and canals; surface water irrigation applied in excess of crop consumption; and subsurface flow from tributary valleys. Discharge components include springs and seeps to the Snake River and wetlands; and consumptive pumping for irrigation, municipalities, and industrial needs. Private residential wells also extract ground water, but the consumptive use of these systems is limited to lawn and garden watering which is partially captured as irrigated lands and does not represent a significant component of the water budget (Goodell, 1988). Annual variation in net aquifer recharge and net Snake River gains for the 1980 through 2002 period (model calibration period) are shown in Figure 1. Figure 1 is presented from the perspective of an aquifer water budget; consequently, river gains are shown as negative values. The net aquifer recharge reflects subtraction of

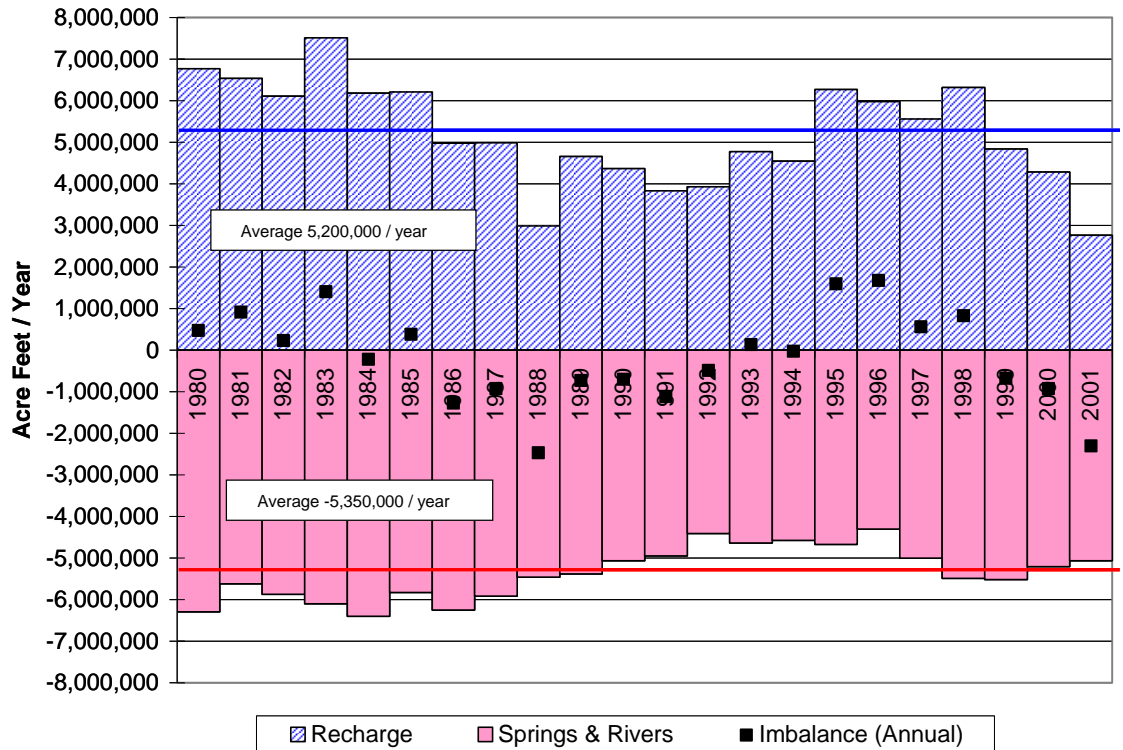


Figure 1. Annual Net Aquifer Recharge and River Gains (ESPAM v1.1)

consumptive irrigation pumping. These values were determined using the GIS Recharge Program. The net Snake River gains are the total of spring discharge to the river minus seepage from the river. The net Snake River gains were determined by summing spring discharge for the Milner to King Hill reach determined by the Kjelstrom method (Kjelstrom, 1995) and reaches above Milner by the IDWR Reach Gain and Loss Program. Figure 1 also shows the annual imbalance between aquifer recharge and discharge (black squares). The graph indicates that in individual years substantial imbalances in recharge and discharge exist, but on average for the entire 22-year period, discharge exceeded recharge by about 150,000 AF/year. This average imbalance is of great importance to understanding future spring discharges and river losses. If water supply and use conditions of the 1980 to 2002 period persist into the future, decreased Snake River gains (spring discharge) and increased losses are expected to total about 150,000 AF/year (207 cfs) over the long term, relative to average gains and losses during the 1980 to 2002 period. This does not identify the impact to individual river reaches or springs which are discussed in a following section on model results.

Figure 1 implies that in some years water is taken into aquifer storage and in some years storage is depleted. The ground water flow model similarly tracks how much water goes into or out of aquifer storage. The model results are a reflection of the water budget presented in Figure 1 with some differences resulting from less than perfect estimates of Snake River gains and losses and model calibration. The cumulative change in storage graph of Figure 2 shows seasonal variation in water going into (+), or coming out of (-), aquifer storage as well as longer term variations during the 1980-2002 period. During the

first few years, a general increase in storage results from a wet weather pattern and healthy irrigation water supply. The downward slope from 1987 to 1994 is a result of dryer conditions. This is again followed by wet conditions from 1994 through 2000,

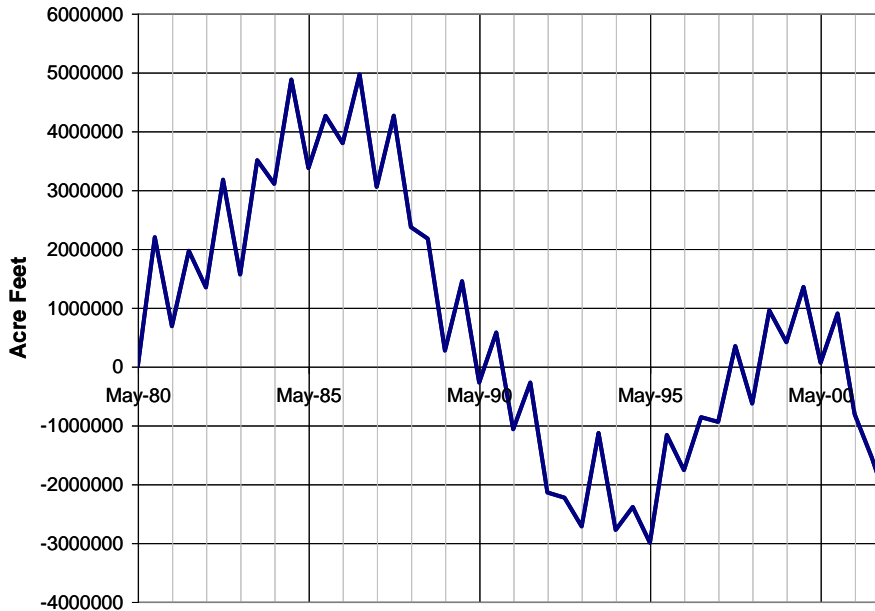


Figure 2. Model Estimated Cumulative Aquifer Storage Relative to May 1, 1980 (ESPAM v1.1)

which is succeeded by a drought to the end of the period. The fact that the cumulative change in storage graph crosses zero at several dates means that, relative to the starting date of May 1, 1980, average aquifer inflows balanced outflows for these periods. The end point (2002) is below zero indicating an average imbalance at the end of the period.

Both Figures 1 and 2 indicate that aquifer discharge may have slightly exceeded recharge when averaged over the entire model period. Although the results of the figures are not directly comparable, due to the use of partially different data sets in their creation, they are in general agreement. It is also important to note that Figure 2 is a reflection of not only balancing the water budget, but also calibration of model parameters to thousands of aquifer water levels and hundreds of Snake River gain observations. Therefore, periods that show no cumulative change in storage are partially reflecting no change in aquifer water levels from the beginning to the end of the period (although offsetting changes in aquifer water levels could exist in different location, on the whole, no change in aquifer storage reflects relatively stable aquifer water levels).

Evaluation of Water Budget Uncertainty (Objective 2)

Water budget analysis for the 1980 to 2002 period indicates that Snake River gains will be depleted and losses increased by about 150,000 AF/year over the long term if water supply and use of this period continues. It is important to recognize, however, that the

water budget analysis, and consequently model simulations based on that water budget, may contain inaccuracies, creating uncertainty in the forecast. Figure 3 shows an estimate of uncertainty relative to the magnitude of recharge and river gains and losses. The uncertainty was estimated by subjective assessment of the uncertainty of individual water budget terms and a statistical treatment of those uncertainties to develop a total estimated uncertainty. The procedure is described in greater detail in Appendix A. Potential water budget errors presented in Figure 4 may be overestimated (errors likely to be smaller than shown) due to the verification process of balancing the water budget. Balancing all aquifer recharge and discharge improves the confidence in, and reliability of, estimates of total recharge and discharge. Although uncertainty in river gains is difficult to estimate, error bars are included on the river gains in Figure 3 to illustrate the magnitude of a potential 5% error in gain estimates.

The estimates of uncertainty shown in Figure 3 substantially exceed the estimate of change that is expected in river gains and losses. This means that we cannot be certain if river gains will increase or decrease from average 1980-2002 levels, but we expect that if water use and supply do not significantly change, we should not experience substantial long term changes in river gains and losses. Drought conditions and other weather variations, however, will continue to cause short-term variations in spring discharge and river gains and losses.

Representing “Average” Conditions With the 1980 to 2002 Period (Objective 3)

It is important to understand the degree to which the 1980-2002 period represents longer term average water supply conditions. Longer term hydrologic records are available for stream flow and precipitation at several stations within the Snake River basin. None of these records provides a perfect reflection of net aquifer recharge and discharge. For example, high Snake River flow in 1997 largely passed downstream as flood flows, without substantially impacting aquifer recharge. Consequently, higher than normal flow of the Snake River does not necessarily mean higher than normal aquifer recharge. Since no hydrologic observation is completely representative of aquifer recharge and discharge, qualitative comparisons are made for two selected measurement stations that have some correlation to recharge: 1) precipitation at Aberdeen, and 2) flow of the Snake River at Heise. A visual correlation of the net aquifer recharge and precipitation at Aberdeen for the 1980 to 2002 period is provided in Figure 4.

The historic record for precipitation at Aberdeen extends from 1914 to present. The average annual precipitation for that period is 8.63 inches per year. During the 1980 to 2002 period, precipitation at Aberdeen averaged 9.14 inches, or 6 % above normal. The average annual flow for the period of record for Snake River flow at Heise (1910 to present), not corrected to natural flow, is 5,054,000 AF/year (Figure 5). The average flow for the May 1, 1980 through April 30, 2002 period was 5,234,000 AF/year, or 3.5% above the longer term average. These data imply that the 1980 to 2002 period was, on average, slightly wetter than normal, and that aquifer recharge during this period may slightly overestimate long term average conditions. The degree of overestimation is not known, but even a 5% difference can result in 250,000 AF/year of recharge. Additional

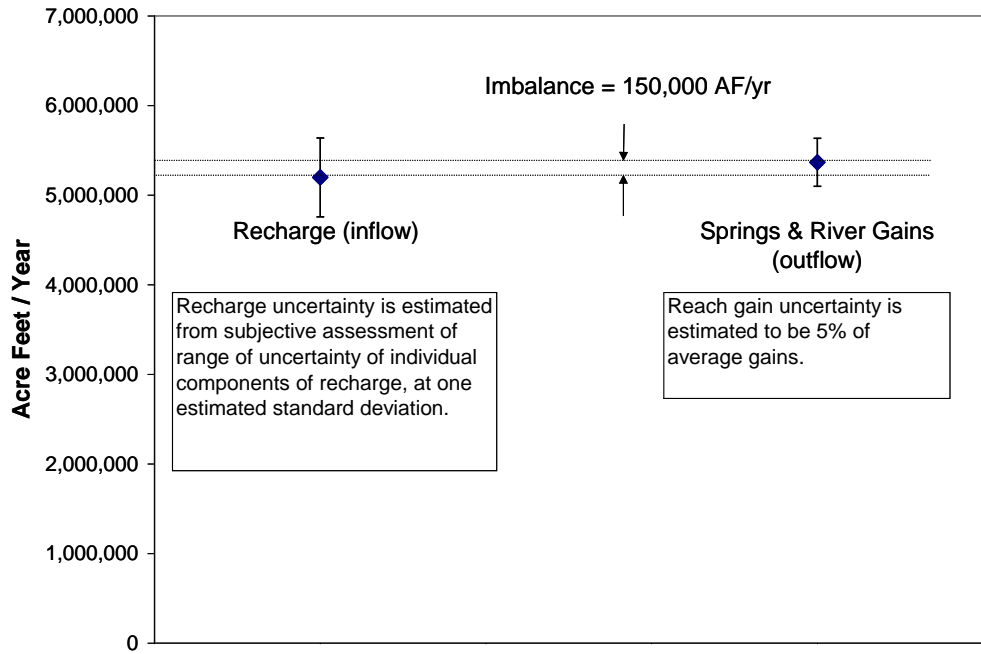


Figure 3. Recharge Uncertainty Estimates Relative to 22-year Average Recharge and Discharge (ESPAM v1.1)

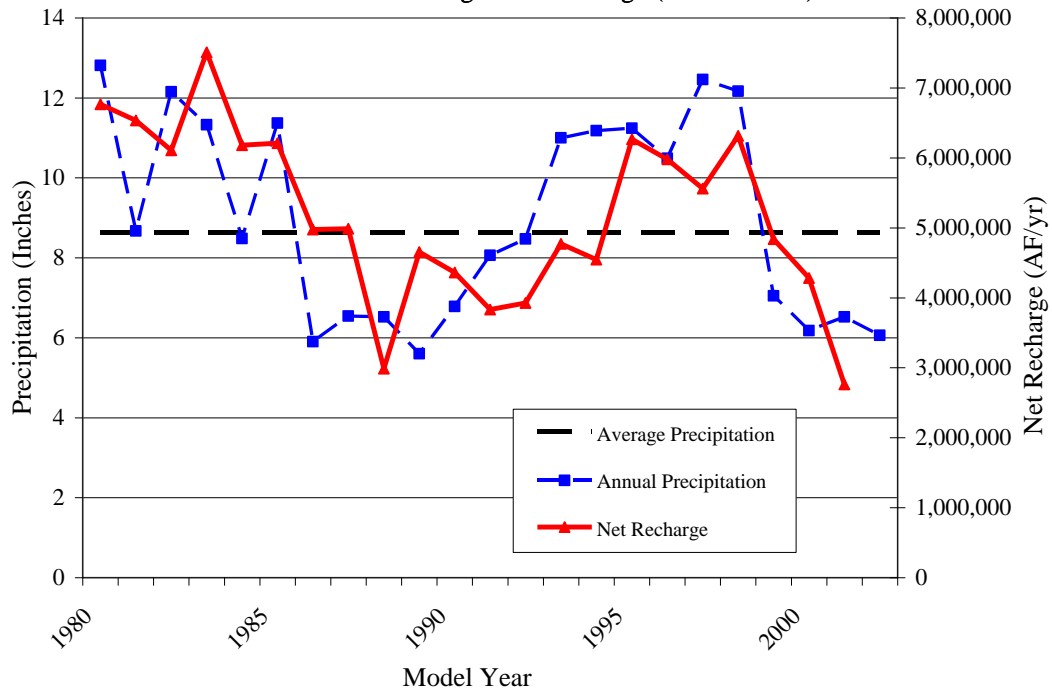


Figure 4. Net Aquifer Recharge and Precipitation at Aberdeen (ESPAM v1.1)

reductions in river gains and spring discharge could result from a return to longer term average hydrologic conditions.

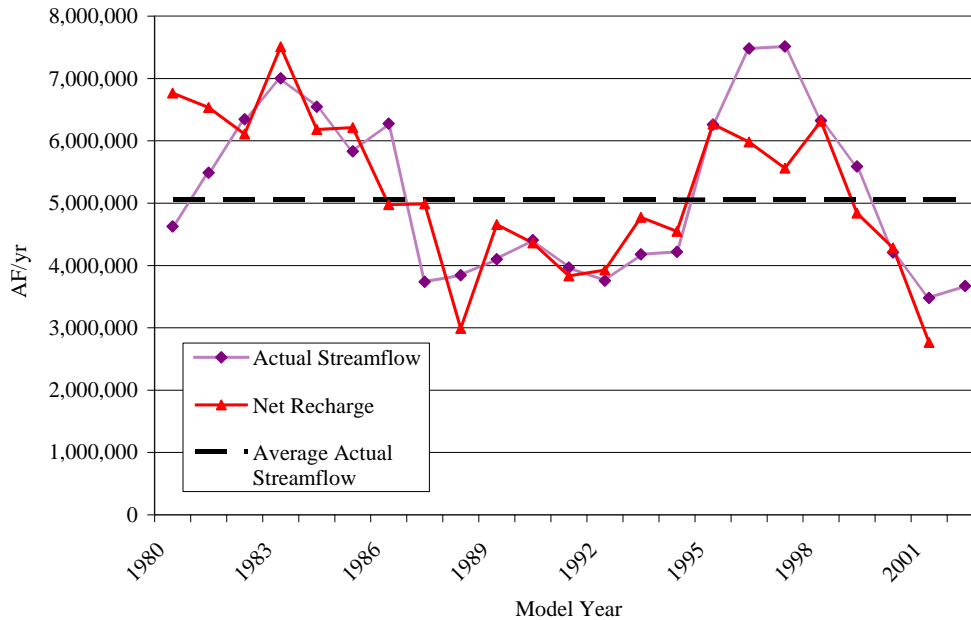


Figure 5. Net Aquifer Recharge and Streamflow near Heise (ESPAM v1.1)

MODELING ANALYSES

Modeling Approach

The modeling approach taken for this scenario has three main components: a) running the steady state simulation of the ESPA model with all recharge and discharge averaged for the 22-year calibration period, b) running the transient ESPA model using the average recharge and discharge, and c) running the transient ESPA model with the full 22-year recharge and discharge run repeatedly for several cycles, using the ending aquifer water levels of each 22-year cycle as the starting heads for the next cycle. Initial aquifer water levels for the transient model runs are the ending aquifer water levels resulting from the 17-year steady state period, as detailed in the final model report.

This approach allows evaluation of a) what the final expected value of spring discharges and river gains would be if current practices and average water supplies were to continue indefinitely, b) what the expected annual average spring discharge and reach gains would look like and how long it would take for the aquifer to come into equilibrium given existing conditions, and c) what the expected highs and lows around those annual averages would be.

A summary of the model results for this scenario is given below. Details of the modeling including input files and programs can be found in the *Addendum Report to Hydrologic Effects of Continued 1980-2002 Water Supply and Use Conditions: Data and Programs*.

Modeling Results (Objectives 4, 5, 6)

The averaged recharge data set was run using a steady state version of the ESPA model. This model run yielded the final expected river reach gains and spring discharges for each river reach, given no changes in water supply or water or land use relative to the 1980 to 2002 period. The results of this simulation are shown in Figures 6 through 16 as a single point on the right-hand side of each graph. The averaged recharge data set was also run using a transient version of the ESPA model with 1-year stress periods and 10 time steps per stress period, to generate the expected average reach gain and spring discharge every 36.5 days. These results are shown in Figures 6 through 16 as a pink line.

The 22-year calibration data set was successively run four times using a transient version of the ESPA model with 44 6-month stress periods. This model configuration was the same configuration used for model calibration. Initial starting aquifer water levels (spring, 1980) were the ending water levels for the calibration steady state version of the ESPA model version 1.1 (which used an average of recharge from May, 1982 through October, 2000; this period was selected to represent a long-term period during which there was minimal change in aquifer water levels). The ending water levels for the first cycle of 22 years were used as the starting water levels for the next 22-year cycle. The 22 years of recharge and discharge were repeated for each 22-year cycle. This scenario was run for 4 cycles or 88 years. The simulation results, shown as dark blue erratic lines for the first 22 years, and lighter blue in subsequent simulations, in Figures 6 through 16 illustrate seasonal and multi-year variations around the average simulated reach gains due to variability of water supply. The color transition, from dark to light blue, approximately corresponds to the transition from the calibration period to subsequent simulations. Because the recharge and discharge of the 1980 to 2002 period were applied to 2002 through 2024, from 2024 through 2046 and again from 2046 through 2068, this simulation should not be considered a forecast.

As previously stated, the initial aquifer water levels were derived from the 17-year steady-state period, which had balanced recharge and discharge. The model results show that within the first 22-year cycle, the spring discharges and river gains/losses have fully compensated for the 150,000 ac-ft/yr imbalance in the 22-year calibration recharge data set. This indicates that the aquifer has reached a dynamic equilibrium some time within the first 22-year cycle.

As can be seen in Figures 6 through 16, by the end of the model calibration period (2002), the basin is close to steady state and fluctuations in reach gains and spring discharges are due mainly to seasonal and inter-year variations in recharge. The simulation results and water budget analysis indicate that, on the average, the aquifer recharge and discharge are reasonably well balanced (but for drought conditions that have continued after 2002) and that the full effect of changes in water use practices (e.g. pumping, conversion to sprinklers) has been realized at the hydraulically connected river reaches. The degree of seasonal and inter-year variation in the figures is largely affected by the proximity of irrigation to each reach.

The initial heads of this analysis, corresponding to 1980, were taken as the results of the calibration steady state simulation. Although these heads are not a perfect representation of 1980 water levels, they have little effect on the outcome of this scenario. The results of this scenario are focused on a period representing 2002 to 2068. Since the 2002 simulated water levels adequately matched observed water levels for the same time period, the extrapolation into the future is unaffected by the use of steady state starting heads in 1980. From 2002 (where observations were matched during model calibration) forward, repeating the same level of recharge and discharge as in the 1980 to 2002 period resulted in little change in hydraulic head or river gains and losses, indicating near steady state conditions.

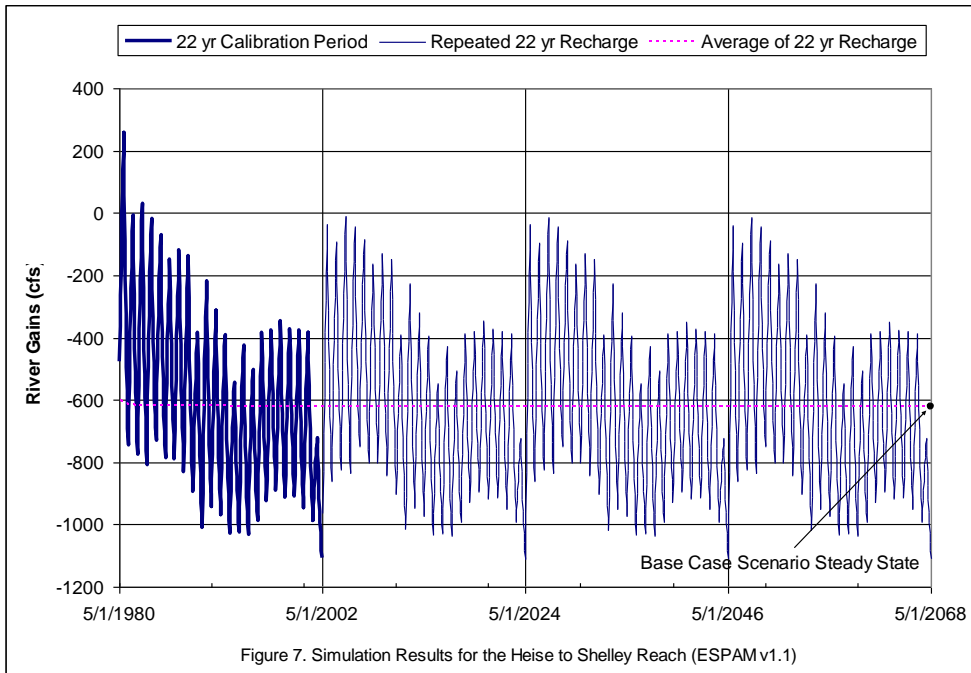
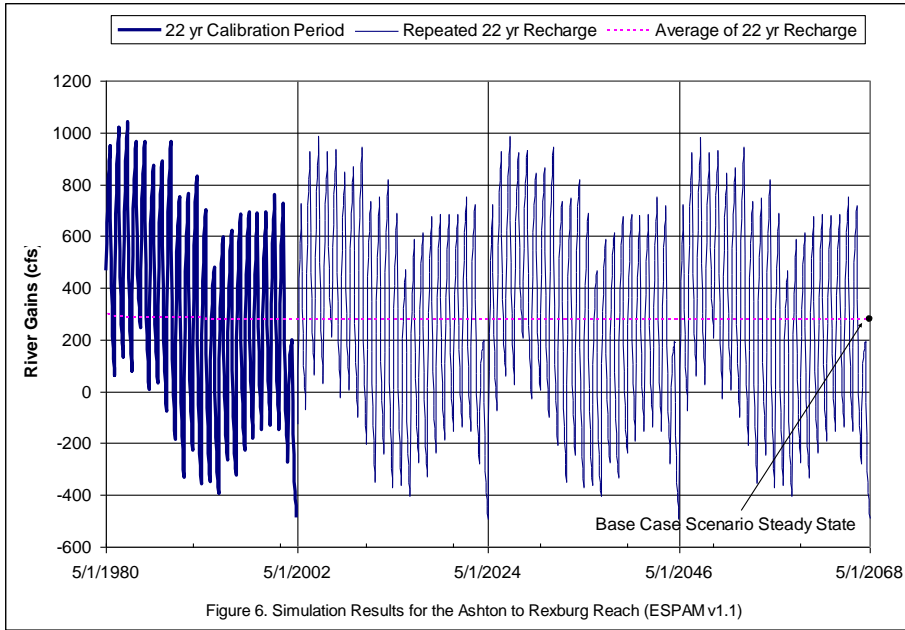
The model results are consistent with the water budget analysis presented in a previous section. Slight differences in the estimated magnitude of the projected changes are due to differences in the evaluation period and the data sets used. Model changes are specified relative to model simulated conditions in 2002. The water budget analysis presents results relative to average aquifer conditions during the 1980 to 2002 period. Both approaches suggest that the Snake River Plain aquifer is approximately in a state of dynamic equilibrium during this period.

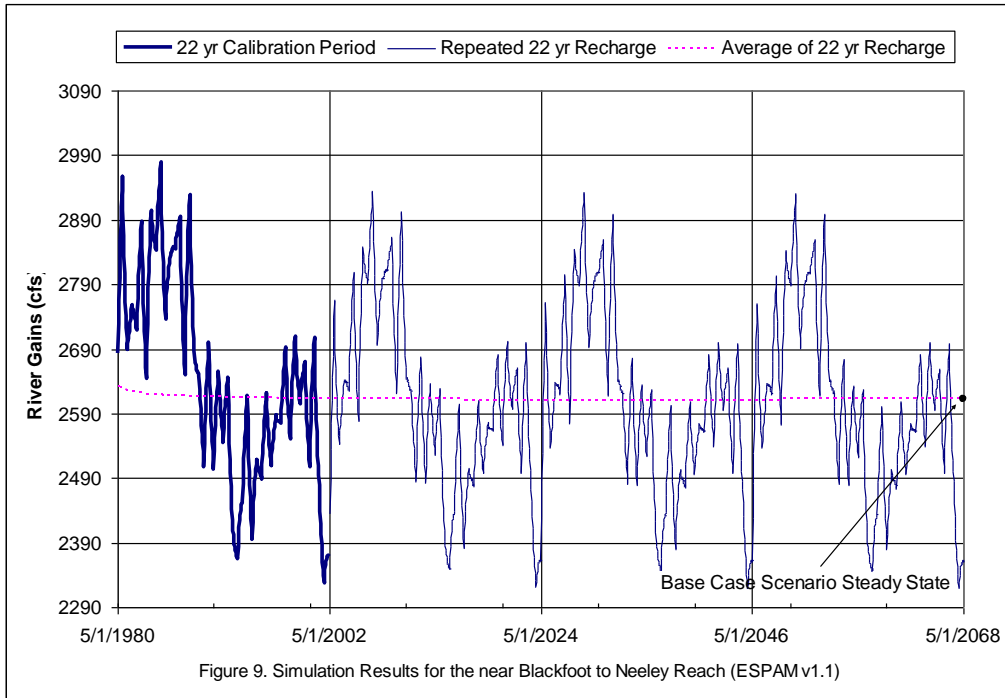
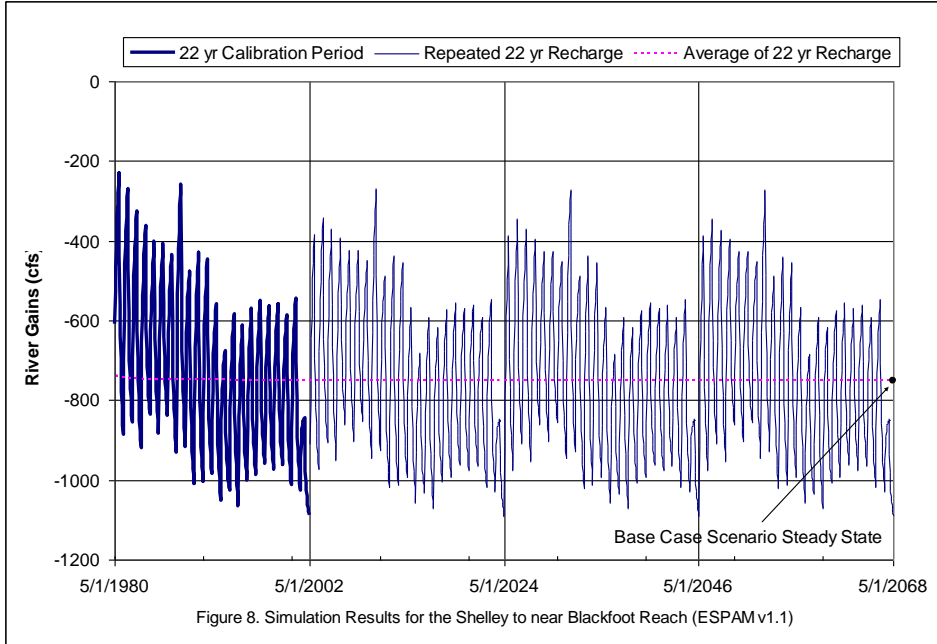
SUMMARY

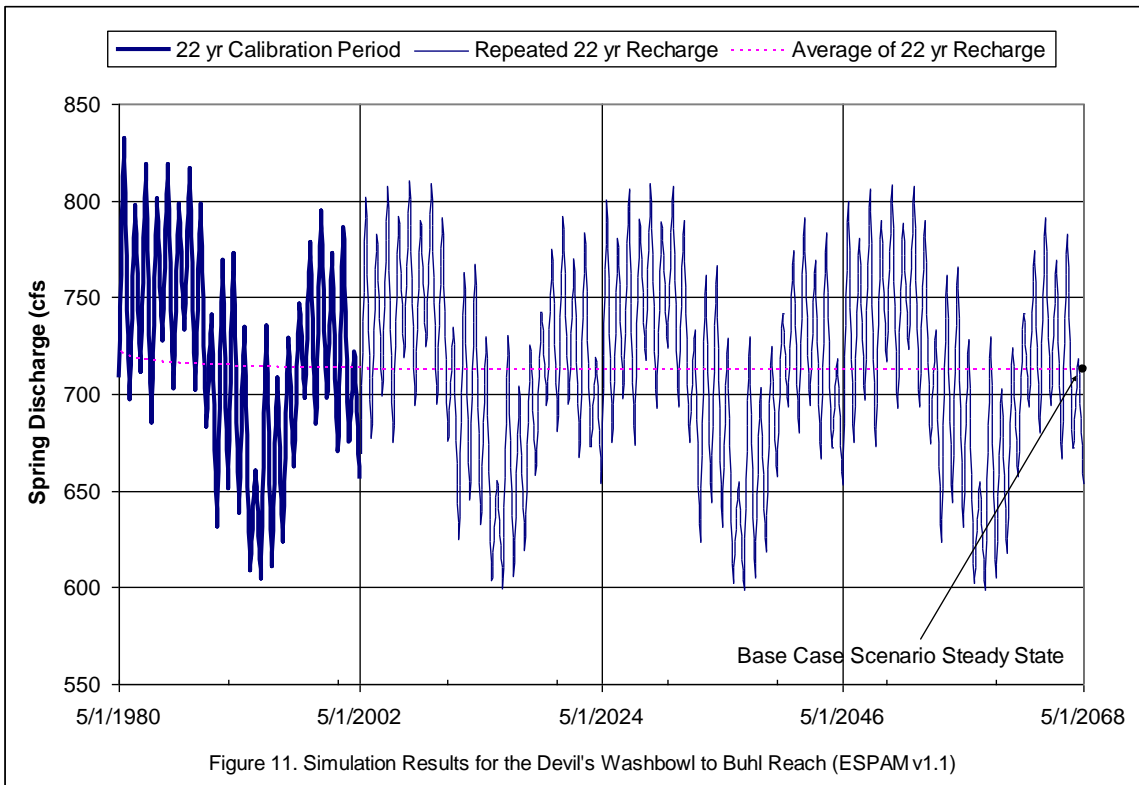
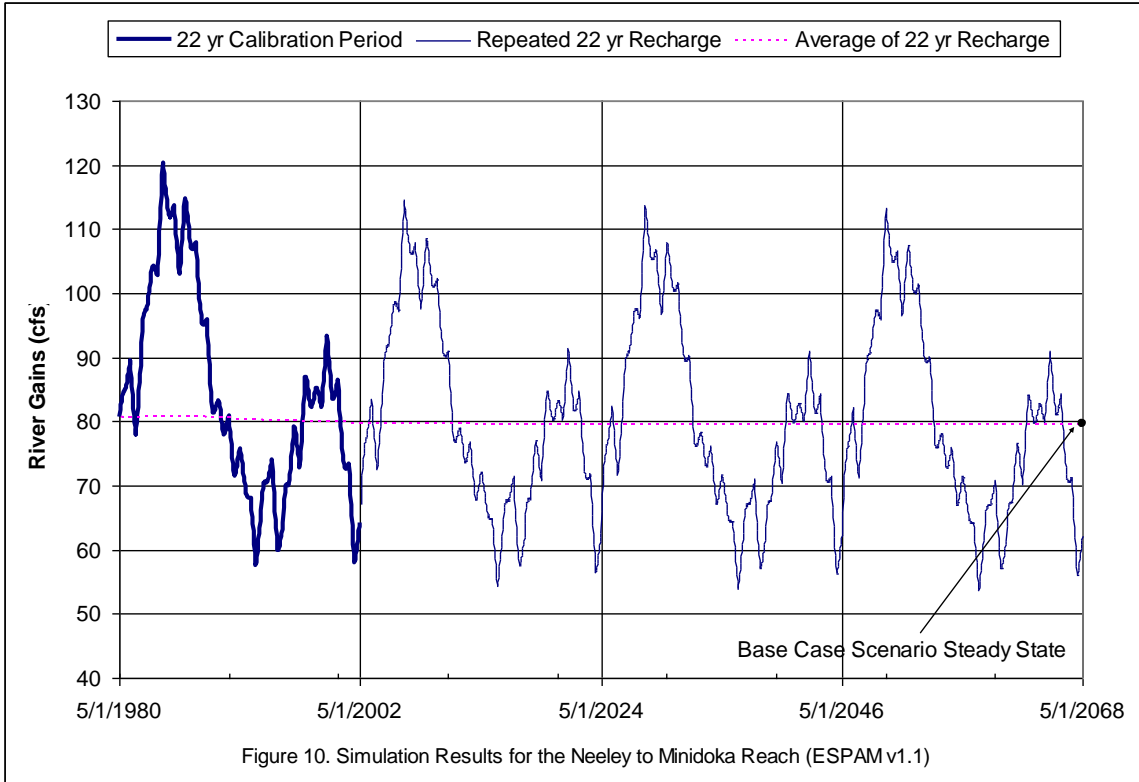
Both the water budget analysis and the model results produced indicate that, as of May 2002, the Snake River Plain aquifer is close to dynamic equilibrium (inexactness of the water budget prohibits making a more definitive statement). This implies that further long-term changes in aquifer water levels and spring discharges are expected to be small, if average water use and supply remain as they were in the 1980-2002. Short-term fluctuations due to weather variation and the resulting changes in irrigation supply would be expected to occur to a degree similar to what has occurred historically. The accuracy of the entire aquifer water budget may be no better than a few hundred thousand acre-feet per year, consequently, accuracy of estimates of the collective changes in river gains and losses are of a similar magnitude.

The projections of spring discharges are based on the assumption that aquifer recharge and discharge will be the same as that estimated for 1980 to 2002. It must be recognized that a) estimates of recharge and discharge for the 1980 to 2002 period contain some degree of error, b) the 1980 to 2002 period is not perfectly representative of average weather conditions and current water use practices, c) projections of changes in spring discharge may be affected by use of steady-state heads as an initial condition, and d) there will be changes in water use practices and weather in the future that are not presently known or captured in this evaluation.

Acknowledging the uncertainty in the model water budget, it should be kept in mind that the model calibration was successful at reflecting long and short-term trends in aquifer water levels and reach gains, indicating a degree of confidence in the model results and the water budget.







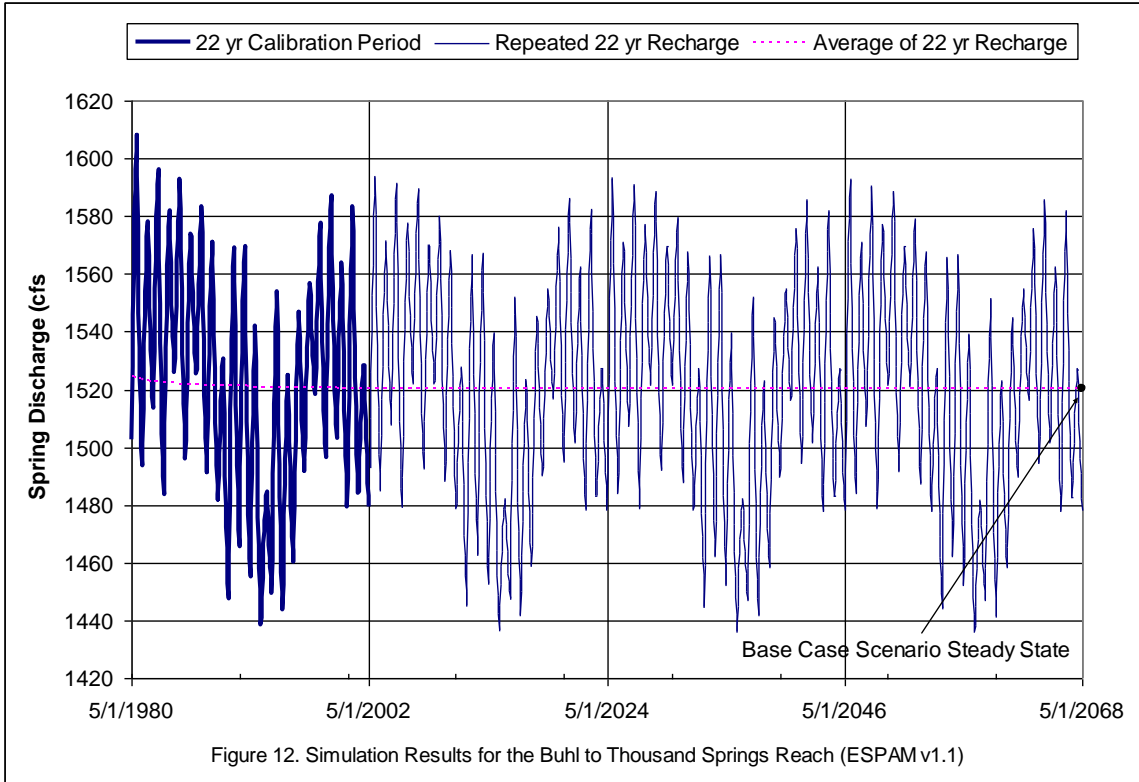


Figure 12. Simulation Results for the Buhl to Thousand Springs Reach (ESPAM v1.1)

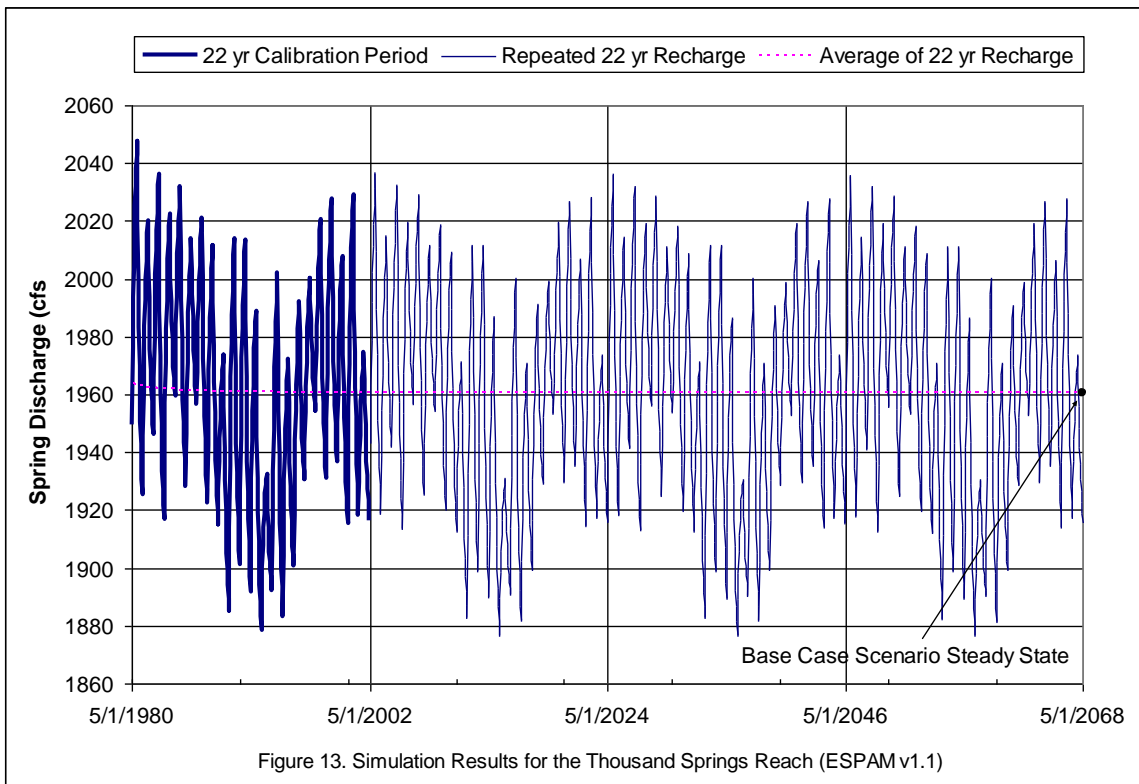


Figure 13. Simulation Results for the Thousand Springs Reach (ESPAM v1.1)

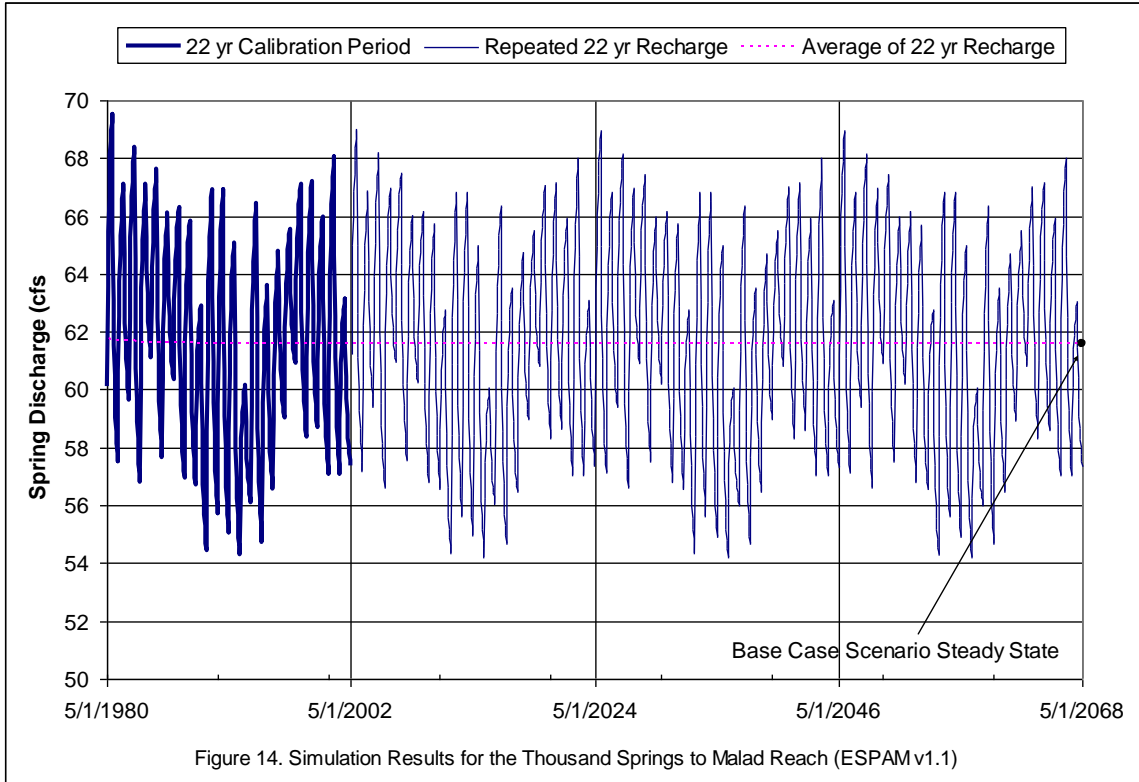


Figure 14. Simulation Results for the Thousand Springs to Malad Reach (ESPAM v1.1)

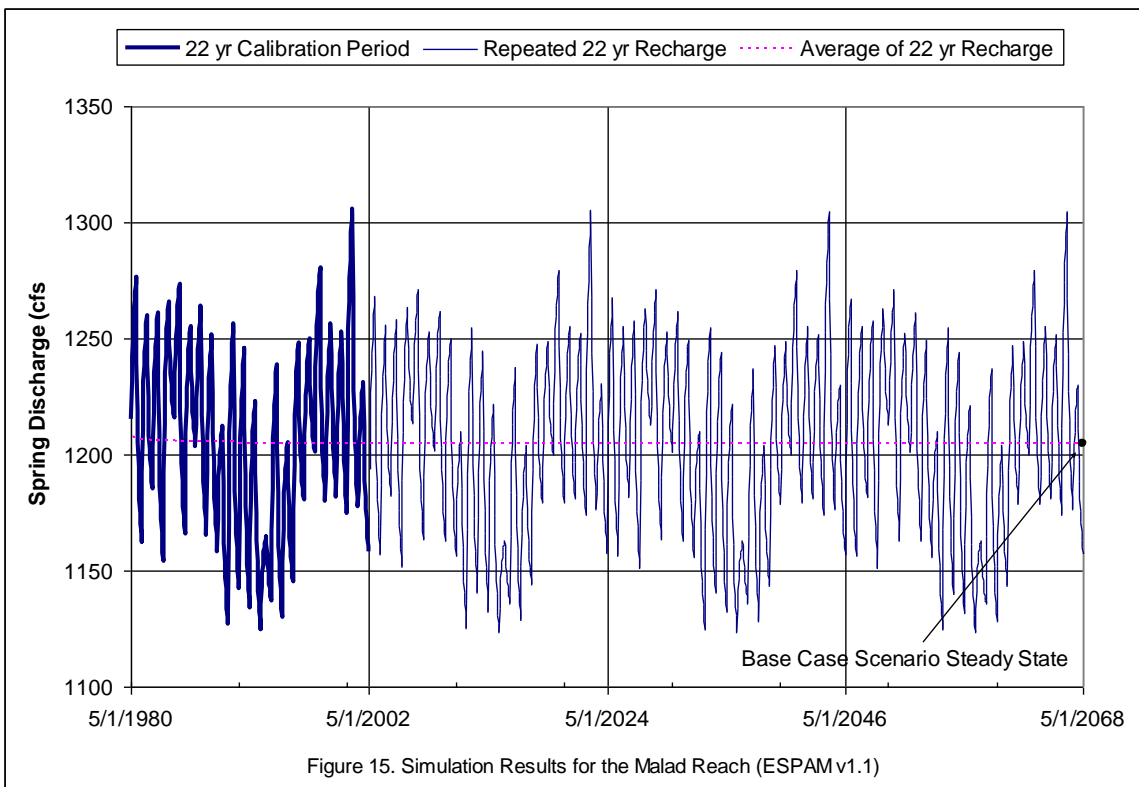


Figure 15. Simulation Results for the Malad Reach (ESPAM v1.1)

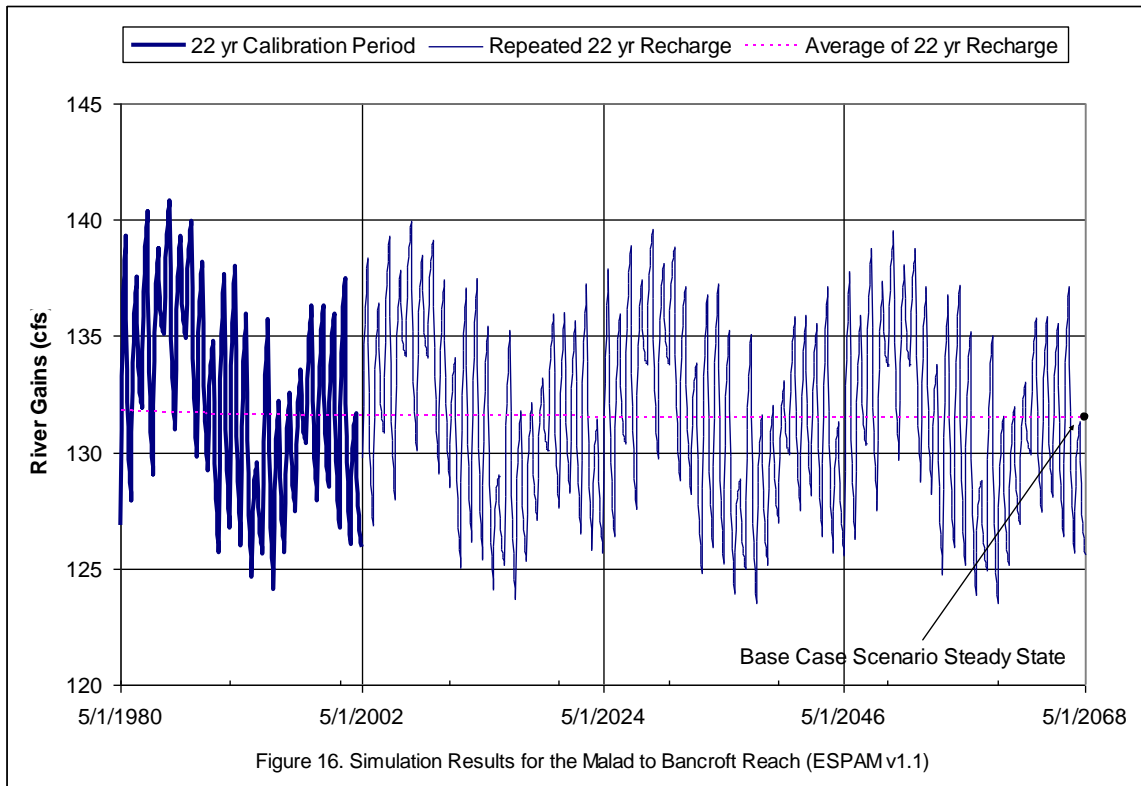


Figure 16. Simulation Results for the Malad to Bancroft Reach (ESPAM v1.1)

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APPENDIX A
Method of estimating confidence bars on water budget

The confidence bars on the water budget are based on the variability *in the estimation and measurement methods* of individual components of the water budget. The variance of the sum of two numbers is the sum of the individual variances, plus twice the covariance (Clemens and Burt 1997). If more than two numbers are summed, all pair-wise covariances need to be considered, along with multi-way covariances. However, multi-way covariances are usually small enough to ignore (Dakins 2003). Covariances can be positive or negative. A negative covariance reduces the overall uncertainty because the components tend to move in opposite directions.

For the water budget analysis, the variance of the water budget components is estimated based on expected accuracy of the estimation methods, expressed as plus or minus a given percentage. Using the percentage, a range is calculated, and the standard deviation is estimated as one-fourth the range. The estimate of standard deviation is squared to give an estimate of variance. Table A1 summarizes the estimated standard deviations.

Table A1
 Estimation of Water-Budget Standard Deviation

Water-Budget Element	Acre Feet/year	Confidence, +/-	Estimated Standard Deviation, acre ft/yr
ET on irrigated lands	-5,370,000	10%	270,000
Recharge on non-irrigated lands	527,000	50%	130,000
Precipitation on irrigated lands	1,720,000	10%	86,000
Net surface-water irrigation deliveries	6,870,000	5%	170,000
Offsite groundwater pumping	-66,000	10%	3,300
Fixed-point withdrawals	-139,000	10%	7,000
Seepage from perched tributaries	274,000	15%	21,000
Tributary valley underflow	924,000	50%	230,000
Canal leakage	459,000	10%	23,000

Though many of the components of the water budget are hydrologically correlated, only precipitation on irrigated lands and recharge from precipitation on non-irrigated lands are considered here to be significantly correlated *in their estimation methods*. Both depend on the same precipitation data set. Because we have no data for repeated measurements of these two components at a single time, we cannot directly calculate the covariance. However, we can intuitively estimate a correlation coefficient. A coefficient of 1.0

suggests that the two estimates would be perfectly correlated, negative -1.0 suggests perfect inverse correlation, and a coefficient of zero suggests no correlation. The definition of the correlation coefficient (Montgomery and Runger 1994) is:

$$r = s_{12} / (s_1 s_2)$$

Where s_{12} is the covariance, s_1 is the standard deviation of the first component, and s_2 is the standard deviation of the second.

This can be rearranged to calculate an estimate of covariance from estimates of correlation and standard deviations:

$$s_{12} = r s_1 s_2$$

Because the irrigated-lands calculation is linear and the non-irrigated lands calculation is non-linear, a correlation of 0.80 is assumed. A correlation of zero is assumed for all other pair-wise and multi-way interactions. The estimated variances of water budget components sum to 1.8×10^{11} (acre ft/year)² and the estimated covariance is 8.9×10^9 (acre ft/year)². The estimated combined variance is 2.0×10^{11} (acre ft/year)² and the estimated standard deviation is 440,000 acre feet per year.

This analysis ignores the reduction in uncertainty that may come from negative correlation that occurs during balancing of the water budget.

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