THE DETERMINATION OF FROZEN GROUND PROBABILITIES FROM CLIMATIC AND HYDROLOGIC DATA

A Thesis

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LIST OF SYMBOLS

ρ: The air density near the ground surface. g/cm^3.

ρ_d: The population autocorrelation coefficient at lag d.

ρ_i: Canonical correlation or autocorrelation coefficient at lag i.

δ_{ij}: Normalized eigenvector.

λ: Dimensionless correction coefficient or latent root.

Σ: The population of a variance-covariance matrix.

μ: The mean of the population.

A: Among group of sum of square and cross products matrix.

C_p: The specific heat of air at constant pressure. 0.24 cal/g-C.

D_a: The estimator of among-group variance-covariance matrix.

D^2_{ij}: Mahalanobis' distance.

D_w: The estimator of within-group variance-covariance matrix.

E: Expectation.

F: The surface freeze index. Degree days F.

$F(n_1,n_2)$: F-value with degrees of freedom n_1,n_2.

g: Number of groups.

K: The turbulence coefficient for thermal conductivity. BTU.hr/ft/F.

K_f: Thermal conductivity of frozen ground. BTU/hr/ft/F/

L: Latent heat of soil. BTU/ft^3.

M: Box's test criterion.

n_1: The number of observation in group 1 or degrees of freedom.

n_2: The number of observation in group 2 or degrees of freedom.

N_i: The number of observation in group i.

$P(H_j|X_i)$: A posteriori probability under the given observation X_i.

$P(H_j)$: A priori probability in a given group H_j.

p: The number of variables.

r_k: The estimator of the autocorrelation coefficient at lag k.

S_j: A pooled within-group variance-covariance matrix of group j.

T: The heat exchange between ground and air. cal/cm^2·sec. or total sum of square and cross products matrix.

Δt: The conventional vertical air temperature difference, C.

t: The latent heat released as pore water freezes to a depth X in time t, hour.

U: Wilk's $U=|W|/|T|$, $|W|$: Determinant of "within-group" sum of square and cross products matrix. $|T|$: Determinants of "total" sum of square and cross products matrix.

Var: Variance.

V_i: Coefficient vector of discriminant variables.

V_s: Temperature of soil surface F.

W: Within-group sum of square and cross products matrix.

X: The depth of frost penetration. ft.

$\overline{X}_j$: The mean of the observations of the variable j.

X_{mjk}: The value of the observation of m discriminant function, k observation, j variable.

y_{mk}: The m-th discriminant function evaluated for observation k. y_{mk} is the first discriminant function.

z_{mk}: The m-th discriminant function evaluated for observation k. z_{mk} is second discriminant function.

ABSTRACT

The prediction of frozen ground occurrences can help the local planner regulate development in areas which are subject to floods. Discriminant analysis was used in the Missouri Flat Creek case in order to study past frozen or unfrozen ground runoff events using eight meteorological factors. These eight factors are number of freezing days in a sequence, freeze index (sum of degree-days below 32°F), precipitation two days before the start of the freeze period, precipitation during the freeze period, precipitation four days after the freeze period, the depth of snowfall during the freeze period, the depth of snow on the ground before and total stream flow 4 days after.

After a series of analyses, the eight meteorological factors used were determined to be sufficient to classify the runoff events into frozen or unfrozen ground events. The freeze index was found to be the single most important factor for estimating the frozen or unfrozen ground condition in the Missouri Flat Creek Basin. Ten of 89 unknown samples are probably frozen ground and the rest of them probably belong to the unfrozen ground class.

CHAPTER I

INTRODUCTION

GENERAL

Frozen ground is one of the important factors causing winter floods in the Pacific Northwest during the winter and early spring months. On February 26, 1948, a flood in the South Fork Palouse River Basin caused extensive damage in Pullman, especially in the mobile home court located southeast of town. This flood was probably triggered by rainfall on frozen ground. In 1962 and 1964, there were two other very severe floods in Idaho. The USGS and SCS reported that the cause of flooding was prolonged rainfall on snow and frozen soil. Johnson and McArthur (1973), in a study of winter flooding in Idaho and surrounding areas for 1955-1972, found that the amount and intensity of the rainfall, the amount of snowmelt, and the imperviousness of the frozen soil combined to affect the flood severity. Because of extensive damage from each flood, millions of dollars have been spent on flood control every year.

Climatological data has been found to be important in studying frozen ground. McCool and Molnau (1974) used the historical climatological record to study both the frequency and the severity of frozen soil conditions. They concluded that climatological data can be very useful in assessing the overall frozen ground hazard for a given location.

The Missouri Flat Creek drainage basin was selected as a test case. The period of record for this investigation is 1960 to 1973.

OBJECTIVE

The prediction of frozen ground occurrences under certain conditions will help local planners regulate development in areas which are subject to floods. For the purposes of this thesis, frozen ground floods or runoff events are defined as rainfall or snowmelt runoff events which occur when there is a crust of frozen soil covering the watershed to such an extent such that the infiltration rate of the watershed is decreased and the runoff is more than that which would occur under conditions of no frozen soil. Discriminant analysis was chosen to help provide a simple technique which can show the probability of occurrence of frozen ground runoff events in the past for a sequence of meteorological events. The number of freezing days, freezing index, precipitation two days before the start of the freeze period, precipitation during the freeze period, precipitation four days after the freeze period, the depth of snowfall during the freeze period, the depth of snow on ground before and stream flow four days after are chosen as discriminators.

The objective of this study is to:

1. Identify some hydrologic parameters that are important in distinguishing frozen from unfrozen ground runoff events.
2. Identify historical frozen ground runoff events using these paramenters.

CHAPTER II

LITERATURE REVIEW

One of the major results of the transformation of water into ice and vice versa in soil is the erosion of the soil. Heavy rainfall over frozen or partially frozen soil, or on snow, often causes drastic soil erosion and winter floods.

Many factors govern whether or not soil is frozen. Bouyoucos (1916) grouped the influencing factors into two categories, (a) intrinsic and (b) external. He considered the intrinsic factors to be those governed by the properties of the soil and its cover, and the external factors to be basically meteorological in nature (Moulton, 1968). Similar classifications have been presented by Pelton, Campbell and Nicholaichuk (1968) and by Linell and Kaplar (1965). Since only meteorological factors are to be considered here, only external factors will be discussed in this literature review.

INFLUENCES OF METEOROLOGICAL FACTORS ON FROZEN GROUND

The meteorological factors affecting the freezing of soil are considered to include the air temperature, solar radiation, wind and precipitation (Moulton, 1968).

The temperature factors exert a direct influence on the transfer of heat to or from the earth. Molga (1962) mentioned that the air layers near the ground are heated during the day by the active surface warmed by solar rays (termed "active surface" by Molga), the thin (several mm thick) air layer adhering to the active surface, called the boundary layer, is warmed by the surface. On winter nights, its thickness usually does not even reach 1 mm. The vertical temperature gradients are the greatest in the vicinity of the active surface and decline with increasing height, rather rapidly at first and more gently higher above the ground. This is due to the increase in the turbulence coefficient with height since the temperature gradient, Δt, is inversely proportional to the turbulence coefficient, K, according to the formula (Molga, 1962):

$$T = C_p * p * K * \Delta t$$

Where: T: The heat exchange between ground and air.

C_p: The specific heat of air under constant pressure, equal to 0.24 cal/g.C.

ρ: The air density near the ground surface, equal to 0.0012-0.0013 g/cm^3.

K: The turbulence coefficient.

Δt: The conventional vertical air temperature gradient, °C.

Frankline (1920) found that wind caused the surface temperature to be lower than the prevailing air temperature, and that this effect became more pronounced as the wind velocity increased or as the relative humidity decreased. In other words, high wind of low relative humidity can have a considerable effect upon subsurface temperatures.

Eno (1929) discussed the important influence of climate and the resultant frost action upon the performance of highways in the United States. Sourwine (1929) in a discussion of Eno's paper suggested a method to relate quantitatively air temperature data to the relative danger of ground freezing in any given locality. He defined three controlling variables that could be evaluated using data available from climatological records. These were:

1. The average minimum temperature.
2. The duration of the cold period.
3. The frequency of occurrence of a cold period of given intensity.

Sourwine concluded that an air temperature of 3°F represented a "critical absolute minimum value" coefficient with a 5 percent frequency of occurrence of ground freezing at the 3 inch level. He was able to relate this absolute minimum air temperature to monthly average minimum temperatures and the duration of the cold period to represent the southerly limits of serious danger to highways from ground freezing. This strongly suggests that the combined effect of low temperature and the duration of the cold period on ground freezing can be an essential variable for estimating frozen ground. He also introduced "degree-hours", with the degrees being measured in °F below the critical initial value of air temperature causing freezing at the surface and the duration being measured in hours of cold period below the critical initial value of air temperature.

Sourwine also suggested that some relationship might exist between this "degree-hour index" and the depth of frost penetration. Casagrande (1931) conducted field tests near the Massachusetts Institute of Technology and concluded that the relationship suggested by Sourwine existed. Later Casagrande found a good correlation between cumulative degree-days of below freezing temperature and the depth of frost penetration. Aldrich (1956) used the Stefan formula and the modified Berggren formula to predict the depth of frost penetration. The Stefan formula is:

$$X = \sqrt{48K_fF/L}$$

Where: X: Depth of frost penetration in ft.
K_f: Thermal conductivity of frozen soil. BTU/hr/ft/F.
F: The surface freezing index, degree days F.
L; Latent heat of pore water, BTU/ft.^{3}

Since this neglects the volumetric heat of the frozen and unfrozen soil and, hence, overestimates frost penetration, it has been modified by Berggren to :

$$X = \lambda\sqrt{2KVst/L}$$

Where: X: Depth of frost penetration in ft.
K: Thermal conductivity of frozen or unfrozen soil, BTU/hr/ft/F.
Vs: Temperature of soil surface °F.
t: Hours.
L: Latent heat of pore water, BTU/ft.^{3}
λ: Dimensionless correction coefficient.

Corrects the Stefan formula for the effects of volumetric heat which it neglected.

Aldrich (1956) also used air temperature in determining the freezing index because data on surface temperatures are almost nonexistent. Both the Stefan and the modified Berggren equations may also be used to predict the depth of thaw of a frozen soil. The "thawing index" used is calculated in degree-days as the mean daily air temperature in excess of 32°F.

Aldrich (1956) described the relationship between air and pavement temperatures and concluded that it was best to use the air freezing index with a correction factor to adjust it to an equivalent pavement surface freezing index. The value of freezing index for a specific location might differ greatly from the general value for the area, and that freezing index might vary widely over a small area (Straub and Wegmann, 1965).

McCool and Molnau (1974) investigated the freezing index as a parameter to predict frozen soil in the Palouse Basin. From an investigation of the 1953 through 1973 data, they found that individual freezing events with freeze index values of less than 100 were generally not associated with large runoff events, and values of 150 and over seldom left any doubt about a frozen ground flood if rain occurred at the end of the freezing event. They also concluded that the freezing index and precipitation combinations can be used, with some discretion, in the prediction of frozen soil runoff events.

They also examined the Missouri Flat Creek streamflow data. They found that many high volume runoff events occur independently of frozen soil conditions. During water years 1953-1973 for periods from 1 to 19 days in length and for runoff totalling greater than 0.5 inch, only 7 of 23 events were definitely associated with rain on frozen soil and one more event was probably associated with runoff on frozen soil. The remaining fifteen were caused by rain and rain on snow. Therefore, we know that runoff is not a single process. It is the result of several separate processes.

McArthur (1971) studied flood runoff from rainfall and snowmelt over a frozen soil mantle in Idaho and concluded that winter flooding is caused by:

1. Antecedent moisture and temperature conditions that produce frozen soil relatively impermeable to water infiltration.
2. A sudden warming period with daily minimum temperatures above 32°F.
3. An accompanying storm producing rainfall for a period of three or more days.

He also presented the following table that shows the hydrologic conditions associated with winter flooding in Idaho.

TABLE 1: HYDROLOGICAL CONDITIONS ASSOCIATED WITH WINTER FLOODING IN IDAHO (McARTHUR, 1971)

Factor	Effect upon flood severity
Excess runoff from	
a. Rainfall	Proportional to total volume
b. Snowmelt	and to the rainfall intensity
c. Both	
Frozen soil	
a. Concrete frost	Increases runoff
b. Porous concrete frost	Can reduce runoff
c. Stalactite frost	Reduces runoff
Snow cover	
a. Deep	Decreases runoff
b. Shallow	Increases runoff
Vegetative cover	
a. Tree, larger type vegetation	Decreases runoff
b. Rangeland, sagebrush and grass	Decreases runoff
c. Cultivated lands	Generally increases runoff
Atmospheric temperature	
Lapse rate	
High	Decreases runoff
Low	Increases runoff (Snowmelt and contributing area)

In Figure 1, the precipitation occurred as snow during January because of the severe cold period. During this period the ground was frozen. A concrete frost was not prevalent because of the antecedent conditions; that is, warmer than usual temperatures and below normal precipitation in November and December. Soil moisture was not enough to produce a concrete, impermeable frost. During January there was a shallow snow cover. Toward the last of January there was a warming period and a storm moved through this area depositing a large volume of rain. So, there was a peak flow probably caused by rainfall on snow.

Merrell (1964) used frequency methods to examine rainfall, snowmelt, and frozen ground conditions in order to compute joint probabilities associated with combinations of these three parameters. He compared joint probabilities with the qualitative record of flooding. As a result, he found that the severity of flooding is not closely related to the joint probability of all three parameters in the Sand Creek watershed. According to the author's study, he did not prove this conclusion is valid for other places.

Turner and Jumikis (1956) found that, when the precipitation was in the form of snow, more of the precipitation ultimately percolated into the ground than if the precipitation was in the form of rain. Snow cover exercises a decisive influence on the frozen ground. The albedo of snow is high, amounting to 75-88% and more on fresh snow, and to approximately 45% on older and moist snow. Thus snow

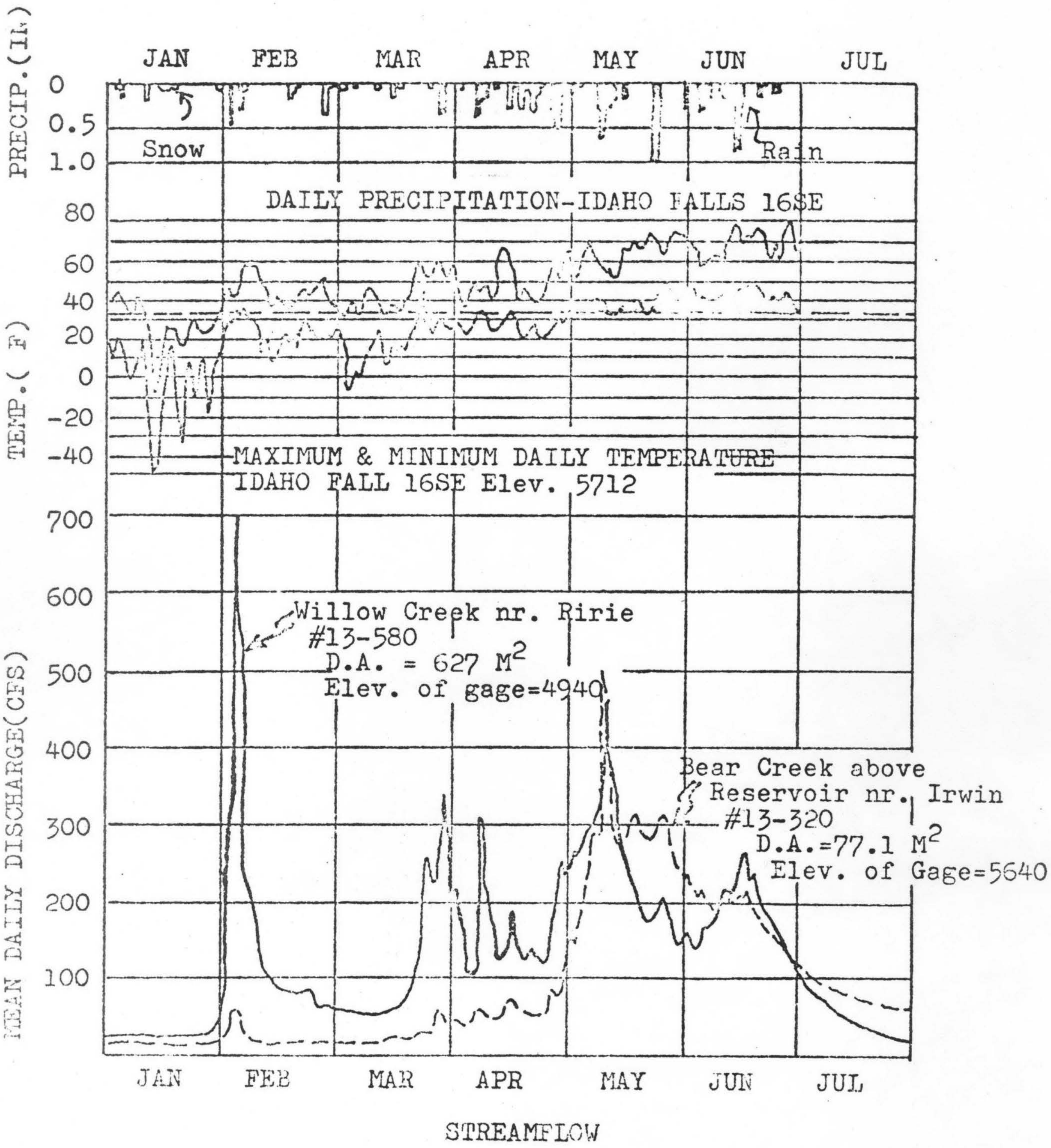


FIGURE 1 Comparison of the runoff hydrograph of Willow Creek with that of Bear Creek

warms up slightly and does not melt quickly under the influence of solar radiation. Even with the high albedo of snow cover, some of the solar rays penetrate its depth and the snow melts from below. If the soil is covered by a snow layer of over 60 cm depth, the soil is completely insulated against the influence of solar radiation (Molga, 1962). The insulating properties of a snow cover also exerts an influence on the runoff in springtime. By protecting the soil from freezing, the snow cover permits water to percolate into the soil and reduces surface runoff. These environmental factors affecting soil freezing can be summarized as follows:

1. Solar radiation: The amount of radiation received by the soil surface depends on:
    a. The angle with which the soil faces the sun, due to latitude, season, time of day, steepness and direction of slope, and the altitude of the location.
    b. The insulation of air, water vapor, clouds, dust, smog, snow, plants, or mulch.
2. Radiation from the sky: The sun's energy is absorbed by the atmosphere and is radiated in all directions. Then soil absorbs from sky indirectly.
3. Conduction and convection of heat from the atmosphere: Convection or wind is necessary to affect the soil temperature significantly since

the conductivity of air is very small.

4. Condensation: Condensation is an exothermic process. Freezing of water generates heat.
5. Evaporation: It is an endothermic process. The greater the rate of evaporation, the more the soil is cooled down. Also thawing of ice absorbs heat.
6. Rainfall: Rainfall, depending on its temperature, can cool or warm the soil.
7. Insulation: The soil can be insulated by plant cover, mulch, snow, and also clouds and fog. In winter, insulated soil is warmer than soil that is directly exposed to the environment.

INFLUENCES OF SOIL FACTORS ON FROZEN GROUND

The permeability and storage of frozen soil depend on the type of soil frost. Soil frost can be divided into four types (Trimble, Sartz, and Pierce, 1958):

1. Concrete frost: This type of frost usually appears in bare agricultural land. It is composed of a great many thin ice lenses, with the ground surface being very hard, like concrete.
2. Granular frost: It is always found in woodland soils which contain organic matter. This type of frozen soil contains small frost crystals mixed with the soil particles, but remain separate from each other and are easily broken.

3. Honeycomb frost: It is often found in highly aggregated soils and it has a loose, porous structure which looks like honeycomb.
4. Stalactite frost: It forms in bare saturated ground and consists of loosely fused, columnar ice crystals and absorbs rainfall-snowmelt like a sponge because of its rough and very porous microrelief. As stalactite frost melts, the infiltration rate decreases.

Haupt (1967) reported that concrete frost reduces the infiltration capacity so that overland flow is increased on burned or sparsely vegetated sites if the rainfall intensity exceeds 3 ins./hr. He also found that snow cover, by cooling rain water, tends to preserve soil frost and keep it visibly intact and that cover, such as plants, litter, and snow, can absorb raindrop energy and increase infiltration. But exposed rock usually accelerates overland flow and erosion. Pelton, Campbell and Nicholaichuk (1968) found that infiltration rates of frozen soil vary widely and depend largely upon the soil moisture content at the time of freezing. They found that the trapping of snow on the soil surface and its subsequent melting and infiltration appear to be dominant factors in conserving winter moisture. When moisture accumulates and freezes near the soil surface, thermally induced-moisture translocation is also a factor contributing to the reduction of infiltration rate. Bloomsburg and Wang (1969) in laboratory studies found that below, then frost depth prediction methods based on an assumed 32°F freezing point will tend to underpredict the depth of the 32°F isotherm. Thermal conductivities of soil-forming minerals, while that of air is about 100 times smaller than that of soil-forming materials (Kohnke, 1968). Kersten (1949) found that the difference between the thermal conductivities of frozen and unfrozen soils was dependent upon their moisture content. As the water content increased the thermal conductivity of the frozen soil became considerably higher than that of the unfrozen soil. Kersten's tests showed that as the quartz content decreased and the clay mineral content increased there was a general decrease in thermal conductivity. He also found that the thermal conductivity decreased with decreasing grain size at a given density and moisture content, and an increase in soil density resulted in an increase in thermal conductivity. Patten (1909) showed that the thermal conductivity of soils increased continuously as the soil moisture content was increased to saturation.

A soil in undisturbed condition has a higher thermal conductivity than when it is recently disturbed. A blocky structure has a higher thermal conductivity than organic soil. But the differences of thermal conductivity due to structure and texture are small compared with those due to moisture changes (Kohnke, 1968).

The ice content of the soil has a very strong effect on meltwater absorption in a basin. The higher the ice the parameter $\phi(1-s)$ (ϕ is porosity and s is saturation) at the start of freezing is directly related to the permeability after freezing by an exponential relationship. This means that the permeability decreases as $\phi(1-s)$ decreases. Also, they found that the value of $\phi(1-s)$ must be less than about 0.13 in silts and sands for the permeability to be zero when the soil is frozen.

The most significant thermal properties to be considered with respect to the determination of frost depth are (1) volumetric heat, (2) latent heat of fusion, and (3) thermal conductivity. The soil temperature is determined by the interaction of numerous factors. All soil heat comes from two sources; radiation from the sun and sky and conduction from the interior of the earth.

The thermal capacity of soil is determined by the specific heat of mineral soil and solution in soil. The specific heat of water is larger than many substances (1.0 cal/gm). The specific heat of ice is 0.5 (Kohnke, 1968). The magnitude of volumetric heat capacity of the soil exerts a relatively small influence on the depth of frost penetration. Also the latent heat of fusion of a soil depends only upon the amount of water in a unit volume of the soil. Practically, in the prediction of the depth of frost penetration, it is assumed that all the soil water freezes at 32°F and that all the latent heat of fusion is liberated at that temperature. If substantial portions of the soil water do not freeze at a temperature of 32°F and content in the soil, the narrower these pores and passages are and, the lower the permeability of the soil. Komarov and Makarovo (1973) reported that the main factors which influence the permeability of the soil prior to snow melting are its ice content and depth of freezing. They also found that the spatial variations of these factors, are very large and are an important factor in the infiltration of meltwater in a basin. Borisovskiy (1973) showed that the temperature of drained soils is much lower throughout the winter than that of undrained soils. In his studies, in the warm period, the termperature of undrained soils is 1.5-2.5°C higher than in drained soils. Conversely, in the cold period, drained soils average 1.5-2° colder than undrained soils.

MULTIVARIATE STATISTICAL ANALYSIS

Multivariate analysis is concerned with analyzing multiple measurements that have been made on N individuals and are considered in combination, as systems. So m measurements which have been made on N individuals can be represented as N points in the m-dimensional space. Therefore, in order to analyze the relative importance of various meteorological factors on frozen soil prediction, it was decided to use some type of multivariate analysis.

Multivariate statistical analysis as applied to hydrologic problems is a new field and still in its developmental and trial stage. Wallis (1965) used the characteristics of the hollow cylinder as an example to discuss and compare the effectiveness of multivariate analyses. He concluded that the different methods of generating prediction equations do not give identical answers, even when the same data are used. Therefore the method that will be most suitable for each specific problem should be selected. He recommended a combination of principal component regression with varimax rotation of the factor weight matrix for an initial analysis of multifactor hydrologic problems. He also introduced cluster analysis in hydrology which can be used if many observations are available. Hastay and Gladwell (1969) used a combination of least squares regression with principal component analysis to evaluate a cloud-seeding program at the streamflow control level. They found that using principal components has the following advantages.

1. Conserving degrees of freedom in the estimate of residual variance.
2. Computational convenience, especially in making statistical tests to identify significant predictors.
3. Superior conditioning of covariance matrices against the effects of multicollinearity, thus increasing the precision of statistical tests.

Rice (1967) said that multivariate methods free the hydrologist from the need to describe a phenomenon as a a single number. So if the hydrologist uses multivariate methods, he can cope with the highly correlated "independent" variables common in hydrology. Also these methods provide a flexible approach that is more in harmony with nature of many hydrologic problems. Wallis (1967) used a discriminant function for testing the identification of marine versus nonmarine sediments based upon microelements. He reported that the prediction of the behavior of one or more variables for one or more of a new population of watersheds were often found to be associated with large errors. According to Wallis, the errors come from the specification of the original model and unwarranted assumptions concerning the degree of similarity between the original and the new group of watersheds. He also discussed whether a prediction equation developed from the watersheds of group 1 is applicable to all or any of the watersheds of group 2. Before answering this question, the following questions must first be answered:

1. Is there a significant difference between the two groups of watersheds?
2. Is a significant difference in the groups relevant to the specific criterion being predicted?

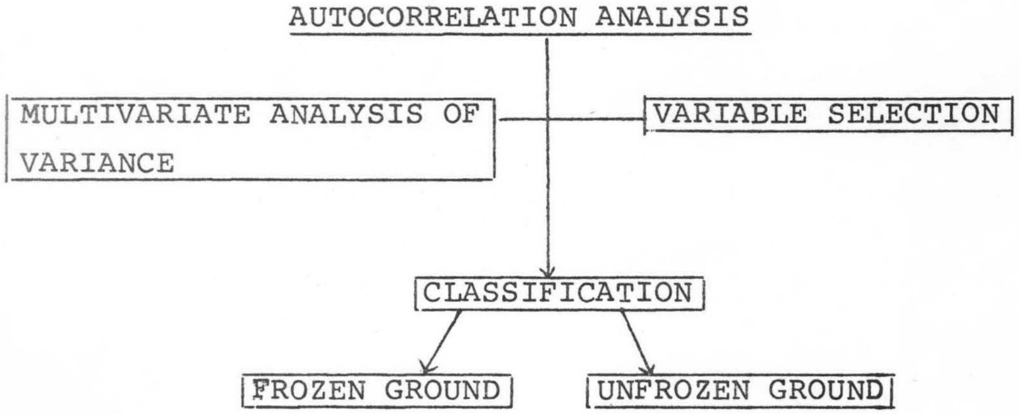
He recommended discriminant analysis to solve this type of uncertainity problem. DeCoursey (1973) used discriminant analysis to analyze factors related to the adequacy of standard slope protection on small dams. His analysis showed that a linear combination of the fetch length, the surface area to length ratio (average width), and the plasticity index of the surface material on the dam could be used to distinguish between the two groups about 85% of the time. Thus a discriminant equation can be used to assign structures to the two groups (those sites with no indication of damage as in group 1 and those sites with observed damage as group 2) such that the additional protection can be most economically provided. Miller (1962) used discriminant analysis to forecast airfield ceiling conditons up to two hours in advance.

But the application of discriminant analysis to determination of frozen soil or unfrozen soil is a new challenge. Although many assumptions in discriminant analysis do not fit the real world perfectly, this method still may be one possible way to classify soild into frozen or unfrozen conditon groups.

CHAPTER III

EXPERIMENTAL METHOD

The computation procedures used in this thesis are shown below and described in the following sections.



AUTOCORRELATION ANALYSIS

Autocorrelation is used for investigating dependence of one observation on another observation. The population autocorrelation coefficient of an observation series, X_i, is defined as

$$\rho_d = \mathrm{cov}(X_i, X_{i+d})/\mathrm{var}(X_i) \qquad (1)$$

Where $\mathrm{cov}(X_i, X_{i+d})$ is the covariance between X_i and X_{i+d}, $\mathrm{var}(X_i)$ is the variance of X_i and d is the lag.

For a discrete series, the value of ρ_d is estimated from a sample of size N and lag k and the estimator is

$$r_k = \mathrm{cov}(X_i, X_{i+k})/[\mathrm{var}(X_i) * \mathrm{var}(X_{i+k})]^{\frac{1}{2}} \qquad (2)$$

For an uncorrelated series, the sampling distribution of r_k has an expected value $E(r_k)$ and a variance $\mathrm{var}(r_k)$ given as

$$E(r_k) = -1/(N-k+1) \tag{3}$$

$$\mathrm{var}(r_k) = [(n-k+1)^3 - 3(N-k+1)^2 + 4]/(N-k+1)^2 \\ [(N-k+1)^2 - 1] \tag{4}$$

If a value of N is larger than 30, the sampling distribution of r_k may be approximated by a normal distribution (Torranin, 1972). The 95% confidence limits of the uncorrelated series can be computed by

$$\rho(k) = E(r_k) \pm 1.96[\mathrm{var}(r_k)]^{\frac{1}{2}} \tag{5}$$

If the sample correlogram lies within the confidence band, and/or if only a small percentage of r_k values defined by the confidence band lies outside these limits, this series can be considered to be serially uncorrelated.

MULTIVARIATE ANALYSIS OF VARIANCE

After the data are divided into two or more groups, it must be determined whether or not these groups have different population means among them and whether or not these groups all come from the same population. The mathematical model of multivariate analysis of variance is

$$\underline{X}_{ki}=\mu+(\mu_k-\mu)+(X_{ki}-\mu_k) \tag{6}$$

Where X_{ki} is the dependent vector variable for the i-th observation in the k-th group.

μ is the vector of total samples means.

μ_k is the centroid for group k.

$(k = 1,2,----,g.\ i = 1,2,----,N_k.\ N = N_1+N_2+--+N_g)$

From multivariate analysis of variance,

$$T = A + W \tag{7}$$

where T is total sum of square and cross products matrix of pxp order (p is the number of variables). A is the among-group sum of square and cross products matrix. W is the within-group sum of square and cross products matrix. The estimator of W based on the pooled within-groups deviations is

$$D_w = W/(N-g) \tag{8}$$

The estimator of A based on the group means vector is

$$D_a = A/(g - 1) \tag{9}$$

Test of Homoscedasticity of Dispersion Matrices

Box (1949) defined the test criterion M for

$H_o\colon \Sigma = \Sigma_k$, k = 1,2,----,g

$$M = (N - g)* \ln|D_w| - \sum_{k=1}^{g}(N_k-1)*\ln|D_k| \tag{10}$$

Required functions of the design parameters are

$$f_1 = [\sum_{k=1}^{g} 1/(N_k-1) - 1/(N-g)]*(2*p^2+3*p-1)/6(g-1)(p+1) \tag{11}$$

$$f_2 = [\sum_{k=1}^{g} 1/(N_k-1)^2 - 1/(N-g)^2]*(p-1)(p-2)/6(g-1) \tag{12}$$

If $f_2 - f_1^2 > 0$, then

$$n_1 = (g-1)p(p+1)/2$$

$$n_2 = (n_1+2)/(f_2-f_1^2)$$

$$b = n_1/[1-f_1-(n_1/n_2)]$$

$$F(n_1,n_2) = M/b \tag{13}$$

If $f_2 - f_1^2 \leq 0$, then

$$n_1 = (g-1)p(p+1)/2$$

$$n_2 = (n_1+2)/(f_1^2-f_2)$$

$$b = n_2/[1-f_1+(2/n_2)]$$

$$F(n_1,n_2) = n_2*M/n1(b-M) \tag{14}$$

If $F(n_1,n_2) < F_o$ at α% significance level and d.f.$=n_1,n_2$, we accept the null hypothesis. Otherwise we reject the null hypothesis.

Test of Equal-Group-Mean Vectors

The null hypothesis is $H_o:\mu_1=\mu_2=\text{-------}=\mu_g$. Wilk's determinant ratio test statistic is defined as

$$U = |W|/|T| \tag{15}$$

For Rao's F approximation it is necessary to compute a set of functions of the design parameters.

$$s = \sqrt{[p^2(g-1)^2-4]/[p^2+(g-1)^2-5]}$$

$$n_1 = (p*g-1)$$

$$n_2 = s[(N-1)-(p+(g-1)+1)/2]-[p(g-1)-2]/2$$

Now let $y=U^{1/s}$

$$F(n_1,n_2) = [(1-y)/y](n_2/n_1) \tag{16}$$

If $F(n_1,n_2) < F_o$ at a significance level of α% and d.f.$=n_1,n_2$, we accept the null hypothesis, H_o at a certain level, and vice versa.

DISCRIMINANT ANALYSIS AND CLASSIFICATION OF DATA

Discriminant analysis is used to find out whether there is a set of the variables that can differentiate between the groups and use these variables to assign future individuals to a group.

In this thesis, the BMD-Biomedical Computer Program (Dixon, 1965) and the Statistical Analysis System (Barr and Goodnight, 1972) were used to analyze the frozen ground events. SAS-Discriminant Analysis was chosen for the analysis of frozen events because it can handle the conditions of unequal within group variance-covariance matrices and unequal priori probabilities between frozen and unfrozen events.

The BMD-Stepwise Discriminant Analysis can be used to select variables which have capability to discriminate the groups and also it can compute canonical correlations and coefficients for discriminant functions so that we can use these first two discriminant functions to plot two-dimensional pictures of the dispersion. The equations for the discriminant equation are as follows

$$(A-\lambda_i W)*V_i=0 \qquad (17)$$

A and W are as defined previously. V_i is the eigenvector (coefficient vector for discriminant function) corresponding to the eigenvaule λ_i. If the eigenvector is normalized we obtain

$$V_i W^{-1} V_j = \delta_{ij} \qquad (18)$$

The canonical correlations of $r_1, r_2, \text{--------}, r_p$ relative to the groups are

$$r_i = \sqrt{\lambda_i/(1+\lambda_i)} \qquad (19)$$

The BMD program was to obtain first two discriminant functions for plotting a two-dimensional scattergram. The equations are:

$$y_{mk} = \sum_{j=1}^{r} v_{j1}*(X_{mkj}-\overline{X}_j)$$

$$z_{mk} = \sum_{j=1}^{r} v_{j2}*(X_{mkj}-\overline{X}_j) \tag{20}$$

The SAS-Discriminant Analysis uses a Bayesian conditional probability interpretation of discriminant analysis and did not reduce the problem to 2 dimension but used the 8 variables directly to classify our data.

The classification of data is concerned with quantitative methods for the assignment of an individual observation to one of several groups on the basis of multiple measurements on that observation. It is assumed that each observation belongs to one of several specified groups or populations. Several relevant characteristics of the individual observation are observed, and on the basis of this information, a classification decision is made. The method used to assign individuals to these groups must have a minimum probability of error.

Let the conditional probability of an observation X_i (a p x 1 vector) under the given group H_j be $P(X_i|H_j)$. According to the Bayesian conditional-probability model, we get

$$P(H_j|X_i)=P(X_i|H_j)*P(H_j)/ \sum_{j=1}^{g} [P(X_i|H_j) *P(H_j)] \tag{21}$$

$i=1,2,$--------$,N$(Number of observation).

$j=1,2,$--------$,g$(Numbers of group).

$P(H_j)$ is called a priori probability in a given group H_j. $P(H_j|X_i)$ is called a posteriori probability under the given observation X_i. The probability that the observation X_i belongs to group H_j is equal to the probability of the observation under group H_j times a priori probability divided by total probability.

Let it be assumed that the a priori probabilities exist and are estimable from the group relative frequencies, that is

$$\hat{P}(H_j) = n_j/N \tag{22}$$

Where: N is the total sample size.

n_j is the sample size in the group j.

Suppose a posteriori probability $P(X_i|H_j)$ has a multivariate normal distribution. Then

$$\hat{P}(X_i|H_j) = (|S_j^{-1}|/2\pi^{p})^{\frac{1}{2}}*\exp[(-\tfrac{1}{2})(X_i-\bar{X}_j)'S_j^{-1}(X_i-\bar{X}_j)]$$

Where S_j is a within group variance-covariance matrix from samples in group j. Then

$$P(H_j|X_i) = \exp(-\chi^2_{ij}/2)/\sum_{j=1}^{g}\exp(-\chi^2_{ij}/2) \tag{24}$$

Where: $\chi^2_{ij}=(X_i-X_j)'S_j^{-1}(X_i-X_j)+\ln|S_j|-2*\ln[P(H_j)]$

If the model of the a posteriori probability $P(X_i|H_j)$ is unknown, a nonparametric method (Pelto, 1969) could be used. Here, it is assumed that the data itself has a multivariate normal distribution, but the author did not verify it.

OTHER TOPICS IN DISCRIMINANT ANALYSIS

Lachenbruch (1968) introduced a method for the determination of the sample size for two groups in discriminant analysis. He said that for the sample discriminant function to have an error rate within some given tolerance limit, n_1 and n_2 observations are necessary. How n_1 and n_2 are obtained depends on p (number of variables), tolerance limit, and $(\mu_1-\mu_2)'\Sigma^{-1}(\mu_1-\mu_2)$, where μ_1 is the population mean in group 1, μ_2 is the population mean in group 2, and Σ is population within-group variance-covariance matrix. Trial and error must be used to make the probability of misclassification less than some specified tolerance. Detail procedures can be obtained from Lachenbruch (1968).

A logical method to select variables in discriminant analysis is desirable because the set of variables available for measurement may be large and may contain redundancies which do not help the discriminant power and would increase the cost of measurement. We would like to select only those variables which have the capability to discriminate between groups in order to proceed to classification analysis. If there are p variables for our selection, then there are 2 to 1 combinations. For each combination of variables a Wilk's determinant ratio U (equation 15) can be calculated. Clearly $0 \leq U \leq 1$ and small values of U indicate "good" discrimination. Thus only those combinations which have U values below a certain value would be used.

For the case of missing observations for some of the variables, Jackson (1968) introduced an iterative method in which an unknown variable is treated as the dependent variable for a regression analysis and all other variables are used as independent variables with mean values substituted for unknowns from the regression equation. A new estimate of each unknown value is estimated and substituted.

Another problem is that the derivation of the discriminant functions is under the assumption of equal variance-covariance matrices for the groups. If these matrices are not all equal, a method proposed by Anderson and Bahadur (1962) can be used for the two group situation. Further discussion of this problem is beyond the scope of this thesis.

CHAPTER IV

DRAINAGE BASIN DESCRIPTION

Missouri Flat Creek is located from 46°48'37" to 46°43'47" north latitude and 116°58'48" to 117°10'30" west longitude. It originates on the northwestern corner of Latah County, Idaho and flows through Pullman, Washington. The total length of this watershed is approximately 12 miles. The drainage area is 27.1 square miles. A map of the basin is shown in Figure 2.

The basin consists of an irregular plateau with rolling topography averaging 2800 ft in elevation. It ranges from 2360 ft in elevation in Pullman to 3300 ft in the headwaters. The slope of this area is from 3% or less to 40% or over. Nearly all of the basin is in cultivation, wheat and peas being the principal crops. The cropping pattern has not changed over the years and cropland occupies 92% of the watershed, pasture 3% and others 5% (Potter and Love, 1942).

The acreage of each soil group and type are shown on Table 3. Dark, medium-textured, deep prairie-land soils with almost no stones cover most of the area. Shallow rocky soils are found in the higher elevations.

The mean annual maximum termperature is 57.4°F and mean annual minimum temperature is 36.4°F. The average monthly maximum and minimum temperatures are listed in Table 2. The mean annual snowfall of this basin is 22.4 inches.

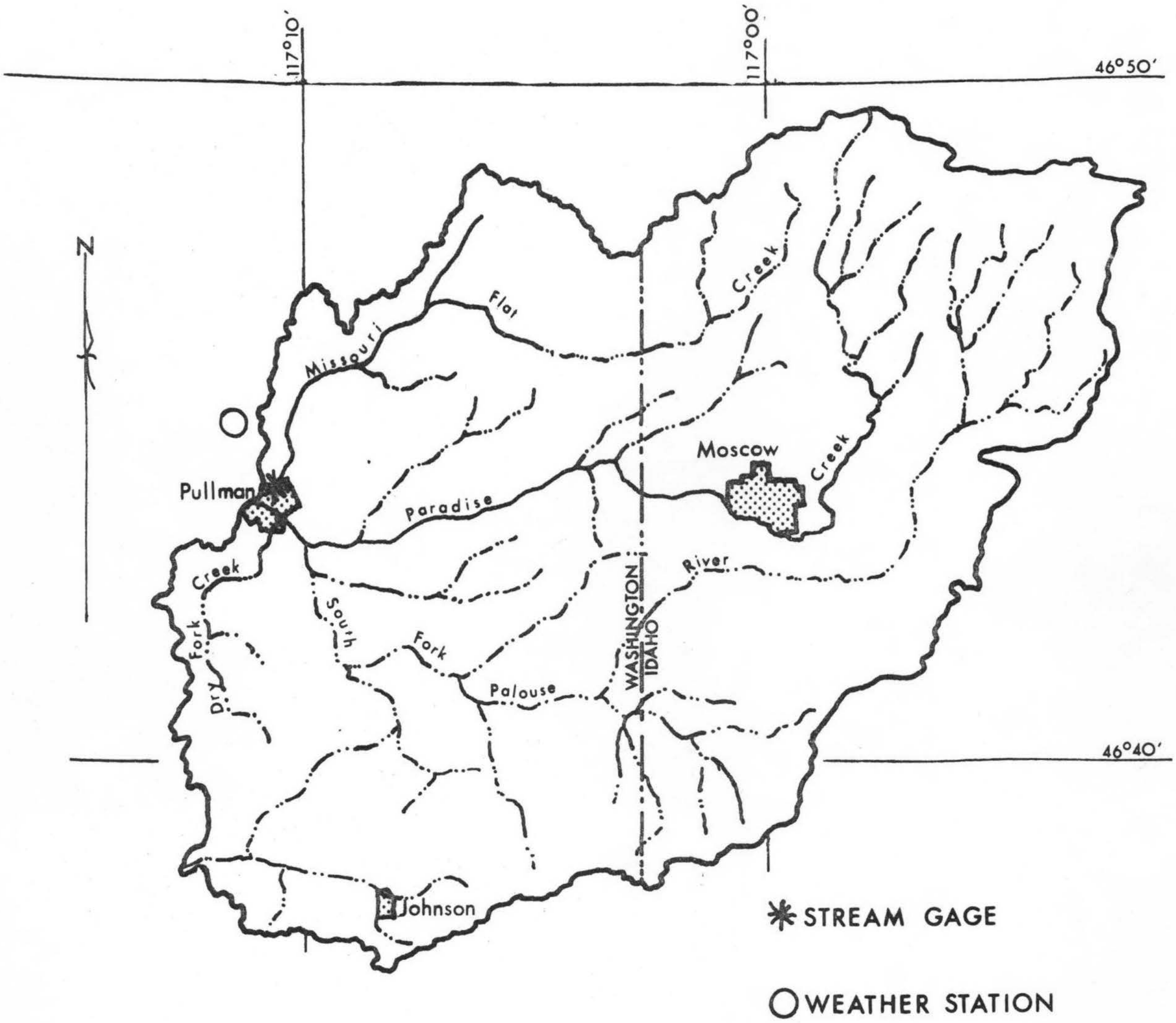


FIGURE 2. Map of Missouri Flat Creek and South Fork Palouse River Drainage Basins.

The total monthly snowfall and precipitation at Pullman from 1940-1971 is listed in Table 2.

TABLE 2 MEAN MONTHLY CLIMATIC FACTORS FOR PULLMAN, WASHINGTON (1940-1971)

Month	Temperature Maximum	Temperature Minimum	Snowfall (in.)	Precipitation (in.)
January	34.4	21.7	7.1	3.13
February	40.6	27.0	3.6	2.16
March	46.0	29.6	3.1	2.01
April	55.8	5.2	0.5	1.54
May	64.6	41.0	0.0	1.55
June	71.0	46.3	0.0	1.70
July	82.2	49.2	0.0	0.46
August	81.4	49.2	0.0	0.75
September	72.3	44.6	0.0	1.15
October	59.7	38.0	0.2	1.94
November	43.9	30.4	2.0	2.94
December	36.3	25.5	5.9	3.05

TABLE 3 SOIL TYPES IN THE MISSOURI FLAT CREEK BASIN

Soil	Area (acres)	Percent
Dark-colored upland soil	16805	0.885
Light-colored upland soil	68	0.004
Soil on terraces	347	0.018
Soil on flood plains	1655	0.087
Rough stony land	111	0.006
Total	18986	100.00

Most of the precipitation is distributed from October through March. Rainfall intensities are usually very low. The 24-hour, 10-year precipitation value is 1.9 inches. This area has moderately cold winters dominated by Pacific maritime air masses with occasionally outbursts of colder continental air from Canada.

The city of Pullman is located at the confluence of Missouri Flat Creek and South Fork of the Palouse River. The population is increasing in this area and as a result, new areas in the flood plain are being developed. Flood induced damages have been increasing at a rapid rate. Basin flood plain zoning has been enacted to help decrease the damages.

CHAPTER V

RESULTS AND DISCUSSION

DATA COLLECTION

The available climatological and stream flow records were obtained from the National Weather Service and the U.S. Geological Survey. Data from October to April, covering 143 runoff events from 1960 to 1973 were used. Fifty-four of the 143 samples were determined to be either frozen or unfrozen ground events. The remaining 89 events could not be classified by an examination of the historical records. These 54 events were then used as a basis for deriving the prediction equation.

On the basis of the literature review, eight readily available variables were chosen as discriminators for a discriminant analysis. These eight variables are listed in Table 4.

DATA ANALYSIS

In this section, the derivation of the classification procedure is shown and applied to three of the unknown observations. This is done to show the procedures used to arrive at the ultimate classification of all 89 unknown observations. The methods used are outlined in Chapter III.

TABLE 4 DISCRIMINATORS FOR FROZEN OR UNFROZEN GROUND ANALYSIS

Discriminators	Notation	Unit
1) Number of days below freezing	X_1	Day
2) Total freezing index	X_2	Degree Day
3) Precipitation, 2 days before freeze period	X_3	Inch
4) Precipitation during freeze period	X_4	Inch
5) Precipitation, 4 days after freeze period	X_5	Inch
6) The depth of snowfall during freeze period	X_6	Inch
7) The depth of snow on ground at start of freeze period	X_7	Inch
8) Stream flow for 4 days after freeze period	X_8	Inch

Autocorrelation Analysis

If the coefficients of autocorrelation are near zero, the sequence being tested is assumed to be random and the notion of the i-th variate being independent of the (i+k)-th variate (k is lag) is supported. The autocorrelation coefficients of the freezing index and stream flow is shown in Figures 3 and 4. It can be seen that most of the observations are within the 95% confidence band. So, it is assumed that the data are independent and random. All

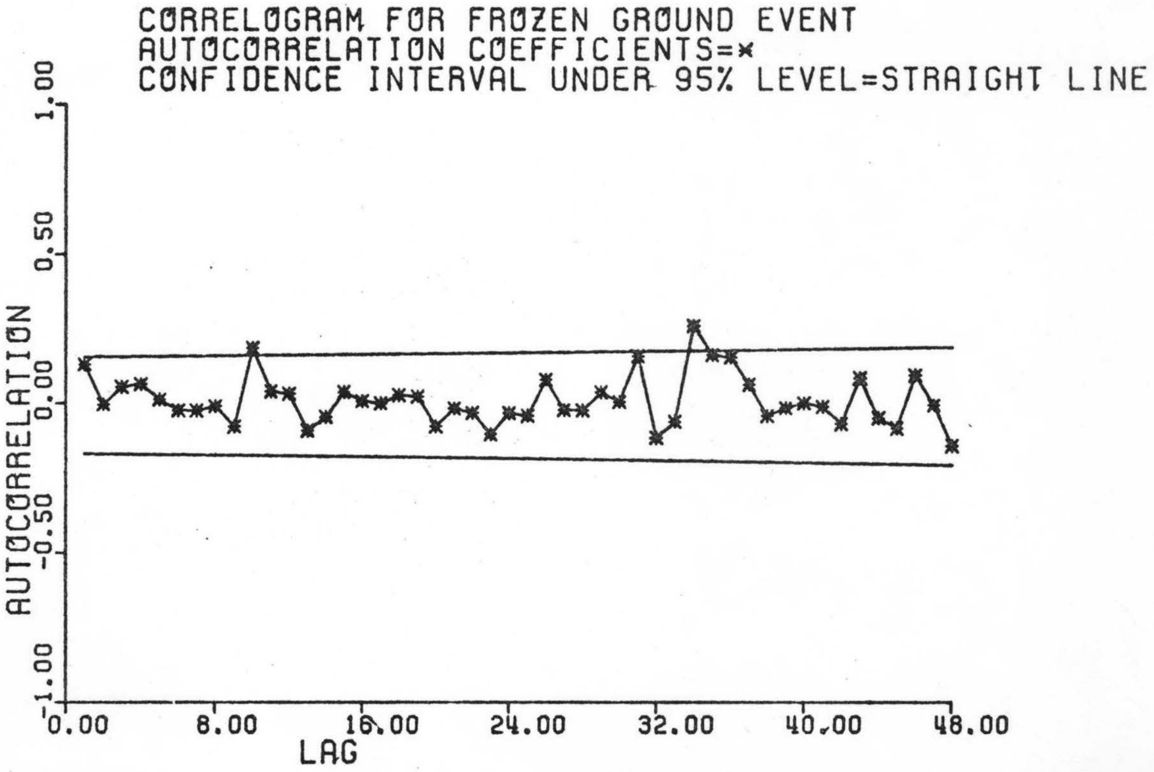


FIGURE 3 Autocorrelation analysis for total freeze index

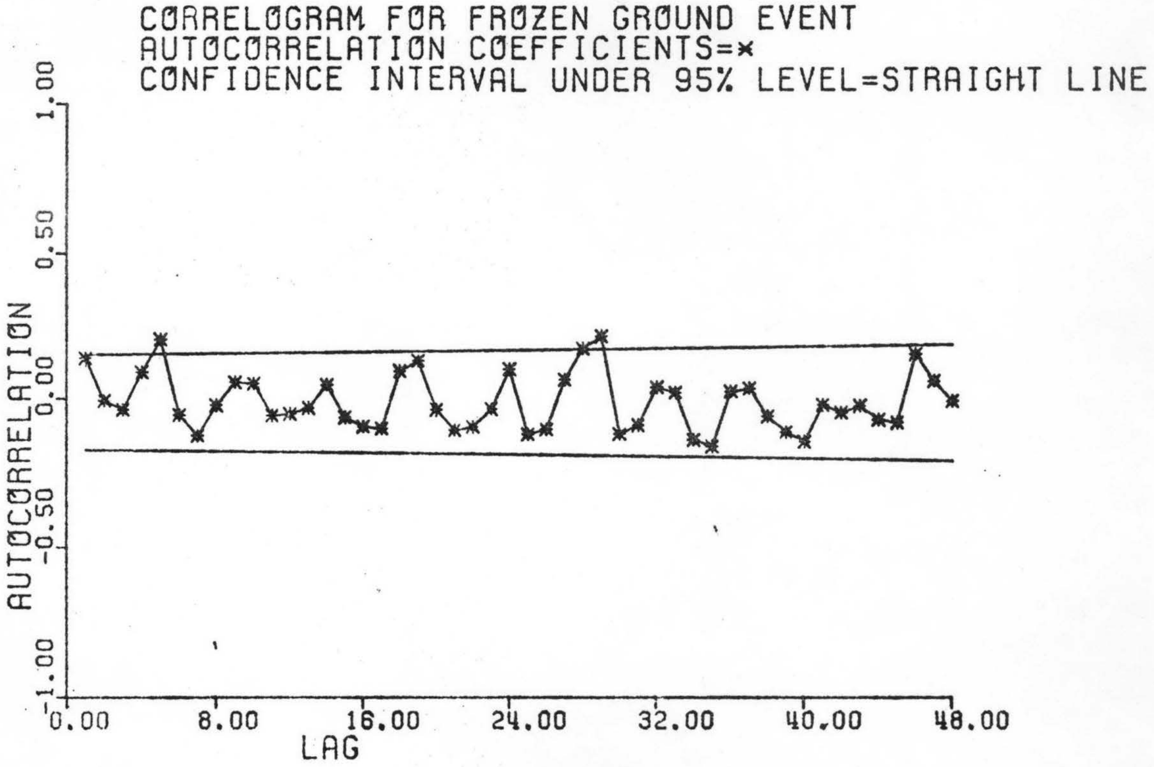


FIGURE 4 Autocorrelation analysis for streamflow, 4 days after freeze period

143 observations are included in this analysis and all 8 variables were checked, but only 2 are shown here for reasons of clarity.

Multivariate Analysis of Variance and Variable Selection

The objective of multivariate analysis of variance in this thesis is to test group differences in a multidimensional measurement space after some combination of variables, which have a small value of Wilk's U, is chosen. Also, our program of a multivariate analysis of variance gives a test of the variance-covariance matrix differences between these two groups for use in classification.

To select the variables, we used the BMD-Stepwise Discriminant analysis program and McCabe and Pohl's optimal method. The BMD method used "F-value to enter or remove" to control variable selection. After the variable is included, Wilk's U is calculated. McCabe's (1975) method computed the Wilk's U-value for each of the 2^p-1 subsets of the variables $X_1, X_2 ------, X_p$. The algorithm first determines the U-value of the optimum set of variables by calculating and comparing the U-values of all possible sets. The order in which the program choose the variables is shown in Table 5. Table 6 shows the BMD method. Next, multivariate analysis of variance was used to test the difference between these two groups for the chosen set of variables.

TABLE 5 MCCABE AND POHL'S STEPWISE SELECTION OF VARIABLES

	Variable											
Stage	1	2	3	4	5	6	7	8	U	F-value	D.F.	Theoretical F(1%)
1		*							0.522	47.62	1-52	7.14
2		*							0.399	38.41	2-51	5.04
3	*	*						*	0.396	25.42	3-50	4.20
4	*	*				*		*	0.394	18.84	4-49	3.73
5	*	*		*		*		*	0.390	15.02	5-48	3.43
6		*	*	*		*	*	*	0.388	12.36	6-47	3.21
7	*	*	*	*		*	*	*	0.382	10.63	7-46	3.06
8	*	*	*	*	*	*	*	*	0.379	9.22	8-45	2.94

The asterisk symbol in Tables 5 and 6 indicates the variables chosen in each stage.

TABLE 6 BMD STEPWISE SELECTION OF VARIABLES

	Variable									
Stage	1	2	3	4	5	6	7	8	F-value to remove	U-value
1		*							47.67	0.522
2		*						*	15.58	0.399
3	*	*						*	0.36	0.396
4	*	*				*		*	0.27	0.394
5	*	*		*		*		*	0.52	0.390
6	*	*		*	*	*		*	0.16	0.389
7	*	*	*	*	*	*		*	0.19	0.388
8	*	*	*	*	*	*	*	*	0.88	0.379

From Table 5 and 6, we can see that the variables entered in the first five stages are the same between these two methods but after that, the variables entered are different. There is nothing in the literature to indicate which method is better. For the purposes of this thesis, all 8 variables were included. An F test for all 8 variables is as follows.

$$
\begin{aligned}
s &= \sqrt{[p^2(g-1)^2-4]/[p^2+(g-1)^2-5]} \\
&= \sqrt{[8^2(2-1)^2-4]/[8^2+(2-1)^2-5]} \\
&= 1
\end{aligned}
$$

$$n_1 = (p*g-1) = 8(2-1) = 8$$

$$
\begin{aligned}
n_2 &= s[(N-1)-(p+(g-1)+1)/2]-[p(g-1)-2]/2 \\
&= [(54-1)-(8+(2-1)+1)/2]-[8(2-1)-2]/2=45
\end{aligned}
$$

$$y = U^{1/s}=0.379$$

$$F(8,45)=(1-y)(n_2/n_2)/y=(1-0.379)(45/8)/0.379=9.22$$

The theoretical F-value is 2.94 under d.f.=8.45 and 1% level. So, $F>F_o$, we reject the $H_o:\mu_1=\mu_2$.

Classification of Data

The SAS method was used for the classification of frozen ground events rather than the BMD method because SAS can handle unequal priori probability and unequal within group variance-covariance matrices.

In order to determine whether or not the frozen and unfrozen ground events come from the same population, Box's method (1949) with M-statistic was used to test $H_o:\Sigma_1=\Sigma_2$, where Σ_1 is the "within group" variance-covariance matrix of the population of frozen ground and that of Σ_2 is unfrozen ground. Doing this, M=189.54 is obtained. This M value is converted to F=3.72, n_1=36 and n_2=1413 degree of freedom. Since F(36,1413) is 1.60 at the 1% level, we reject the null hypothesis $H_o:\Sigma_1=\Sigma_2$ and accept $H:\Sigma_1\neq\Sigma_2$. This means that frozen ground and unfrozen ground events do not come from the same population. Thus the "within group" variance-covariance matrices are unequal between these two groups. This will have to be considered later in the classification analysis and may restrict use of the discriminant functions as a classification tool.

A Bayesian conditional-probability model was used to identify the probability of frozen or unfrozen ground occurring during a given freeze event. Suppose the population means and within group variance-covariance matrices of each group can be estimated from that of the samples. Prior probabilities can also be estimated from the group relative frequencies. Then, if three observations are obtained from historical data without knowing whether or not these observations are for frozen ground events, the following procedure may be used to assign a frozen or unfrozen probability to each observation.

The means and the "within group" variance-covariance matrices which were calculated from the original 54 observations (42 are unfrozen and 12 are frozen) are taken as given.

TABLE 7 THE MEANS OF EACH GROUP

Group	$\overline{X}_1$	$\overline{X}_2$	$\overline{X}_3$	Variable $\overline{X}_4$	$\overline{X}_5$	$\overline{X}_6$	$\overline{X}_7$	$\overline{X}_8$
Frozen	10.83	143.13	0.23	0.63	1.00	5.54	0.42	0.54
Unfrozen	3.57	17.57	0.23	0.25	0.48	2.36	1.07	0.15

TABLE 8 THREE ADDITIONAL SAMPLE OBSERVATIONS

Observation	X_1	X_2	X_3	Variable X_4	X_5	X_6	X_7	X_8
1	1.00	2.00	0.52	0.05	1.74	0.00	1.00	0.05
2	13.00	84.50	0.00	0.80	0.44	5.80	0.00	0.12
3	12.00	70.00	0.25	1.21	0.94	12.25	0.00	0.60

First, Mahalanobis' distance, $D^2_{ij} = (X_i-\overline{X}_j)'S^{-1}_j(X_i-\overline{X}_j)$, must be calculated. Where the vector of observation 1, X_1, is

$$(1.00\ 2.00\ 0.52\ 0.05\ 1.74\ 0.00\ 1.00\ 0.05)'$$

$(X_i-\overline{X}_j)'$ is the transpose of $(X_i-\overline{X}_j)$. $\overline{X}_1$, the mean vector of group 1 (frozen ground) is

$$(10.83\ 143.13\ 0.23\ 0.63\ 1.00\ 5.54\ 0.42\ 0.54)'$$

and $\overline{X}_2$, the mean vector of group 2 (unfrozen ground) is

$$(3.57\ 17.57,\ 0.23,\ 0.25\ 0.48\ 2.36\ 1.07\ 0.15)'$$

Then, D^2_{11} is the value of observation 1 in frozen ground

$$=[(1.00-10.83)\ (2.00-143.13)\ (0.52-0.23)\ (0.05-0.63)\ (1.74-1.00)\ (0.00-5.54)\ (1.00-0.42)\ (0.05-0.54)]'*$$

$$\begin{bmatrix} 0.65 & -0.02 & 2.90 & -0.21 & 0.69 & -0.30 & -0.67 & -1.00 \\ -0.02 & 0.00 & -0.09 & -0.01 & -0.03 & 0.01 & -0.67 & 0.06 \\ 2.90 & -0.09 & 36.00 & 9.50 & 2.80 & -2.60 & -8.30 & -1.60 \\ -0.20 & -0.01 & 9.50 & 33.00 & -0.42 & -2.00 & -4.80 & -14.00 \\ 0.69 & -0.03 & 2.80 & -0.42 & 2.40 & -0.35 & -0.59 & -4.70 \\ -0.30 & 0.01 & -2.60 & -2.00 & -0.35 & 0.32 & 0.88 & 1.40 \\ -0.67 & 0.02 & -8.30 & -4.80 & -0.59 & 0.88 & 5.00 & 2.50 \\ -1.00 & 0.06 & -1.60 & -14.00 & -4.70 & 1.40 & 2.50 & 29.00 \end{bmatrix}$$

$$*[(1.00-10.83)\ (2.00-143.13)\ (0.52-0.23)\ (0.05-0.63)\ (1.74-1.00)\ (0.00-5.54)\ (1.00-0.42)\ (0.05-0.54)]$$

So, $D^2{}_{11}=12.01$. $D^2{}_{12}$, $D^2{}_{21}$, $D^2{}_{22}$, $D^2{}_{31}$, and $D^2{}_{32}$ are calculated similarly and are listed in Table 9.

TABLE 9 THE MAHALANOBIS' DISTANCE $D^2{}_{ij}$

Observation	Frozen ground	Unfrozen ground
1	12.01	18.16
2	18.47	23.02
3	5.51	24.36

$$\hat{P}(X_i|H_j)\hat{P}(H_j)=\hat{P}(H_j)\exp[-(X_i-\overline{X}_j)'S^{-1}{}_j(X_i-\overline{X}_j)/2]/\sqrt{2\pi^p|S_j|}$$

$$=\exp[\frac{-(X_i-\overline{X}_j)'S^{-1}{}_j(X_i-\overline{X}_j)}{2}-\ln|S_j|+2*\ln P(H_j)]$$

Then, let Chi-square $=[(X_i-\overline{X}_j)'S^{-1}{}_j(X_i-\overline{X}_j)+\ln|S_j|-2*\ln\hat{P}(H_j)]$

$$=[(D^2{}_{ij}+\ln|S_j|-2*\ln\hat{P}(H_j)]$$

Where $\ln|S_1|=3.18$, $\ln|S_2|=-1.02$ and $2*\ln\hat{P}(H_i)=2*\ln(0.22)=-3.03$,

$2*\ln\hat{P}(H_2)=2*\ln 0.78=0.50$

The Chi-square values are shown in Table 10.

TABLE 10 CHI-SQUARE VALUES

Observation	Frozen ground	Unfrozen ground
1	18.22	17.64
2	24.68	22.50
3	11.72	23.84

TABLE 11 CONDITIONAL PROBABILITIES FOR 3 OBSERVATIONS

Observation	Frozen ground	Unfrozen ground
1	0.430	0.570
2	0.250	0.750
3	0.998	0.002

From Table 10, it is seen that Chi-square for observation 1 for frozen ground is greater than that of observation 1 for unfrozen ground. This means that observation 1 is closer to the centroid of the known unfrozen ground observation that that of frozen ground. In Table 11, the probability of frozen ground in observation 1 is less than that of unfrozen ground in observation 1 since the probabilities are inversely proportional to the distance. The probabilities of frozen ground of 143 observations in the data were calculated by a computer program using the above procedures. All the results are given in the appendix.

In the original data, forty-two of the 54 observations were unfrozen ground events and twelve were frozen ground. In order to test the discriminatory power on the basis of these eight variables, these 54 given observations were classified with the Bayesian method. It was found that 2 of the observations of frozen ground events which had been thought to be frozen ground events were classified into the unfrozen ground group and 1 of 42 observations of unfrozen ground were misclassified into the frozen ground group. This is not too surprising since the original classification was made using somewhat sparse data. In matrix notation this misclassification is indicated by the "confusion matrix".

$$C=\begin{bmatrix} 41 & 1 \\ 2 & 10 \end{bmatrix}$$

There is a method to test the discriminatory power on the basis of their variables for this confusion matrix. Massy (1965) used the Chi-square statistic for testing this discriminatory power. The null hypothesis is that the function does not have the ability to classify correctly. If we reject the null hypothesis, it means that we have a relatively high degree of confidence in its abiltiy to discriminate properly. The statistic is

$$\text{Chi-square}=(N-nk)^2/[N(k-1)]$$

Where N is total number of observations.

n is the number of correct classifications.

k is the number of populations.

So, Chi-square=$(54-51*2)^2/54(2-1)=42.67$. Its degree of freedom is one. The theoretical Chi-square at the 1% level and 1 d.f. is 3.84. Since 42.67 is greater than the theoretical Chi-square, the null hypothesis is rejected. We are confident that the Bayesian method was able to distinguish between frozen and unfrozen ground events.

Results and Discussion

Using the 89 unknown observations in the Bayesian classification model, it was found that 10 of the 89 unknown observations were classified into frozen ground events and the remaining 79 were classified as unfrozen. Table 12 shows 20 observations sampled at random from the 89 events.

A frozen ground event is caused by the total effects of the eight factors. In Table 12, it can be seen that the freeze index of some observations is not very large and the soil still is assigned a frozen condition because the other seven variables carry enough weight to assign the event to frozen ground. A trait of multivariate analysis is to consider all eight variables as a system. A frozen ground event is caused by the combined effects of these eight correlated factors, not by a singe variable. Also from the data in Table 12, it can be seen that McCool and Molnau (1974) could not correctly classify all runoff events

TABLE 12 THE PROBABILITY OF FROZEN GROUND FOR 20 RANDOM EVENTS 1960 TO 1973; MISSOURI FLAT CREEK

Month/Date/ Year	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Prob. of frozen ground events
1) 12/15/60	3.0	13.5	0.05	0.15	0.91	1.70	0	0.05	0.06%
2) 12/04/62	1.0	2.0	0.68	0.00	0.04	0	0	0.01	4.89%
3) 2/13/69 2/15/69	3.0	13.5	0.46	0.00	0.00	0	0	0.18	1.57%
4) 1/28/70 1/30/70	3.0	4.5	0.36	0.01	0.21	0	0	0.16	0.98%
5) 11/22/60	1.0	2.0	0.52	0.0.	1.74	0	1	0.05	42.70%
6) 12/23/60 1.04/61	13.0	84.5	0.00	0.80	0.44	5.8	0	0.12	25.09%
7) 11/24/61 11/26/61	3.0	12.0	1.45	0.86	0.11	8.0	2	0.01	40.00%
8) 12/6/61 12/16/61	11.0	137.5	0.23	1.10	1.31	10.1	0	0.02	100.00%
9) 11/15/61 11/21/61	7.0	59.0	0.03	0.71	2.29	3.7	0	0.02	99.45%
10) 1/09/62 1/24/62	16.0	247.0	0.37	0.70	0.01	8.8	0	0.02	100.00%
11) 2/21/62 3/04/62	12.0	123.0	0.00	0.96	0.05	11.6	0	0.06	98.52%
12) 1/10/63 1/14/63	5.0	115.5	0.00	0.05	0.03	0.5	0	0.00	100.00%
13) 12/25/69 1/08/70	15.0	144.0	0.17	0.31	1.11	4.9	2	0.02	99.35%
14) 1/10/70 1/13/70	4.0	13.0	0.48	1.08	0.60	1.5	2	0.16	100.00%
15) 1/17/70 1/19/70	3.0	14.5	0.18	1.59	1.16	6.2	0	0.73	100.00%
16) 2/26/71 3/09/71	12.0	70.0	0.25	1.21	0.94	12.5	0	0.60	99.77%
17) 12/27/68 1/04/69	9.0	199.5	0.04	1.44	1.42	21.5	0	0.90	100.00%
18) 12/22/62 12/26/62	5.0	46.5	0.04	0.00	0.09	0.0	0	0.01	1.19%
19) 1/20/73 1/23/73	4.0	5.0	0.04	0.00	0.00	0.0	0	0.03	0.54%
20) 11/28/62 11/29/62	2.0	6.5	0.34	0.00	1.12	0.0	0	0.01	0.74%

on the basis of freeze index along since several of the observations have a freeze index below 100.

The BMD method uses a certain number of discriminant functions with equal prior probabilities among groups and equal within-group variance-covariance matrices. The first discriminant function usually takes care of a major part of the variance, the second one takes care of a major part of the remainder, and so on. The first two discriminant variables for the fifty-four observations known to be frozen or unfrozen are listed as follows

$$\begin{aligned} y_{mk}=&-0.11*(X_{mk1})-5.19)+0.01*(X_{mk2}-45.47)+0.91*\\ &(X_{mk3}-0.23)+1.07*(X_{mk4}-0.33)+0.21*(X_{mk5}-0.59)\\ &-0.13*(X_{mk6}-3.07)-0.10*(X_{mk7}-0.93)+1.79*(X_{mk8}-0.24)\\ z_{mk}=&-0.57*(X_{mk1}-5.19)+0.03*(X_{mk2}-45.47)-1.41*(X_{mk3}-0.23)\\ &-2.16*(X_{mk4}-0.33)-.20*(X_{mk5}-0.59)\ +0.27*(X_{mk6}-3.07)\\ &+0.13*(X_{mk7}-0.93)+0.20*(X_{mk8}-0.24) \end{aligned}$$

These two functions, when plotted in two dimensional scattergrams, result in Figure 5 for fifty-four observations and Figure 6 for all the observations. It is obvious that the two groups are well separated in general, but there are some overlapping. The resulting classification is also somewhat different from that by the SAS method. The actual classification of events was done using the SAS program for the initial 54 observations and a program based on the SAS method for all additional observations (appendix C) because it can handle unequal prior probabilities and unequal within group variance-covariance matrices.

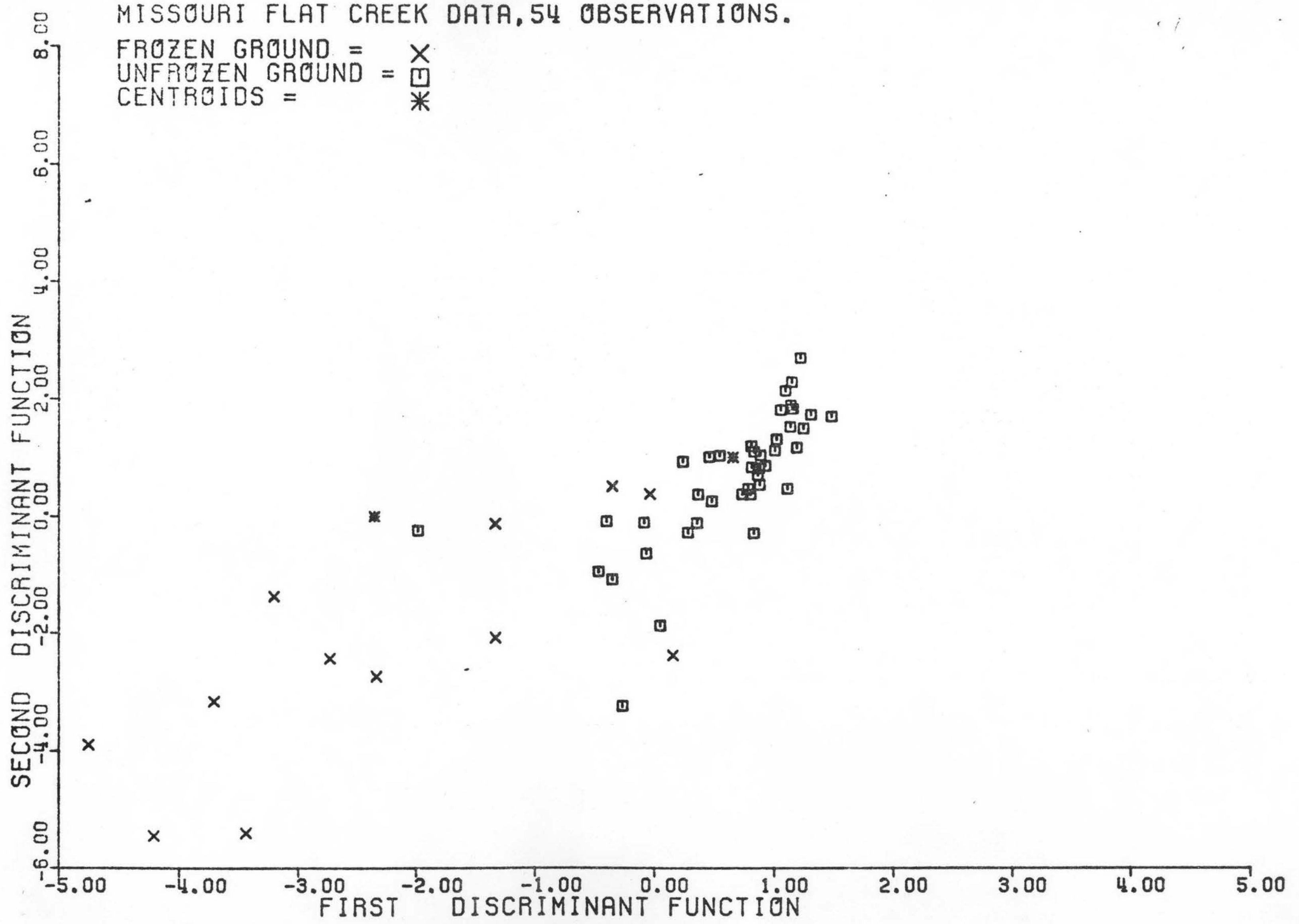


FIGURE 5 Scattergram of 54 observations

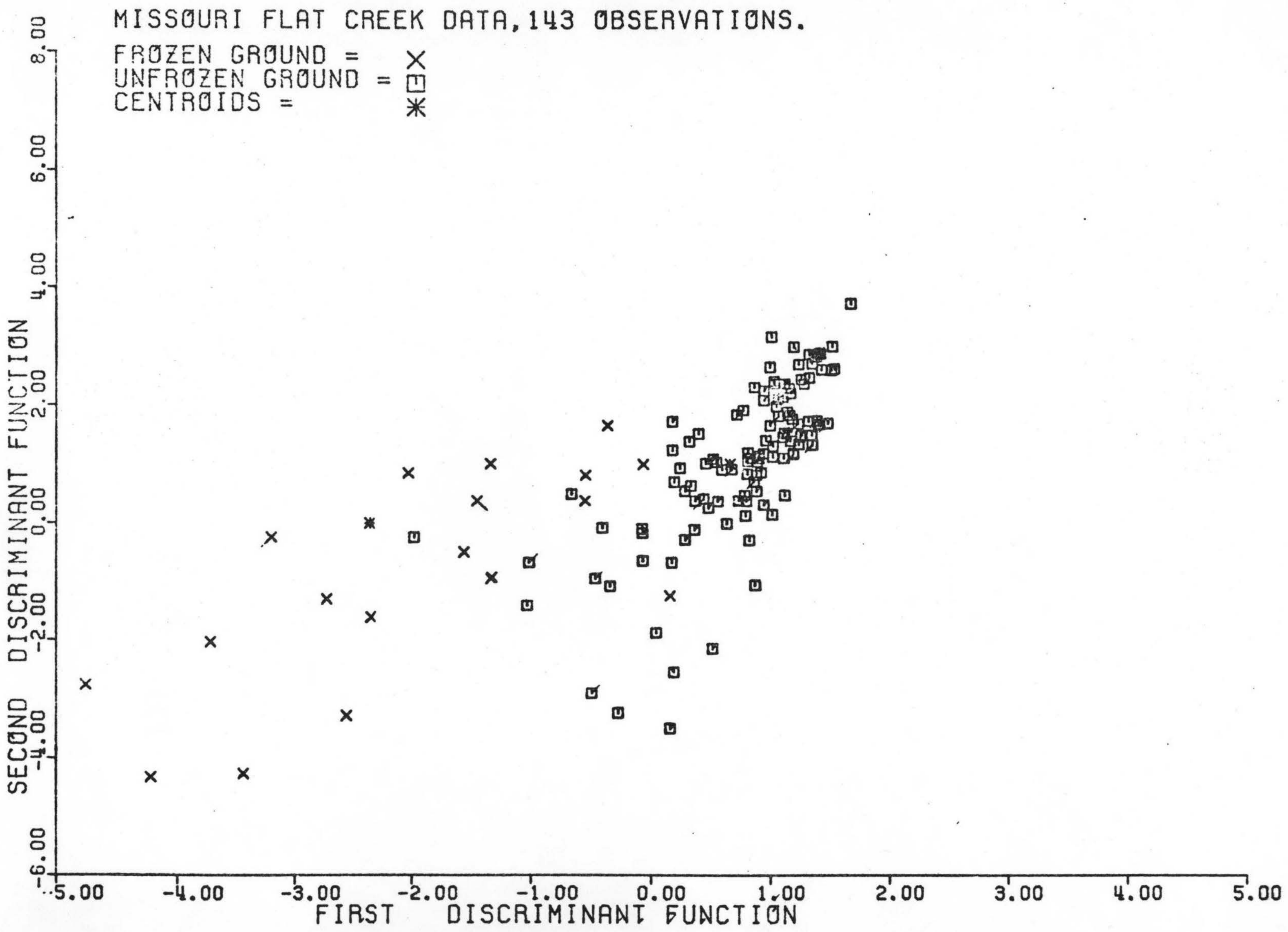


FIGURE 6 Scattergram of 143 observations

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The main purposes of this thesis were to introduce a discriminant method to analyze the frozen ground problem in small watersheds and to identify historical frozen ground floods from past data. Although it is not known whether the data fits some of the assumptions of discriminant analysis or not, the combination of use of discriminant function and Bayesian classification appears to be a good way to classify frozen or unfrozen ground events.

The classification of observations can lead to results having widely different probabilities of error. For some observations, the decision of their assignment can be made with confidence. But in others the best decision will be subject to great uncertainity. Overall and Klett (1972) suggested:

1. Classify the observations into group I if the probability of its belonging to group II is smaller than some prespecified value, 0.10.
2. Classify the observation into group II if the probability of its belonging to group I is smaller than some prespecified value, 0.5.
3. Avoid making a definite classification pending examination of additional information.

There are several methods for variable selection. McCabe (1975) said that the selection of an optimal subset of variables should not be considered as a final solution but rather as a starting point for further analysis.

Autocorrelation analysis can be used to show the independence or dependence of the data. Although some of the variables have dependent relations among the observations, their mutual independence is still assumed.

Mahalanobis' generalized distance is applicable only to groups in which the measurements are normally distributed. For our data we assume it is a multivariate normal distribution in order to simplify the analysis but much further analysis would have to be done in order to prove this.

Unequal within group variance-covariance matrices between these two groups cause an inaccurate discriminant function. This is why we used the 8 variables directly in the Bayesian classification model which can deal with unequal within group variance-covariance matrices.

CONCLUSIONS

The following conclusions can be drawn from this study:

1. The eight variables used in this thesis are sufficient to classify the data into frozen or unfrozen ground runoff events.
2. Using discriminant analysis, it was revealed that 10 of 89 unknown observations are probably frozen ground and the remaining 79 probably are not frozen ground runoff events.

3. The freeze index is the most important of the eight factors studied for estimating the probability of frozen ground in Missouri Flat Creek. This is seen by the stepwise selection of variables (Table 5).
4. The occurrence of a crust of frozen soil on a watershed at the start of a historical snowmelt or rainfall event cannot be studied by relying upon only one variable such as the freeze index, but several variables must be used to reflect the interaction that results in the physical world.
5. Precipitation on frozen ground does not necessarily cause high runoff.

RECOMMENDATIONS FOR FUTURE STUDIES

The following recommendations are given in the hope that the next step in this project will be able to build upon the results of this thesis. These recommendations are:

1. Check the classification of frozen ground with independent data from other sources to verify the classification.
2. Use other methods, such as a nonparametric method, to analyze the data and compare the events with the result of the method used in this thesis.

3. Use additional factors which may influence the occurrence of frozen ground events and try to use discriminant analysis again.
4. Use this method in another area where freeze-thaw cycles are prominant in the winter runoff region, such as Spokane or Pendleton.

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APPENDIX A

DATA INPUT LIST

1	8	44.5	0	0.01	1.54	0.1	0	160
1	3	6.5	0.75	0.13	0.31	0.9	0	174
1	2	9.0	0.43	0.03	0.20	0.3	0	32.1
1	5	14.5	0.05	0.26	0	3.2	0	46.4
1	1	1.5	0.01	0	0.01	0	0	9.2
1	3	14.0	0.10	0.95	0.48	7.5	4	39.6
1	4	17.5	0.03	0.54	0.04	5.3	0	33.5
1	1	1.0	0	0	0.47	0	0	40.6
1	2	2.0	0.35	0.10	0.59	0	1	88.0
1	3	5.5	0.04	0.41	0.42	2.9	0	94.0
1	7	28.0	0.34	0.28	0	2.6	2	63.6
1	5	16.5	0	0.17	0.72	0	1	58.0
1	1	2.5	0.42	0	1.21	0	1	1178
1	3	4.0	0.20	0.02	0.08	0.3	0	122
1	4	16.5	0.30	0	0.66	0	0	137
1	2	1.5	0.26	0.16	0.26	0.6	0	216
1	1	1.0	0.28	0	0.46	0	0	114
1	3	8.0	0.07	1.42	1.62	14.7	0	23.0
1	1	0.5	1.75	0.25	0.97	0	18	63.0
1	11	98.0	0.02	0.49	0.03	5.0	0	7.5
1	9	53.5	0.12	0.67	0.29	5.7	0	16.5
1	4	18.5	0.04	0	0.52	0	0	25.0
1	2	4.0	0.20	0.27	0	0.3	0	60.5
1	3	10.0	0.12	0.09	1.19	0.5	1	43.4
1	2	2.0	0.11	0.17	1.05	1.3	0	330
1	4	15.0	0.25	0.07	0.88	0.8	0	298
1	2	2.0	0.11	0.18	0.27	1.8	0	38.4
1	2	1.5	0.26	0	0	0	0	20.9
1	3	5.5	0.08	0.14	0.63	1.5	1	23.3
1	4	9.5	0.10	0.11	0.25	0	4	36.9
1	2	10.5	0.15	0.12	0.80	0	0	8.5
1	7	58.5	0.25	0.53	0.66	3.50	0	61.0
1	4	6.0	0	0.07	0	0.3	0	87
1	1	4.0	0	0.25	0.14	3.0	0	174
1	1	4.5	0.26	0	0.26	0	2	54.2
1	2	4.5	0.03	0.14	1.67	3.8	0	80.3
1	10	88.5	1.49	2.15	0.29	26.5	4	24.6

1	7	65.5	0	0.19	0.14	3.8	0	42.9
1	3	40.5	0	0.14	0.15	3.0	0	37.6
1	3	27.5	0	0	0.67	0	0	175
1	1	2.0	0.20	0	0.10	0	0	170
1	4	12.0	0.45	0	0.06	0	6	76.6
2	16	205.0	0.03	1.04	1.29	6.6	0	814
2	9	123.0	0.12	0.95	4.18	9.5	0	739.9
2	11	146.5	0.01	0.90	1.20	3.2	0	353.6
2	5	22.0	1.06	0.52	0.48	11.1	0	213
2	25	323.0	0.38	1.50	0.16	23.2	0	54
2	5	20.0	0	0	0.28	0	0	475
2	8	90.5	0.49	0.30	0.43	0.30	2	227
2	4	22.0	0.12	0.79	1.08	1.8	1	586
2	6	20.5	0.01	0.11	0.84	0.5	0	228
2	12	234.0	0.07	0.38	0.14	4.5	0	165
2	14	306.5	0.43	0.34	0.87	1.2	1	231
2	15	204.5	0.06	0.73	1.05	4.6	1	523
1	1	2	0.52	0.05	1.74	0	1	33.7
1	4	11.5	0.99	0	0.39	0	1	7.1
1	7	51.0	0.19	0	0.05	0	0	2.9
1	3	13.5	0.05	0.15	0.91	1.7	0	38.5
1	13	84.5	0	0.80	0.44	5.8	0	82.3
1	1	6.0	0.08	0.14	0	0.8	0	1.3
1	3	12.0	1.45	0.86	0.11	8.0	2	6.1
1	1	5.0	0.55	0.04	0.97	0.5	5	27.5
1	1	1.5	0.32	0.08	0.49	0	0	37.5
1	1	0.5	0	0.06	0.22	0	0	1.9
1	2	2.0	0.09	0.16	0.56	0.7	0	4.8
1	2	6.5	0.34	0	1.12	0	0	8.4
1	1	2.0	0.68	0	0.04	0	0	5.5
1	2	5.0	0	0	1.06	0	0	13.2
1	5	46.5	0.04	0	0.09	0	0	4.9
1	2	7.0	0	0	0	0	0	2.5
1	4	29.5	0	0	1.04	0	0	4.7
1	8	67.5	0.02	0.63	0.76	5.4	0	4.1
1	2	6.0	0.76	0	0.19	0	4	6.4
1	2	4.0	0	0.19	0.16	0	3	5.4

1	1	2.0	0.09	0.14	0.88	1.5	0	16.0
1	9	40.5	0.84	0.56	0.85	5.5	1	6.6
1	2	3.0	0.37	0.48	0.16	3.5	3	10.3
1	1	2.0	1.37	0	0.26	0	4	38.5
1	1	0.5	0	0	0.36	0	2	44.9
1	1	2.0	0.10	0	0.07	0	1	46.0
1	2	6.5	0.07	0	0.03	0	0	38.2
1	1	3.0	0.01	0	0.02	0	0	34.0
1	1	3.5	0.02	0	0.20	0	0	23.0
1	3	14.5	0.01	0.06	0	0	0	113.0
1	6	29.5	0	0	1.83	0	0	8.2
1	1	0.5	0	0.10	1.88	0	0	10.6
1	2	4.0	0.15	0.27	1.06	2.5	1	21.7
1	12	108.0	2.36	0.84	0.53	6.9	0	39.0
1	2	6.0	0.39	0.34	0.49	2.8	1	272
1	2	4.5	0.46	0	0	0	1	44.6
1	3	26.5	0	0	0.23	0	0	19.1
1	1	1.0	0	0.05	0.22	0	0	18.3
1	6	33.0	0.23	0.32	0	2.2	0	20.4
1	1	3.5	0.26	0.13	0.19	0.8	0	0.4
1	8	45.5	0.10	0	0.10	0	0	0.4
1	6	37.5	0	0.21	0.20	3.0	0	2.1
1	1	0.5	0.01	0	0.02	0	2	13.0
1	1	2.5	0	0	0.32	0	0	19.1
1	1	3.0	0	0	0.36	0	0	5.6
1	1	3.5	0.04	0	0.49	0	0	2.0
1	1	1.5	0.36	0	0.04	0	0	5.2
1	4	7.5	0.04	0	0.48	0	0	16.1
1	1	0.5	0.11	0.11	0.43	0	0	22.8
1	3	11.0	0.34	0.09	0.09	1.2	0	11.2
1	2	2.5	0.05	0	0	0	0	9.3
1	1	0.5	0.15	0.29	0	4.0	0	33.0
1	6	20.5	0.15	0.28	0.23	1.8	0	1.3
1	3	7.0	0.17	0.03	0.03	0.3	1	1.8
1	1	2.5	0	0	0.31	0	0	6.9
1	8	56.5	0.19	0.51	0.21	6.0	2	5.5
1	7	47.5	0	0.02	0.50	0	0	36.2

1	2	6.0	0	0	1.21	0	0	43.2
1	1	1.5	0.11	0	0.54	0	1	29.9
1	2	2.0	0.31	0	0.45	0	0	24.2
1	3	13.5	0.46	0	0	0	0	126
1	15	75.5	0	0.68	0.20	4.1	0	6.5
1	3	4.5	0.36	0.01	0.21	0	0	114
1	6	32.5	0	0.53	0.31	5.0	0	197
1	2	26.5	0	0.11	0.69	0	0	9.6
1	2	3.5	0.44	0.25	0.14	0.7	3	11.5
1	3	10.0	0.04	0.21	0.02	0.80	2	19.8
1	1	4.5	0	0.02	0.25	0	0	5.9
1	3	6.0	0.04	0.23	0.08	2.5	0	5.8
1	12	102.5	0.08	0.37	0.58	3.6	2	14.6
1	1	1.0	0.42	0.02	0	0	3	152
1	3	5.5	0.02	0	0	0	2	170
1	3	22.0	0.11	0.12	0.51	1.5	1	6.6
1	1	1	0.22	0.05	0.35	0	1	3.7
1	1	1	0	0	0.43	0	0	5.9
1	4	5.0	0.04	0	0	0	0	18.6
1	3	14.5	0	0	0.08	0	0	11.1
1	1	0.5	0.08	0	0.01	0	1	10.9
1	5	18.5	0.01	0	0.45	0	0	10.3
2	7	59.0	0.03	0.71	2.29	3.7	0	15.3
2	11	137.5	0.23	1.10	1.31	10.1	0	12.0
2	16	247.0	0.37	0.70	0.01	8.8	0	10.9
2	12	123.0	0	0.96	0.05	11.6	0	43.5
2	5	115.5	0	0.05	0.03	0.5	0	2.1
2	9	199.5	0.04	1.44	1.42	21.5	0	640
2	15	144.0	0.17	0.31	1.11	4.9	2	12.1
2	4	13.0	0.48	1.08	0.60	1.5	2	113
2	3	14.5	0.18	1.59	1.16	6.2	0	523
2	12	70.0	0.25	1.21	0.94	12.5	0	432

APPENDIX B

MULTIVARIATE ANALYSIS OF VARIANCE OUTPUT

```
GROUP  1NO. OF OBS.     42
MEANS FOR GROUP     1
      3.57      17.57       0.23        0.25        0.48        2.36        1.07        0.15
STANDARD DEVIATIONS
      2.54      23.62       0.36        0.41        0.47        4.74        3.02        0.26
DISPERSION DETERMINANT=     0.3681E+00
-----------------------------------------------
GROUP  2NO. OF OBS.     12
MEANS FOR GROUP     2
     10.83     143.12       0.23        0.63        1.00        5.54        0.42        0.54
STANDARD DEVIATIONS
      6.07     111.85       0.31        0.43        1.08        6.63        0.67        0.34
DISPERSION DETERMINANT=     0.2426E+02
-----------------------------------------------
MEANS FOR TOTAL SAMPLE
      5.19      45.47       0.23        0.33        0.59        3.07        0.93        0.24
      3.59      55.56       0.35        0.42        0.65        5.20        2.70        0.28
T MATRIX
ROW=  1         0.12E+04       0.18E+05       0.45E+00       0.66E+02       0.22E+02
               0.75E+03      -0.70E+02      0.16E+02
ROW=  2         0.31E+06       0.15E+02       0.91E+03       0.46E+03       0.99E+04
              -0.93E+03       0.31E+03
ROW=  3         0.63E+01       0.21E+01      -0.61E+00       0.32E+02       0.33E+02
              -0.24E+00
ROW=  4         0.10E+02       0.39E+01       0.11E+03       0.41E+01       0.84E+00

ROW=  5         0.24E+02       0.25E+02       0.91E+00       0.61E+01
ROW=  6         0.15E+04      -0.20E+01      -0.24E+01
ROW=  7         0.38E+03      -0.43E+01
ROW=  8         0.54E+01
A MATRIX
ROW=  1         0.49E+03       0.85E+04       0.18E+00       0.26E+02       0.35E+02
               0.22E+03      -0.44E+02      0.26E+02
ROW=  2         0.15E+06       0.31E+01       0.45E+03       0.61E+03       0.37E+04
              -0.77E+03       0.45E+03
ROW=  3         0.64E-04       0.93E-02       0.13E-01       0.78E-01      -0.16E-01
               0.94E-02
ROW=  4         0.13E+01       0.18E+01       0.11E+02      -0.23E+01       0.14E+01

ROW=  5         0.25E+01       0.15E+02      -0.32E+01       0.19E+01
ROW=  6         0.94E+02      -0.19E+02       0.11E+02
ROW=  7         0.40E+01      -0.24E+01
ROW=  8         0.14E+01
W MATRIX
ROW=  1         0.67E+03       0.90E+04       0.28E+00       0.40E+02      -0.13E+02
               0.54E+03      -0.26E+02     -0.10E+02
ROW=  2         0.16E+06       0.12E+02       0.47E+03      -0.15E+03       0.62E+04
              -0.16E+03      -0.14E+03
ROW=  3         0.63E+01       0.21E+01      -0.62E+00       0.31E+02       0.33E+02
              -0.25E+00
ROW=  4         0.90E+01       0.21E+01       0.10E+03       0.64E+01      -0.52E+00

ROW=  5         0.22E+02       0.93E+01       0.41E+01       0.42E+01
ROW=  6         0.14E+04       0.17E+02      -0.14E+02
ROW=  7         0.38E+03      -0.20E+01
ROW=  8         0.40E+01
DISPERSION DETERMINANT=     0.3418E+02
FOR TEST OF H1(EQUALITY OF DISPERSIONS),M=    189.541  AND F=      3.717
FOR F, NDF1= 36    AND NDF2= 1413
UNIVARIATE F-RATIOS,WITH NDF1=  1  AND NDF2=      52
VARIABLE AMONG MEAN SQ     WITHIN MEAN SQ     F-RATIO          ETA SQUARE
    1          492.20               12.88             38.20          .4235
    2       147127.87             3086.56             47.67          .4783
    3            0.00                0.12              0.00          .0000
    4            1.35                0.17              7.74          .1296
    5            2.54                0.42              6.08          .1047
    6           94.37               26.99              3.50          .0630
    7            4.00                7.26              0.55          .0105
    8            1.38                0.08             17.99          .2570
WILKS LAMBDA= 0.3805  GENERALIZED CORRELATION RATIO   , ETA SQUARE=.6195
F-RATIO FOR H2, OVERALL DISCRIMINATION,=        9.16
NDF1=  8  AND NDF2=         45
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APPENDIX C

CLASSIFICATION OUTPUT

```
VAR-COVAR MATRIX=  1
   6.446   54.129     0.042     0.500    -0.117     5.898    -0.505    -0.150
  54.129  557.958     0.829     4.888    -1.236    61.010    -3.908    -1.155
   0.042    0.829     0.126     0.052     0.006     0.572     0.801     0.007
   0.500    4.888     0.052     0.170     0.014     1.890     0.174    -0.021
  -0.117   -1.237     0.006     0.014     0.216     0.139     0.123     0.039
   5.898   61.010     0.572     1.890     0.139    22.440     0.900    -0.210
  -0.505   -3.908     0.801     0.174     0.123     0.900     9.092    -0.044
  -0.150   -1.155     0.007    -0.021     0.038    -0.210    -0.044     0.067
VAR-COVAR MATRIX=  2
  36.879  616.250    -0.130     1.798    -0.757    26.709    -0.470    -0.380
 616.2502511.367    -1.983    24.092    -9.203   331.703    -0.148    -8.139
  -0.130   -1.983     0.099    -0.002    -0.073     0.729     0.039    -0.048
   1.798   24.092    -0.002     0.188     0.137     2.226    -0.063     0.030
  -0.757   -9.203    -0.078     0.137     1.168     0.327    -0.104     0.241
  26.708  331.708     0.729     2.226     0.327    43.946    -1.773    -0.475
  -0.470   -0.148     0.039    -0.063    -0.104    -1.773     0.447    -0.016
  -0.380   -8.139    -0.048     0.030     0.241    -0.475    -0.016     0.114
INVERSE VAR-COVAR MATRIX=  1
  0.92E+00-0.90E-01 0.12E+00-0.25E+01-0.95E-01 0.22E+00 0.32E-01 0.46E+00
 -0.90E-01 0.11E-01 0.39E-02 0.31E+00 0.26E-01-0.34E-01-0.34E-02-0.31E-01
  0.12E+00 0.38E-02 0.27E+02 0.10E+02 0.24E+01-0.16E+01-0.24E+01-0.71E+01
 -0.25E+01 0.31E+00 0.10E+02 0.12E+03-0.62E-02-0.10E+02-0.21E+01 0.19E+01
 -0.97E-01 0.27E-01 0.24E+01-0.15E-01 0.56E+01-0.17E+00-0.29E+00-0.40E+01
  0.22E+00-0.34E-01-0.16E+01-0.10E+02-0.17E+00 0.37E+00 0.24E+00 0.17E+00
  0.33E-01-0.34E-02-0.24E+01-0.21E+01-0.29E+00 0.24E+00 0.35E+00 0.74E+00
  0.46E+00-0.31E-01-0.71E+01 0.19E+01-0.39E+01 0.16E+00 0.74E+00 0.20E+02
INVERSE VAR-COVAR MATRIX=  2
  0.65E+00-0.23E-01 0.29E+01-0.21E+00 0.69E+00-0.30E+00-0.67E+00-0.10E+01
 -0.23E-01 0.99E-03-0.87E-01-0.82E-02-0.25E-01 0.10E-01 0.19E-01 0.56E-01
  0.29E+01-0.87E-01 0.36E+02 0.95E+01 0.28E+01-0.26E+01-0.83E+01-0.16E+01
 -0.21E+00-0.82E-02 0.95E+01 0.33E+02-0.42E+00-0.20E+01-0.48E+01-0.14E+02
  0.69E+00-0.25E-01 0.28E+01-0.42E+01 0.24E+01-0.35E+00-0.59E+00-0.47E+01
 -0.30E+00 0.10E-01-0.26E+01-0.20E+01-0.35E+00 0.37E+00 0.88E+00 0.34E+01
 -0.67E+00 0.19E-01-0.83E+01-0.48E+01-0.59E+00 0.88E+00 0.50E+01 0.25E+01
 -0.10E+01 0.56E-01-0.16E+01-0.14E+02-0.47E+01 0.14E+01 0.25E+01 0.28E+02
ABSOLUTE VALUE OF THE DETERMINANT FOR GROUP 1=  0.36ABSOLUTE VALUE OF THE DETERMINANT FOR GROUP 2= 24.30

OBSERVATION NO.=  1
       3.00        44.50          0.00          0.01          1.54          0.10          0.00          0.22
CHI-SQUARE VALUE,GROUP 1=    12.959        GROUP 2=    24.468
OBSERVATION NO.=  2
       3.00         6.50          0.75          0.13          0.31          0.90          0.00          0.24
CHI-SQUARE VALUE,GROUP 1=    10.461        GROUP 2=    18.284
OBSERVATION NO.=  3
       2.00         9.00          0.43          0.03          0.20          0.30          0.00          0.04
CHI-SQUARE VALUE,GROUP 1=     3.840        GROUP 2=    13.468
OBSERVATION NO.=  4
       5.00        14.50          0.05          0.26          0.00          3.20          0.00          0.06
CHI-SQUARE VALUE,GROUP 1=     4.893        GROUP 2=    14.053
OBSERVATION NO.=  5
       1.00         1.50          0.01          0.00          0.01          0.00          0.00          0.01
CHI-SQUARE VALUE,GROUP 1=     2.711        GROUP 2=    20.328
OBSERVATION NO.=  6
       3.00        14.00          0.10          0.35          0.48          7.50          4.00          0.06
CHI-SQUARE VALUE,GROUP 1=    12.808        GROUP 2=   123.833
OBSERVATION NO.=  7
       4.00        17.50          0.03          0.54          0.04          5.30          0.00          0.05
CHI-SQUARE VALUE,GROUP 1=     2.449        GROUP 2=    21.649
OBSERVATION NO.=  8
       1.00         1.00          0.00          0.00          0.47          0.00          0.00          0.06
CHI-SQUARE VALUE,GROUP 1=     1.699        GROUP 2=    19.055
OBSERVATION NO.=  9
       2.00         2.00          0.35          0.10          0.59          0.00          1.00          0.12
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CHI-SQUARE VALUE,GROUP 1=    1.577      GROUP 2=   12.522
OBSERVATION NO.= 10
      3.00        5.50        0.04        0.41        0.42        2.90        0.00        0.13
CHI-SQUARE VALUE,GROUP 1=    1.865      GROUP 2=   17.131
OBSERVATION NO.= 11
      7.00       28.00        0.34        0.28        0.00        2.60        2.00        0.09
CHI-SQUARE VALUE,GROUP 1=    6.373      GROUP 2=   19.493
OBSERVATION NO.= 12
      5.00       16.50        0.00        0.17        0.72        0.00        1.00        0.08
CHI-SQUARE VALUE,GROUP 1=    3.109      GROUP 2=   17.155
OBSERVATION NO.= 13
      1.00        2.50        0.42        0.00        1.21        0.00        1.00        1.65
CHI-SQUARE VALUE,GROUP 1=   33.834      GROUP 2=   55.352
OBSERVATION NO.= 14
      3.00        4.00        0.20        0.02        0.08        0.30        0.00        0.17
CHI-SQUARE VALUE,GROUP 1=    1.886      GROUP 2=   11.253
OBSERVATION NO.= 15
      4.00       16.50        0.30        0.00        0.66        0.00        0.00        0.19
CHI-SQUARE VALUE,GROUP 1=    1.948      GROUP 2=   12.549
OBSERVATION NO.= 16
      2.00        1.50        0.26        0.16        0.26        0.60        0.00        0.30
CHI-SQUARE VALUE,GROUP 1=    1.812      GROUP 2=   11.032
OBSERVATION NO.= 17
      1.00        1.00        0.28        0.00        0.46        0.00        0.00        0.16
CHI-SQUARE VALUE,GROUP 1=    1.901      GROUP 2=   12.759
OBSERVATION NO.= 18
      3.00        8.00        0.07        1.42        1.62       14.70        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=   19.128      GROUP 2=   61.643
OBSERVATION NO.= 19
      1.00        0.50        1.75        0.25        0.97        0.00       18.00        0.09
CHI-SQUARE VALUE,GROUP 1=   33.371      GROUP 2= 1187.650
OBSERVATION NO.= 20
     11.00       98.00        0.02        0.49        0.03        5.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   14.807      GROUP 2=   14.631
OBSERVATION NO.= 21
      9.00       53.50        0.12        0.57        0.29        5.70        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=    6.640      GROUP 2=   17.552
OBSERVATION NO.= 22
      4.00       18.50        0.04        0.00        0.52        0.00        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=    0.734      GROUP 2=   15.010
OBSERVATION NO.= 23
      2.00        4.00        0.20        0.27        0.00        0.30        0.00        0.08
CHI-SQUARE VALUE,GROUP 1=    6.684      GROUP 2=   16.322
OBSERVATION NO.= 24
      3.00       10.00        0.12        0.09        1.19        0.50        1.00        0.06
CHI-SQUARE VALUE,GROUP 1=    3.219      GROUP 2=   18.296
OBSERVATION NO.= 25
      2.00        2.00        0.11        0.17        1.05        1.30        0.00        0.46
CHI-SQUARE VALUE,GROUP 1=    2.142      GROUP 2=   10.997
OBSERVATION NO.= 26
      4.00       15.00        0.25        0.07        0.88        0.80        0.00        0.42
CHI-SQUARE VALUE,GROUP 1=    2.155      GROUP 2=    9.522
OBSERVATION NO.= 27
      2.00        2.00        0.11        0.18        0.27        1.80        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=    0.679      GROUP 2=   16.591
OBSERVATION NO.= 28
      2.00        1.50        0.26        0.00        0.00        0.00        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=    2.756      GROUP 2=   13.574
OBSERVATION NO.= 29
      3.00        5.50        0.08        0.14        0.63        1.50        1.00        0.03
CHI-SQUARE VALUE,GROUP 1=    1.172      GROUP 2=   19.794
OBSERVATION NO.= 30
      4.00        9.50        0.10        0.11        0.25        0.00        4.00        0.05
CHI-SQUARE VALUE,GROUP 1=    4.868      GROUP 2=   78.272
OBSERVATION NO.= 31
      2.00       10.50        0.15        0.12        0.80        0.00        0.00        0.01
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CHI-SQUARE VALUE,GROUP 1=    3.988     GROUP 2=   16.843
OBSERVATION NO.= 32
      7.00      58.50        0.25        0.53        0.66        3.50        0.00        0.09
CHI-SQUARE VALUE,GROUP 1=   11.008     GROUP 2=   15.582
OBSERVATION NO.= 33
      4.00       6.00        0.00        0.07        0.00        0.30        0.00        0.12
CHI-SQUARE VALUE,GROUP 1=    3.559     GROUP 2=   13.415
OBSERVATION NO.= 34
      1.00       4.00        0.00        0.25        0.14        3.00        0.00        0.24
CHI-SQUARE VALUE,GROUP 1=    4.044     GROUP 2=   25.819
OBSERVATION NO.= 35
      1.00       4.50        0.26        0.00        0.26        0.00        2.00        0.08
CHI-SQUARE VALUE,GROUP 1=    1.972     GROUP 2=   30.742
OBSERVATION NO.= 36
      2.00       4.50        0.03        0.14        1.67        3.80        0.00        0.11
CHI-SQUARE VALUE,GROUP 1=   14.554     GROUP 2=   21.492
OBSERVATION NO.= 37
     10.00      88.50        1.49        2.15        0.29       26.50        4.00        0.03
CHI-SQUARE VALUE,GROUP 1=   32.609     GROUP 2=  119.396
OBSERVATION NO.= 38
      7.00      65.50        0.00        0.19        0.14        3.80        0.00        0.06
CHI-SQUARE VALUE,GROUP 1=    8.855     GROUP 2=   13.588
OBSERVATION NO.= 39
      3.00      40.50        0.00        0.14        0.15        3.00        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=    9.704     GROUP 2=   22.055
OBSERVATION NO.= 40
      3.00      27.50        0.00        0.00        0.67        0.00        0.00        0.24
CHI-SQUARE VALUE,GROUP 1=    3.129     GROUP 2=   13.149
OBSERVATION NO.= 41
      1.00       2.00        0.20        0.00        0.10        0.00        0.00        0.24
CHI-SQUARE VALUE,GROUP 1=    1.959     GROUP 2=   13.776
OBSERVATION NO.= 42
      4.00      12.00        0.45        0.00        0.06        0.00        6.00        0.11
CHI-SQUARE VALUE,GROUP 1=    7.559     GROUP 2=  145.603
OBSERVATION NO.= 43
      1.00       2.00        0.57        0.05        1.74        0.00        1.00        0.05
CHI-SQUARE VALUE,GROUP 1=   17.636     GROUP 2=   18.224
OBSERVATION NO.= 44
      4.00      11.50        0.99        0.00        0.39        0.00        1.00        0.01
CHI-SQUARE VALUE,GROUP 1=   20.404     GROUP 2=   26.239
OBSERVATION NO.= 45
      7.00      51.00        0.19        0.00        0.05        0.00        0.00        0.00
CHI-SQUARE VALUE,GROUP 1=    4.683     GROUP 2=   16.361
OBSERVATION NO.= 46
      3.00      13.50        0.05        0.15        0.91        1.70        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=    1.505     GROUP 2=   16.286
OBSERVATION NO.= 47
     13.00      84.50        0.00        0.80        0.44        5.80        0.00        0.12
CHI-SQUARE VALUE,GROUP 1=   22.496     GROUP 2=   24.684
OBSERVATION NO.= 48
      1.00       6.00        0.08        0.14        0.00        0.80        0.00        0.00
CHI-SQUARE VALUE,GROUP 1=    3.540     GROUP 2=   21.005
OBSERVATION NO.= 49
      3.00      12.00        1.45        0.86        0.11        8.00        2.00        0.01
CHI-SQUARE VALUE,GROUP 1=   33.451     GROUP 2=   34.262
OBSERVATION NO.= 50
      1.00       5.00        0.55        0.04        0.97        0.50        5.00        0.04
CHI-SQUARE VALUE,GROUP 1=    5.629     GROUP 2=  106.515
OBSERVATION NO.= 51
      1.00       1.50        0.32        0.08        0.49        0.00        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=    3.904     GROUP 2=   14.921
OBSERVATION NO.= 52
      1.00       0.50        0.00        0.06        0.22        0.00        0.00        0.00
CHI-SQUARE VALUE,GROUP 1=    2.000     GROUP 2=   20.507
OBSERVATION NO.= 53
      2.00       2.00        0.09        0.16        0.56        6.70        0.00        0.01
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CHI-SQUARE VALUE,GROUP 1=   1.165       GROUP 2=   17.337
OBSERVATION NO.= 54
      2.00        6.50        0.34        0.00        1.12        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   7.197       GROUP 2=   17.003
OBSERVATION NO.= 55
      1.00        2.00        0.68        0.00        0.04        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=  11.287       GROUP 2=   17.221
OBSERVATION NO.= 56
      2.00        5.00        0.00        0.00        1.06        0.00        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=   3.482       GROUP 2=   18.723
OBSERVATION NO.= 57
      5.00       46.50        0.04        0.00        0.09        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   5.090       GROUP 2=   13.932
OBSERVATION NO.= 58
      2.00        7.00        0.00        0.00        0.00        0.00        0.00        0.00
CHI-SQUARE VALUE,GROUP 1=   1.844       GROUP 2=   18.081
OBSERVATION NO.= 59
      4.00       29.50        0.00        0.00        1.04        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   4.813       GROUP 2=   17.790
OBSERVATION NO.= 60
      8.00       67.50        0.02        0.63        0.76        5.40        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   8.824       GROUP 2=   17.742
OBSERVATION NO.= 61
      2.00        6.00        0.76        0.00        0.19        0.00        4.00        0.01
CHI-SQUARE VALUE,GROUP. 1=  4.889       GROUP 2=   54.272
OBSERVATION NO.= 62
      2.00        4.00        0.00        0.19        0.16        0.00        3.00        0.01
CHI-SQUARE VALUE,GROUP 1=   4.863       GROUP 2=   61.696
OBSERVATION NO.= 63
      1.00        2.00        0.09        0.14        0.88        1.50        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=   3.110       GROUP 2=   19.715
OBSERVATION NO.= 64
      9.00       40.50        0.84        0.56        0.85        5.50        1.00        0.01
CHI-SQUARE VALUE,GROUP 1=  21.248       GROUP 2=   31.279
OBSERVATION NO.= 65
      2.00        3.00        0.37        0.48        0.16        3.50        3.00        0.01
CHI-SQUARE VALUE,GROUP 1=   2.850       GROUP 2=   53.557
OBSERVATION NO.= 65
      1.00        2.00        1.37        0.00        0.26        0.00        4.00        0.05
CHI-SQUARE VALUE,GROUP 1=  24.644       GROUP 2=   46.305
OBSERVATION NO.= 67
      1.00        0.50        0.00        0.00        0.36        0.00        2.00        0.06
CHI-SQUARE VALUE,GROUP 1=   3.422       GROUP 2=   43.478
OBSERVATION NO.= 68
      1.00        2.00        0.10        0.00        0.07        0.00        1.00        0.06
CHI-SQUARE VALUE,GROUP 1=   2.310       GROUP 2=   23.120
OBSERVATION NO.= 69
      2.00        6.50        0.07        0.00        0.03        0.00        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=   1.317       GROUP 2=   15.667
OBSERVATION NO.= 70
      1.00        3.00        0.01        0.00        0.02        0.00        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=   2.727       GROUP 2=   19.957
OBSERVATION NO.= 71
      1.00        3.50        0.02        0.00        0.20        0.00        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=   2.232       GROUP 2=   19.497
OBSERVATION NO.= 72
      3.00       14.50        0.01        0.06        0.00        0.00        0.00        0.16
CHI-SQUARE VALUE,GROUP 1=   1.289       GROUP 2=   13.885
OBSERVATION NO.= 73
      6.00       29.50        0.00        0.00        1.83        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=  15.296       GROUP 2=   28.625
OBSERVATION NO.= 74
      1.00        0.50        0.00        0.10        1.88        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=  14.735       GROUP 2=   24.206
OBSERVATION NO.= 75
      2.00        4.00        0.15        0.27        1.06        2.50        1.00        0.03
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CHI-SQUARE VALUE,GROUP 1=     2.502       GROUP 2=    22.298
OBSERVATION NO.= 76
     12.00      108.00         2.36         0.84         0.53         6.90         0.00         0.05
CHI-SQUARE VALUE,GROUP 1=   161.253       GROUP 2=   216.094
OBSERVATION NO.= 77
      2.00        6.00         0.39         0.34         0.49         2.80         1.00         0.38
CHI-SQUARE VALUE,GROUP 1=     1.659       GROUP 2=    14.215
OBSERVATION NO.= 78
      2.00        4.50         0.46         0.00         0.00         0.00         1.00         0.06
CHI-SQUARE VALUE,GROUP 1=     3.066       GROUP 2=    12.095
OBSERVATION NO.= 79
      3.00       26.50         0.00         0.00         0.23         0.00         0.00         0.03
CHI-SQUARE VALUE,GROUP 1=     2.830       GROUP 2=    16.167
OBSERVATION NO.= 80
      1.00        1.00         0.00         0.05         0.22         0.00         0.00         0.03
CHI-SQUARE VALUE,GROUP 1=     1.905       GROUP 2=    20.068
OBSERVATION NO.= 81
      6.00       33.00         0.23         0.32         0.00         2.20         0.00         0.03
CHI-SQUARE VALUE,GROUP 1=     2.940       GROUP 2=    14.406
OBSERVATION NO.= 82
      1.00        3.50         0.26         0.13         0.19         0.80         0.00         0.00
CHI-SQUARE VALUE,GROUP 1=     3.219       GROUP 2=    16.914
OBSERVATION NO.= 83
      8.00       45.50         0.10         0.00         0.10         0.00         0.00         0.00
CHI-SQUARE VALUE,GROUP 1=     6.017       GROUP 2=    20.131
OBSERVATION NO.= 84
      6.00       37.50         0.00         0.21         0.20         3.00         0.00         0.00
CHI-SQUARE VALUE,GROUP 1=     2.398       GROUP 2=    14.941
OBSERVATION NO.= 85
      1.00        0.50         0.01         0.00         0.02         0.00         2.00         0.02
CHI-SQUARE VALUE,GROUP 1=     4.526       GROUP 2=    44.587
OBSERVATION NO.= 86
      1.00        2.50         0.00         0.00         0.32         0.00         0.00         0.03
CHI-SQUARE VALUE,GROUP 1=     2.039       GROUP 2=    19.856
OBSERVATION NO.= 87
      1.00        3.00         0.00         0.00         0.36         0.00         0.00         0.01
CHI-SQUARE VALUE,GROUP 1=     2.198       GROUP 2=    20.184
OBSERVATION NO.= 88
      1.00        3.50         0.04         0.00         0.49         0.00         0.00         0.00
CHI-SQUARE VALUE,GROUP 1=     2.306       GROUP 2=    19.194
OBSERVATION NO.= 89
      1.00        1.50         0.36         0.00         0.04         0.00         0.00         0.01
CHI-SQUARE VALUE,GROUP 1=     4.151       GROUP 2=    14.701
OBSERVATION NO.= 90
      4.00        7.50         0.04         0.00         0.48         0.00         0.00         0.02
CHI-SQUARE VALUE,GROUP 1=     2.508       GROUP 2=    15.758
OBSERVATION NO.= 91
      1.00        0.50         0.11         0.11         0.43         0.00         0.00         0.03
CHI-SQUARE VALUE,GROUP 1=     2.392       GROUP 2=    17.698
OBSERVATION NO.= 92
      3.00       11.00         0.34         0.09         0.09         1.20         0.00         0.02
CHI-SQUARE VALUE,GROUP 1=     2.145       GROUP 2=    13.101
OBSERVATION NO.= 93
      2.00        2.50         0.05         0.00         0.00         0.00         0.00         0.01
CHI-SQUARE VALUE,GROUP 1=     1.687       GROUP 2=    16.565
OBSERVATION NO.= 94
      1.00        0.50         0.15         0.29         0.00         4.00         0.00         0.05
CHI-SQUARE VALUE,GROUP 1=     4.177       GROUP 2=    24.594
OBSERVATION NO.= 95
      6.00       20.50         0.15         0.28         0.23         1.80         0.00         0.00
CHI-SQUARE VALUE,GROUP 1=     4.122       GROUP 2=    16.700
OBSERVATION NO.= 96
      3.00        7.00         0.17         0.03         0.08         0.30         1.00         0.00
CHI-SQUARE VALUE,GROUP 1=     1.302       GROUP 2=    16.254
OBSERVATION NO.= 97
      1.00        2.50         0.00         0.00         0.31         0.00         0.00         0.01
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CHI-SQUARE VALUE,GROUP 1=    2.142       GROUP 2=   20.168
OBSERVATION NO.= 98
      8.00       56.50        0.19        0.51        0.21        6.00        2.00        0.01
CHI-SQUARE VALUE,GROUP 1=    4.356       GROUP 2=   28.052
OBSERVATION NO.= 99
      7.00       47.50        0.00        0.02        0.50        0.00        0.00        0.05
CHI-SQUARE VALUE,GROUP 1=    3.728       GROUP 2=   16.848
OBSERVATION NO.=100
      2.00        6.00        0.00        0.00        1.21        0.00        0.00        0.06
CHI-SQUARE VALUE,GROUP 1=    4.344       GROUP 2=   18.181
OBSERVATION NO.=101
      1.00        1.50        0.11        0.00        0.54        0.00        1.00        0.04
CHI-SQUARE VALUE,GROUP 1=    1.718       GROUP 2=   21.872
OBSERVATION NO.=102
      2.00        2.00        0.31        0.00        0.45        0.00        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=    2.374       GROUP 2=   13.881
OBSERVATION NO.=103
      3.00       13.50        0.46        0.00        0.00        0.00        0.00        0.18
CHI-SQUARE VALUE,GROUP 1=    3.744       GROUP 2=   12.027
OBSERVATION NO.=104
     15.00       75.50        0.00        0.68        0.20        4.10        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=   39.428       GROUP 2=   47.558
OBSERVATION NO.=105
      3.00        4.50        0.36        0.01        0.21        0.00        0.00        0.16
CHI-SQUARE VALUE,GROUP 1=    2.618       GROUP 2=   11.844
OBSERVATION NO.=106
      6.00       32.50        0.00        0.53        0.31        5.00        0.00        0.28
CHI-SQUARE VALUE,GROUP 1=    3.366       GROUP 2=   13.002
OBSERVATION NO.=107
      2.00       26.50        0.00        0.11        0.69        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=    9.013       GROUP 2=   19.256
OBSERVATION NO.=108
      2.00        3.50        0.44        0.25        0.14        0.70        3.00        0.02
CHI-SQUARE VALUE,GROUP 1=    3.053       GROUP 2=   39.858
OBSERVATION NO.=109
      3.00       10.00        0.04        0.21        0.02        0.80        2.00        0.03
CHI-SQUARE VALUE,GROUP 1=    2.642       GROUP 2=   34.298
OBSERVATION NO.=110
      1.00        4.50        0.00        0.02        0.25        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=    2.435       GROUP 2=   20.503
OBSERVATION NO.=111
      3.00        6.00        0.04        0.23        0.08        2.50        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=    1.780       GROUP 2=   17.849
OBSERVATION NO.=112
     12.00      102.50        0.08        0.37        0.58        3.60        2.00        0.02
CHI-SQUARE VALUE,GROUP 1=   18.835       GROUP 2=   26.774
OBSERVATION NO.=113
      1.00        1.00        0.42        0.02        0.00        0.00        3.00        0.21
CHI-SQUARE VALUE,GROUP 1=    2.704       GROUP 2=   46.266
OBSERVATION NO.=114
      3.00        5.50        0.02        0.00        0.00        0.00        2.00        0.24
CHI-SQUARE VALUE,GROUP 1=    5.276       GROUP 2=   34.194
OBSERVATION NO.=115
      3.00       22.00        0.11        0.12        0.51        1.50        1.00        0.01
CHI-SQUARE VALUE,GROUP 1=    1.416       GROUP 2=   20.095
OBSERVATION NO.=116
      1.00        1.00        0.22        0.05        0.35        0.00        1.00        0.01
CHI-SQUARE VALUE,GROUP 1=    1.689       GROUP 2=   18.928
OBSERVATION NO.=117
      1.00        1.00        0.00        0.00        0.43        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=    1.968       GROUP 2=   19.965
OBSERVATION NO.=118
      4.00        5.00        0.04        0.00        0.00        0.00        0.00        0.03
CHI-SQUARE VALUE,GROUP 1=    4.205       GROUP 2=   14.633
OBSERVATION NO.=119
      3.00       14.50        0.00        0.00        0.08        0.00        0.00        0.02
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CHI-SQUARE VALUE,GROUP 1=     1.098     GROUP 2=    16.020
OBSERVATION NO.=120
        1.00        0.50        0.08        0.00        0.01        0.00        1.00        0.02
CHI-SQUARE VALUE,GROUP 1=     2.682     GROUP 2=    24.390
OBSERVATION NO.=121
        5.00       18.50        0.01        0.00        0.45        0.00        0.00        0.01
CHI-SQUARE VALUE,GROUP 1=     2.316     GROUP 2=    16.384
OBSERVATION NO.=122
       16.00      205.00        0.03        1.04        1.29        6.60        0.00        1.14
CHI-SQUARE VALUE,GROUP 1=   186.802     GROUP 2=    13.918
OBSERVATION NO.=123
        9.00      123.00        0.12        0.95        4.18        9.50        0.00        1.04
CHI-SQUARE VALUE,GROUP 1=   127.397     GROUP 2=    16.099
OBSERVATION NO.=124
       11.00      146.50        0.01        0.90        1.20        3.20        0.00        0.50
CHI-SQUARE VALUE,GROUP 1=   138.755     GROUP 2=    13.972
OBSERVATION NO.=125
        5.00       22.00        1.06        0.52        0.48       11.10        0.00        0.30
CHI-SQUARE VALUE,GROUP 1=    37.763     GROUP 2=    15.630
OBSERVATION NO.=126
       25.00      323.00        0.38        1.50        0.16       23.20        0.00        0.08
CHI-SQUARE VALUE,GROUP 1=   241.886     GROUP 2=    15.561
OBSERVATION NO.=127
        5.00       20.00        0.00        0.00        0.28        0.00        0.00        0.66
CHI-SQUARE VALUE,GROUP 1=     8.644     GROUP 2=    12.273
OBSERVATION NO.=128
        8.00       90.50        0.49        0.30        0.43        0.30        2.00        0.32
CHI-SQUARE VALUE,GROUP 1=    36.295     GROUP 2=    13.075
OBSERVATION NO.=129
        4.00       22.00        0.12        0.79        1.08        1.80        1.00        0.82
CHI-SQUARE VALUE,GROUP 1=    49.438     GROUP 2=    14.293
OBSERVATION NO.=130
        6.00       20.50        0.01        0.11        0.84        0.50        0.00        0.32
CHI-SQUARE VALUE,GROUP 1=     5.034     GROUP 2=    12.684
OBSERVATION NO.=131
       12.00      234.00        0.07        0.38        0.14        4.50        0.00        0.23
CHI-SQUARE VALUE,GROUP 1=   262.328     GROUP 2=    12.803
OBSERVATION NO.=132
       14.00      306.50        0.43        0.34        0.87        1.20        1.00        0.32
CHI-SQUARE VALUE,GROUP 1=   557.989     GROUP 2=    13.304
OBSERVATION NO.=133
       15.00      204.50        0.06        0.73        1.05        4.60        1.00        0.73
CHI-SQUARE VALUE,GROUP 1=   168.368     GROUP 2=     8.999
OBSERVATION NO.=134
        7.00       59.00        0.03        0.71        2.29        3.70        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=    42.586     GROUP 2=    32.197
OBSERVATION NO.=135
       11.00      137.50        0.23        1.10        1.31       10.10        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=    64.493     GROUP 2=    20.913
OBSERVATION NO.=136
       16.00      247.00        0.37        0.70        0.01        8.80        0.00        0.02
CHI-SQUARE VALUE,GROUP 1=   205.888     GROUP 2=    10.953
OBSERVATION NO.=137
       12.00      123.00        0.00        0.96        0.05       11.60        0.00        0.06
CHI-SQUARE VALUE,GROUP 1=    26.279     GROUP 2=    17.879
OBSERVATION NO.=138
        5.00      115.50        0.00        0.05        0.03        0.50        0.00        0.00
CHI-SQUARE VALUE,GROUP 1=    87.463     GROUP 2=    19.435
OBSERVATION NO.=139
        9.00      199.50        0.04        1.44        1.42       21.50        0.00        0.90
CHI-SQUARE VALUE,GROUP 1=   215.266     GROUP 2=   118.227
OBSERVATION NO.=140
       15.00      144.00        0.17        0.31        1.11        4.90        2.00        0.02
CHI-SQUARE VALUE,GROUP 1=    44.924     GROUP 2=    34.853
OBSERVATION NO.=141
        4.00       13.00        0.48        1.08        0.60        1.50        2.00        0.16
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CHI-SQUARE VALUE,GROUP 1=   93.538       GROUP 2=   36.764
OBSERVATION NO.=142
      3.00        14.50          0.18           1.59           1.16          6.20          0.00          0.73
CHI-SQUARE VALUE,GROUP 1=  132.204       GROUP 2=   46.685
OBSERVATION NO.=143
     12.00        70.00          0.25           1.21           0.94         12.50          0.00          0.60
CHI-SQUARE VALUE,GROUP 1=   23.842       GROUP 2=   11.716
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      1      0.99684     0.00316
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      2      0.98038     0.01962
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      3      0.99195     0.00805
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      4      0.98985     0.01015
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      5      0.99985     0.00015
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      6      1.00000     0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      7      0.99993     0.00007
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      8      0.99983     0.00017
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
      9      0.99582     0.00418
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     10      0.99952     0.00048
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     11      0.99859     0.00141
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     12      0.99911     0.00089
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     13      0.99998     0.00002
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     14      0.99084     0.00916
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     15      0.99504     0.00496
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     16      0.99014     0.00985
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     17      0.99563     0.00437
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     18      1.00000     0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     19      1.00000     0.0
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     20      0.47802     0.52198
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     21      0.99575     0.00425
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     22      0.99921     0.00079
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     23      0.99199     0.00801
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     24      0.99947     0.00053
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     25      0.98820     0.01180
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     26      0.97548     0.02452
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     27      0.99965     0.00035
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     28      0.99653     0.00347
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     29      0.99991     0.00009
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
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        30      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        31      0.99839      0.00161
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        32      0.90781      0.09219
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        33      0.99281      0.00719
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        34      0.99998      0.00002
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        35      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        36      0.96980      0.03020
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        37      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        38      0.91426      0.08574
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        39      0.99792      0.00208
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        40      0.99337      0.00663
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        41      0.99729      0.00271
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        42      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        43      0.57301      0.42700
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        44      0.94870      0.05130
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        45      0.99710      0.00290
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        46      0.99938      0.00062
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        47      0.74915      0.25085
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        48      0.99984      0.00016
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        49      0.60001      0.39999
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        50      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        51      0.99596      0.00404
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        52      0.99990      0.00010
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        53      0.99969      0.00031
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        54      0.99263      0.00737
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        55      0.95106      0.04894
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        56      0.99951      0.00049
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        57      0.98813      0.01188
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        58      0.99970      0.00030
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        59      0.99848      0.00152
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        60      0.98856      0.01144
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        61      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        62      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
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        63      0.99975      0.00025
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        64      0.99341      0.00659
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        65      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        66      0.99998      0.00002
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        67      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        68      0.99997      0.00003
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        69      0.99924      0.00076
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        70      0.99982      0.00018
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        71      0.99982      0.00018
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        72      0.99816      0.00184
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        73      0.99873      0.00127
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        74      0.99130      0.00870
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        75      0.99995      0.00005
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        76      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        77      0.99813      0.00187
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        78      0.98917      0.01083
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        79      0.99873      0.00127
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        80      0.99989      0.00011
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        81      0.99677      0.00323
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        82      0.99894      0.00106
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        83      0.99914      0.00086
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        84      0.99811      0.00189
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        85      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        86      0.99986      0.00014
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        87      0.99988      0.00012
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        88      0.99979      0.00022
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        89      0.99491      0.00509
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        90      0.99867      0.00133
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        91      0.99953      0.00047
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        92      0.99584      0.00416
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        93      0.99941      0.00059
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        94      0.99996      0.00004
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
        95      0.99815      0.00185
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
```

```
     96      0.99943      0.00057
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     97      0.99988      0.00012
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     98      0.99999      0.00001
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
     99      0.99859      0.00141
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    100      0.99901      0.00099
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    101      0.99996      0.00004
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    102      0.99684      0.00316
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    103      0.98435      0.01565
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    104      0.98312      0.01688
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    105      0.99018      0.00982
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    106      0.99198      0.00802
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    107      0.99407      0.00593
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    108      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    109      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    110      0.99988      0.00012
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    111      0.99968      0.00032
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    112      0.98147      0.01853
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    113      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    114      1.00000      0.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    115      0.99991      0.00009
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    116      0.99982      0.00018
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    117      0.99988      0.00012
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    118      0.99459      0.00541
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    119      0.99943      0.00057
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    120      0.99998      0.00002
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    121      0.99912      0.00088
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    122      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    123      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    124      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    125      0.00002      0.99998
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    126      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    127      0.85991      0.14009
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    128      0.00001      0.99999
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    129      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    130      0.97865      0.02135
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    131      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    132      0.0          1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    133      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    134      0.00552      0.99448
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    135      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    136      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    137      0.01477      0.98523
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    138      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    139      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    140      0.00646      0.99354
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    141      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    142      0.00000      1.00000
OBSERVATION PROB. OF GROUP 1  PROB. OF GROUP 2
    143      0.00232      0.99768
"
```

APPENDIX D

STATISTICAL ANALYSIS SYSTEM-DISCRIMINANT ANALYSIS PROGRAM OUTPUT

```
DATA FRT;
INPUT GROUP 1 X1 2-6 X2 7-12 X3 13-18 X4 19-24 X5 25-30 X6 31-36
X7 37-42 X8 43-48;
CARDS


  54 OBSERVATIONS IN DATA SET FRT                9  VARIABLES

PROC DISCRIM S WCOV WCORR PCOV PCORR LIST PROP POOL=TEST SLPOOL=0.05;
CLASSES GROUP;
TITLE 'DISCRIMINANT ANALYSIS';
```

```
                    DISCRIMINANT ANALYSIS

                    DISCRIMINANT ANALYSIS

GROUP                   FREQUENCY        PRIOR PROBABILITY

    1                      42              0.77777778

    2                      12              0.22222222
-----                      --              ----------
TOTAL                      54              1.00000000
```

```
                                            DISCRIMINANT ANALYSIS

                              DISCRIMINANT ANALYSIS     SIMPLE STATISTICS

                                                  GROUP = 1

                                                                                                    STANDARD
VARIABLE          N                 SUM                  MEAN                VARIANCE              DEVIATION

X1               42        150.00000000            3.57142857              6.44599303             2.53889603

X2               42        738.00000000           17.57142857            557.95818815            23.62113659

X3               42          9.62000000            0.22904762              0.12620883             0.35525882

X4               42         10.51000000            0.25023810              0.16995848             0.41226021

X5               42         20.09000000            0.47833333              0.21623374             0.46500940

X6               42         99.20000000            2.36190476             22.43948897             4.73703377

X7               42         45.00000000            1.07142857              9.09233449             3.01534981

X8               42          6.37000000            0.15166667              0.06712154             0.25907826
--------------------------------------------------------------------------------------------------------------

                                                  GROUP = 2

X1               12        130.00000000           10.83333333             36.87878788             6.07279076

X2               12       1717.50000000          143.12500000          12511.36931818           111.85423246

X3               12          2.78000000            0.23166667              0.09885152             0.31440661

X4               12          7.56000000            0.63000000              0.18825455             0.43388310

X5               12         12.00000000            1.00000000              1.16739091             1.08049032

X6               12         66.50000000            5.54166667             43.94628788             6.62919964

X7               12          5.00000000            0.41666667              0.44696970             0.66855792

X8               12          6.45000000            0.53833333              0.11372424             0.33723025
```

```
                                        DISCRIMINANT ANALYSIS

                              DISCRIMINANT ANALYSIS      WITHIN COVARIANCE MATRICES
DF =   41                                               GROUP = 1
VARIABLE             X1               X2               X3               X4               X5               X6
X1           6.44599303      54.12891986       0.04153310       0.50034843      -0.11707317       5.89790941
                     X7               X8
            -0.50522648      -0.15073171
----------------------------------------------------------------------------------------------------------------
                     X1               X2               X3               X4               X5               X6
X2          54.12891986     557.95818815       0.82872822       4.88839721      -1.23646341      61.01010453
                     X7               X8
            -3.90766551      -1.15817073
----------------------------------------------------------------------------------------------------------------
                     X1               X2               X3               X4               X5               X6
X3           0.04153310       0.82372822       0.12620383       0.05238560       0.00586423       0.57159698
                     X7               X8
             0.80055749       0.00698943
----------------------------------------------------------------------------------------------------------------
                     X1               X2               X3               X4               X5               X6
X4           0.50034843       4.88839721       0.05238560       0.16995348       0.01360772       1.89015563
                     X7               X8
             0.17412892      -0.02078089
----------------------------------------------------------------------------------------------------------------
                     X1               X2               X3               X4               X5               X6
X5          -0.11707317      -1.23646341       0.00586423       0.01360772       0.21623374       0.13920325
                     X7               X8
             0.12768293       0.03783943
----------------------------------------------------------------------------------------------------------------
                     X1               X2               X3               X4               X5               X6
X6           5.89790941      61.01010453       0.57159698       1.89015563       0.13920325      22.43548897
                     X7               X8
             0.90034843      -0.21113008
```

```
                                                        DISCRIMINANT ANALYSIS

                                         DISCRIMINANT ANALYSIS     WITHIN COVARIANCE MATRICES

DF =   41                                                     GROUP = 1

VARIABLE                   X1              X2                 X3                 X4                 X5                 X6

X7                -0.50522648     -3.90766551         0.80055749         0.17412892         0.12768293         0.90034843

                           X7              X8

                   9.09233449     -0.04158537
--------------------------------------------------------------------------------------------------------------------------

                           X1              X2                 X3                 X4                 X5                 X6

X8                -0.15073171     -1.15817073         0.00698943        -0.02078089         0.03783943        -0.21113008

                           X7              X8

                  -0.04158537      0.06712154
--------------------------------------------------------------------------------------------------------------------------
```

```
                                                       DISCRIMINANT ANALYSIS

                                       DISCRIMINANT ANALYSIS     WITHIN COVARIANCE MATRICES

DF =   11                                                    GROUP = 2

VARIABLE                    X1                  X2                  X3                  X4                  X5                  X6

X1                 36.87878788        616.25000000         -0.12969697          1.79818182         -0.75727273         26.70757576

                            X7                  X8

                   -0.46969697         -0.37575758
----------------------------------------------------------------------------------------------------------------------------------

                            X1                  X2                  X3                  X4                  X5                  X6

X2                616.25000000      12511.36931818         -1.98340909         24.09227273         -9.20318182        331.70795455

                            X7                  X8

                   -0.14772727         -8.12113636
----------------------------------------------------------------------------------------------------------------------------------

                            X1                  X2                  X3                  X4                  X5                  X6

X3                 -0.12969697         -1.98340909          0.09885152         -0.00150000         -0.07840909          0.72919697

                            X7                  X8

                    0.03924242         -0.04802424
----------------------------------------------------------------------------------------------------------------------------------

                            X1                  X2                  X3                  X4                  X5                  X6

X4                  1.79818182         24.09227273         -0.00150000          0.18825455          0.13748182          2.22627273

                            X7                  X8

                   -0.06272727          0.03071818
----------------------------------------------------------------------------------------------------------------------------------

                            X1                  X2                  X3                  X4                  X5                  X6

X5                 -0.75727273         -9.20318182         -0.07840909          0.13748182          1.16789091          0.32736364

                            X7                  X8

                   -0.10363636          0.24259091
----------------------------------------------------------------------------------------------------------------------------------

                            X1                  X2                  X3                  X4                  X5                  X6

X6                 26.70757576        331.70795455          0.72919697          2.22627273          0.32736364         43.94628788

                            X7                  X8

                   -1.77343485         -0.46365152
```

```
                                        DISCRIMINANT ANALYSIS

                              DISCRIMINANT ANALYSIS     WITHIN COVARIANCE MATRICES
DF =    11                                            GROUP = 2
VARIABLE              X1              X2              X3              X4              X5              X6
X7           -0.46969697     -0.14772727      0.03924242     -0.06272727  .  -0.10363636     -1.77348485
                      X7              X8
              0.44696970     -0.01651515
--------------------------------------------------------------------------------------------------------
                      X1              X2              X3              X4              X5              X6
X8           -0.37575758     -8.12113636     -0.04802424      0.03071818      0.24259091     -0.46365152
                      X7              X8
             -0.01651515      0.11372424
--------------------------------------------------------------------------------------------------------
```

```
                                   DISCRIMINANT ANALYSIS

             DISCRIMINANT ANALYSIS      WITHIN CORRELATION COEFFICIENTS   /   PROBABILITY > |R|

                                              GROUP = 1

VARIABLE           X1          X2          X3          X4          X5          X6          X7          X8

X1           1.000000    0.902576    0.046047    0.478031   -0.099163    0.490396   -0.065994   -0.229155
               0.0000      0.0001      0.7692      0.0017      0.5390      0.0013      0.6811      0.1407

X2           0.902576    1.000000    0.098757    0.501989   -0.112569    0.545248   -0.054863   -0.189252
               0.0001      0.0000      0.5407      0.0010      0.5154      0.0004      0.7299      0.2280

X3           0.046047    0.098757    1.000000    0.357681    0.035498    0.339655    0.747326    0.075939
               0.7692      0.5407      0.0000      0.0190      0.8180      0.0262      0.0001      0.6379

X4           0.478031    0.501989    0.357681    1.000000    0.070983    0.967876    0.140075   -0.194564
               0.0017      0.0010      0.0190      0.0000      0.6594      0.0001      0.6202      0.2146

X5          -0.099163   -0.112569    0.035498    0.070983    1.000000    0.063195    0.091061    0.314088
               0.5390      0.5154      0.8180      0.6594      0.0000      0.6933      0.5730      0.0404

X6           0.490396    0.545248    0.339655    0.967876    0.063195    1.000000    0.063033   -0.172033
               0.0013      0.0004      0.0262      0.0001      0.6933      0.0000      0.6940      0.2755

X7          -0.065994   -0.054863    0.747326    0.140075    0.091061    0.063033    1.000000   -0.053232
               0.6811      0.7299      0.0001      0.6202      0.5730      0.6940      0.0000      0.7371

X8          -0.229155   -0.189252    0.075939   -0.194564    0.314088   -0.172033   -0.053232    1.000000
               0.1407      0.2280      0.6379      0.2146      0.0404      0.2755      0.7371      0.0000
```

```
                                         DISCRIMINANT ANALYSIS

                  DISCRIMINANT ANALYSIS       WITHIN CORRELATION COEFFICIENTS   /   PROBABILITY > |R|

                                                     GROUP = 2

VARIABLE            X1          X2          X3          X4          X5          X6          X7          X8

X1            1.000000    0.907227   -0.067928    0.682453   -0.115389    0.663415   -0.115689   -0.183482
                0.0000      0.0001      0.8277      0.0140      0.7205      0.0180      0.7198      0.5736

X2            0.907227    1.000000   -0.056399    0.496424   -0.076135    0.447345   -0.001975   -0.215297
                0.0001      0.0000      0.8557      0.0980      0.8084      0.1422      0.9910      0.5073

X3           -0.067928   -0.056399    1.000000   -0.010996   -0.230767    0.349858    0.186692   -0.452942
                0.8277      0.8557      0.0000      0.9717      0.5238      0.2644      0.5668      0.1366

X4            0.682453    0.496424   -0.010996    1.000000    0.293205    0.774006   -0.216244    0.209941
                0.0140      0.0980      0.9717      0.0000      0.6429      0.0034      0.5054      0.5132

X5           -0.115389   -0.076135   -0.230767    0.293205    1.000000    0.045695   -0.143441    0.665651
                0.7205      0.8084      0.5238      0.6429      0.0000      0.8825      0.6595      0.0175

X6            0.663415    0.447345    0.349858    0.774006    0.045695    1.000000   -0.400154   -0.207398
                0.0180      0.1422      0.2644      0.0034      0.8825      0.0000      0.1955      0.5235

X7           -0.115689   -0.001975    0.186692   -0.216244   -0.143441   -0.400154    1.000000   -0.073252
                0.7198      0.9910      0.5668      0.5054      0.6595      0.1955      0.0000      0.8152

X8           -0.183482   -0.215297   -0.452942    0.209941    0.665651   -0.207398   -0.073252    1.000000
                0.5736      0.5073      0.1366      0.5132      0.0175      0.5235      0.8152      0.0000
```

```
                                          DISCRIMINANT ANALYSIS

DF =    52                       DISCRIMINANT ANALYSIS      POOLED COVARIANCE MATRIX

VARIABLE                  X1              X2              X3              X4              X5              X6

X1               12.98369963    173.03914835      0.00531136      0.77489011     -0.25250000     10.29995421

                          X7              X8

                 -0.49771062     -0.19333333
-----------------------------------------------------------------------------------------------------------

                          X1              X2              X3              X4              X5              X6

X2              173.03914835   3086.56438874      0.23385302      8.95075549     -2.92173077    118.27311126

                          X7              X8

                 -3.11229396     -2.63110577
-----------------------------------------------------------------------------------------------------------

                          X1              X2              X3              X4              X5              X6

X3                0.00531136      0.23385302      0.12042170      0.04098672     -0.01196282      0.60493544

                          X7              X8

                  0.63951007     -0.00464808
-----------------------------------------------------------------------------------------------------------

                          X1              X2              X3              X4              X5              X6

X4                0.77489011      8.95075549      0.04098672      0.17382880      0.03981186      1.96125733

                          X7              X8

                  0.12402473     -0.00988636
-----------------------------------------------------------------------------------------------------------

                          X1              X2              X3              X4              X5              X6

X5               -0.25250000     -2.92173077     -0.01196282      0.03981186      0.41754583      0.17900641

                          X7              X8

                  0.07875000      0.08115224
-----------------------------------------------------------------------------------------------------------

                          X1              X2              X3              X4              X5              X6

X6               10.29995421    118.27311126      0.60493544      1.96125733      0.17900641     26.98900412

                          X7              X8

                  0.33472965     -0.26454808
-----------------------------------------------------------------------------------------------------------
```

```
                                         DISCRIMINANT ANALYSIS

DF =    52                    DISCRIMINANT ANALYSIS     POOLED COVARIANCE MATRIX

VARIABLE                 X1           X2           X3           X4           X5           X6

X7              -0.49771062  -3.11229396   0.63951007   0.12402473   0.07875000   0.33472985

                         X7           X8

                 7.26350733  -0.03628205
--------------------------------------------------------------------------------------------

                         X1           X2           X3           X4           X5           X6

X8              -0.19833333  -2.63110577  -0.00464308  -0.00988636   0.08115224  -0.26454808

                         X7           X8

                -0.03628205   0.07697981
--------------------------------------------------------------------------------------------
```

```
                                          DISCRIMINANT ANALYSIS

DISCRIMINANT ANALYSIS      PARTIAL CORRELATION COEFFICIENTS COMPUTED FROM POOLED COVARIANCE MATRIX    /    PROB > |R|

    VARIABLE           X1          X2          X3          X4          X5          X6          X7          X8

    X1           1.000000    0.867734    0.004264    0.517796   -0.108865    0.552359   -0.051450   -0.199153
                   0.0000      0.0001      0.9744      0.0002      0.5562      0.0001      0.7154      0.1492

    X2           0.867734    1.000000    0.012130    0.386421   -0.081386    0.409784   -0.020786   -0.170692
                   0.0001      0.0000      0.9288      0.0045      0.5692      0.0027      0.8773      0.2195

    X3           0.004264    0.012130    1.000000    0.283289   -0.053349    0.335555    0.683789   -0.048276
                   0.9744      0.9288      0.0000      0.0375      0.7060      0.0134      0.0001      0.7312

    X4           0.517796    0.386421    0.283289    1.000000    0.147775    0.905483    0.110376   -0.085469
                   0.0002      0.0045      0.0375      0.0000      0.2911      0.0001      0.5628      0.5498

    X5          -0.108865   -0.081386   -0.053349    0.147775    1.000000    0.053324    0.045219    0.452647
                   0.5562      0.5692      0.7060      0.2911      0.0000      0.7061      0.7465      0.0010

    X6           0.552359    0.409784    0.335555    0.905483    0.053324    1.000000    0.023907   -0.183537
                   0.0001      0.0027      0.0134      0.0001      0.7061      0.0000      0.8593      0.1853

    X7          -0.051450   -0.020786    0.683789    0.110376    0.045219    0.023907    1.000000   -0.048521
                   0.7154      0.8773      0.0001      0.5628      0.7465      0.8593      0.0000      0.7300

    X8          -0.199153   -0.170692   -0.048276   -0.085469    0.452647   -0.183537   -0.048521    1.000000
                   0.1492      0.2195      0.7312      0.5498      0.0010      0.1853      0.7300      0.0000
```

```
                         DISCRIMINANT ANALYSIS

DISCRIMINANT ANALYSIS        WITHIN COVARIANCE MATRIX INFORMATION

GROUP     RANK OF COVARIANCE MATRIX      NATURAL LOG OF THE DETERMINANT
                                            OF THE COVARIANCE MATRIX

    1                 8                          -0.99468787

    2                 8                           3.16945640

POOLED                8                           3.53281769
```

```
                         DISCRIMINANT ANALYSIS

DISCRIMINANT ANALYSIS      TEST OF HOMOGENEITY OF WITHIN COVARIANCE MATRICES

      NOTATION:          K     =     NUMBER OF GROUPS

                         P     =     NUMBER OF VARIABLES

                         N     =     TOTAL NUMBER OF OBSERVATIONS

                         N(I)  =     NUMBER OF OBSERVATIONS IN THE I'TH GROUP

                                                                 N(I)/2
                                       TT  |WITHIN SS MATRIX(I)|
                         V     =     ----------------------------------
                                                           N/2
                                           |POOLED SS MATRIX|

                                                                      2.
                                     |-           1            1   -|  2P  + 3P - 1
                         RHO   =   1.0 - |  SUM -------   -   ---    |  ------------
                                     |_         N(I)-1        N-K  _|  6(P+1)(K-1)

                         DF    =     .5(K-1)P(P+1)

                                  |-    PN/2               -|
                                  |    N           V        |
UNDER NULL HYPOTHESIS:  -2 RHO LN |  ------------------     |  IS DISTRIBUTED APPROXIMATELY AS CHI-SQUARE(DF)
                                  |    __         PN(I)/2   |
                                  |_   TT  N(I)            _|

TEST CHI-SQUARE VALUE =             142.36846474     WITH         36 DF     PROB > CHI-SQ = 0.0001

SINCE THE CHI-SQUARE VALUE IS SIGNIFICANT AT THE 0.0500 LEVEL, THE WITHIN COVARIANCE MATRICES WILL BE
USED IN THE DISCRIMINANT FUNCTION.

REFERENCE: KENDALL,M.G. AND STUART,A. THE ADVANCED THEORY OF STATISTICS VOL.3, P266 & 282.
```

```
                            DISCRIMINANT ANALYSIS

 DISCRIMINANT ANALYSIS     PAIRWISE SQUARED GENERALIZED DISTANCES BETWEEN GROUPS

         2                      -1
                    _   _            _   _
 WHERE: D (I|J) = ( X - X )' COV   ( X - X )  +  LN |COV |  -  2 LN PRIOR
                     I   J      J     I   J             J                J

                    GENERALIZED SQUARED DISTANCE TO GROUP

             FROM GROUP                    1                   2

                    1           -0.49205901         71.70813752

                    2           14.87855452          6.17761119
```

```
                                            DISCRIMINANT ANALYSIS

                           DISCRIMINANT ANALYSIS      CLASSIFICATION RESULTS FOR EACH OBSERVATION
               GIVING GENERALIZED SQUARED DISTANCE TO EACH GROUP   /   POSTERIOR PROBABILITY OF MEMBERSHIP IN EACH GROUP

         2                   -1                                                                       2                       2
WHERE: D (X) = ( X - X )' COV   ( X - X )  +  LN |COV |  -  2 LN PRIOR       AND      PR(J|X) = EXP(-.5 D (X)) / SUM EXP(-.5 D (X))
        J            J       J        J               J                J                                  J          K            K

                                          GENERALIZED SQUARED DISTANCE TO GROUP

                          OBS        FROM    CLASSIFIED                 1                2
                                     GROUP   INTO GROUP

                            1          1          1             12.935399        24.777567
                                                                   0.9973           0.0027

                            2          1          1             10.326322        18.289670
                                                                   0.9817           0.0183

                            3          1          1              3.847199        13.528990
                                                                   0.9922           0.0078

                            4          1          1              4.923170        14.178049
                                                                   0.9903           0.0097

                            5          1          1              2.744549        20.418493
                                                                   0.9999           0.0001

                            6          1          1             12.977099       124.916269
                                                                   1.0000           0.0000

                            7          1          1              2.474798        21.783300
                                                                   0.9999           0.0001

                            8          1          1              1.688878        19.036582
                                                                   0.9998           0.0002

                            9          1          1              1.576131        12.572584
                                                                   0.9959           0.0041

                           10          1          1              1.893061        17.275311
                                                                   0.9995           0.0005

                           11          1          1              6.430258        19.489257
                                                                   0.9985           0.0015

                           12          1          1              3.147143        17.272594
                                                                   0.9991           0.0009

                           13          1          1             33.002755        56.074785
                                                                   1.0000           0.0000

                           14          1          1              1.940459        11.223435
                                                                   0.9904           0.0096

                           15          1          1              1.966196        12.587629
                                                                   0.9951           0.0049
```

```
                                  DISCRIMINANT ANALYSIS

                  DISCRIMINANT ANALYSIS      CLASSIFICATION RESULTS FOR EACH OBSERVATION
GIVING GENERALIZED SQUARED DISTANCE TO EACH GROUP   /   POSTERIOR PROBABILITY OF MEMBERSHIP IN EACH GROUP

                              GENERALIZED SQUARED DISTANCE TO GROUP

          OBS       FROM     CLASSIFIED                   1                2
                    GROUP    INTO GROUP

           16         1          1                 1.781334        11.013286
                                                     0.9902           0.0098

           17         1          1                 1.901937        12.724216
                                                     0.9956           0.0044

           18         1          1                19.217417        62.641169
                                                     1.0000           0.0000

           19         1          1                33.389658      1189.856481
                                                     1.0000           0.0000

           20         1          2                14.838607        14.714694
                                                     0.4845           0.5155

           21         1          1                 6.642013        17.768379
                                                     0.9962           0.0038

           22         1          1                 0.762468        15.163327
                                                     0.9993           0.0007

           23         1          1                 6.633738        16.435963
                                                     0.9926           0.0074

           24         1          1                 3.192285        18.396065
                                                     0.9995           0.0005

           25         1          1                 2.166141        10.956954
                                                     0.9878           0.0122

           26         1          1                 2.222423         9.476114
                                                     0.9741           0.0259

           27         1          1                 0.711625        16.716412
                                                     0.9997           0.0003

           28         1          1                 2.277366        13.544857
                                                     0.9964           0.0036

           29         1          1                 1.174411        19.921607
                                                     0.9999           0.0001

           30         1          1                 4.842068        78.429224
                                                     1.0000           0.0000

           31         1          1                 4.011712        16.377548
                                                     0.9985           0.0015
```

```
                              DISCRIMINANT ANALYSIS


          DISCRIMINANT ANALYSIS     CLASSIFICATION RESULTS FOR EACH OBSERVATION
GIVING GENERALIZED SQUARED DISTANCE TO EACH GROUP   /   POSTERIOR PROBABILITY OF MEMBERSHIP IN EACH GROUP

                         GENERALIZED SQUARED DISTANCE TO GROUP

         OBS          FROM     CLASSIFIED                        1                 2
                     GROUP     INTO GROUP

          32             1              1                11.000187         15.496706
                                                            0.9045            0.0955

          33             1              1                 3.580966         13.435840
                                                            0.9928            0.0072

          34             1              1                 4.026994         25.931109
                                                            1.0000            0.0000

          35             1              1                 1.979179         30.769296
                                                            1.0000            0.0000

          36             1              1                14.306782         21.603303
                                                            0.9746            0.0254

          37             1              1                32.741203        120.013361
                                                            1.0000            0.0000

          38             1              1                 8.836561         13.573141
                                                            0.9144            0.0356

          39             1              1                 9.692490         22.120466
                                                            0.9980            0.0020

          40             1              1                 3.148359         13.147575
                                                            0.9933            0.0067

          41             1              1                 1.990927         13.735411
                                                            0.9972            0.0028

          42             1              1                 7.591054        145.745810
                                                            1.0000            0.0000

          43             2              2               187.193111         13.943135
                                                            0.0000            1.0000

          44             2              2               128.032699         16.047269
                                                            0.0000            1.0000

          45             2              2               139.659960         13.824966
                                                            0.0000            1.0000

          46             2              2                38.509279         15.616264
                                                            0.0000            1.0000

          47             2              2               241.766596         15.530083
                                                            0.0000            1.0000
```

```
                                   DISCRIMINANT ANALYSIS

                   DISCRIMINANT ANALYSIS     CLASSIFICATION RESULTS FOR EACH OBSERVATION
GIVING GENERALIZED SQUARED DISTANCE TO EACH GROUP   /   POSTERIOR PROBABILITY OF MEMBERSHIP IN EACH GROUP

                            GENERALIZED SQUARED DISTANCE TO GROUP

                    OBS        FROM    CLASSIFIED                1                 2
                               GROUP   INTO GROUP

                     48          2          1             8.533729         12.145670
                                                          0.8589            0.1411

                     49          2          2            36.219628         13.025212
                                                          0.0000            1.0000

                     50          2          2            49.373878         14.334518
                                                          0.0000            1.0000

                     51          2          1             5.074779         12.676181
                                                          0.9781            0.0219

                     52          2          2           262.942251         12.757794
                                                          0.0000            1.0000

                     53          2          2           559.466029         13.283007
                                                          0.0000            1.0000

                     54          2          2           168.871437          8.947232
                                                          0.0000            1.0000
```

```
                                   DISCRIMINANT ANALYSIS

DISCRIMINANT ANALYSIS     SUMMARY OF CLASSIFICATION PERFORMANCE USING GENERALIZED SQUARED DISTANCE

                2                         -1
          WHERE: D (X) = ( X - X̄ )' COV  ( X - X̄ )  +  LN |COV |  -  2 LN PRIOR
                  J             J      J        J             J                J

                        NUMBER OF OBSERVATIONS CLASSIFIED INTO GROUP

                          FROM GROUP          1         2

                                  1          41         1

                                  2           2        10
```

APPENDIX E

BIOMEDICAL COMPUTER PROGRAMS-STEPWISE DISCRIMINANT ANALYSIS OUTPUT

```
BMD07M - STEPWISE DISCRIMINANT ANALYSIS - REVISED SEPT 14, 1970
UNIVERSITY OF IDAHO
COMPUTER SERVICES


PROBLEM CODE        FRNGRD

NUMBER OF VARIABLES     8

NUMBER OF GROUPS        2

NUMBER OF CASES IN EACH GROUP    42     12

PRIOR PROBABILITIES         0.5000     0.5000

VARIABLE FORMAT          (1X,F5.0,7F6.0)


DATA INPUT FROM CARDS

MEANS  (THE LAST COLUMN CONTAINS THE GRAND MEANS OVER THE GROUPS USED IN THE ANALYSIS)

          GROUP
              A           B
VARIABLE
  1       3.57143    10.83333    5.18518
  2      17.57143   143.12500   45.47221
  3       0.22905     0.23167    0.22963
  4       0.25024     0.63000    0.33463
  5       0.47833     1.00000    0.59426
  6       2.36190     5.54166    3.06851
  7       1.07143     0.41667    0.92593
  8       0.15167     0.53833    0.23759

STANDARD DEVIATIONS

          GROUP
              A           B
VARIABLE
  1       2.53889     6.07278
  2      23.62111   111.85413
  3       0.35526     0.31441
  4       0.41226     0.43388
  5       0.46501     1.08069
  6       4.73702     6.62919
  7       3.01534     0.66856
  8       0.25908     0.33723

WITHIN GROUPS COVARIANCE MATRIX

          VARIABLES
              1            2            3            4            5            6            7            8
VARIABLE
  1      12.88366
  2     173.03860   3086.55762
  3       0.00531      0.23386      0.12042
  4       0.77489      8.95073      0.04099      0.17383
  5      -0.25250     -2.92173     -0.01196      0.03981      0.41754
  6      10.29994    118.27290      0.60493      1.96125      0.17901     26.98891
  7      -0.49771     -3.11226      0.63951      0.12402      0.07875      0.33473      7.26343
  8      -0.19833     -2.63110     -0.00465     -0.00989      0.08115     -0.26455     -0.03628      0.07698

WITHIN GROUPS CORRELATION MATRIX
```

```
              VARIABLES
                  1            2            3            4            5            6            7            8
VARIABLE
    1         1.00000
    2         0.86773      1.00000
    3         0.00426      0.01213      1.00000
    4         0.51780      0.38642      0.28329      1.00000
    5        -0.10887     -0.08139     -0.05335      0.14777      1.00000
    6         0.55236      0.40978      0.33556      0.90548      0.05332      1.00000
    7        -0.05145     -0.02079      0.68379      0.11038      0.04522      0.02391      1.00000
    8        -0.19915     -0.17069     -0.04828     -0.08547      0.45265     -0.18354     -0.04852      1.00000
```

```
SUBPROBLEM                    1
F-LEVEL FOR INCLUSION    0.0100
F-LEVEL FOR DELETION     0.0050
TOLERANCE LEVEL          0.0001
CONTROL VALUES         11111111

*****************************************************************************************************

STEP NUMBER          0
VARIABLE ENTERED

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   52

  1  38.2031          3   0.0005          5   6.0830          7   0.5509
  2  47.6673          4   7.7435          6   3.4966          8  18.1273

*****************************************************************************************************

STEP NUMBER          1
VARIABLE ENTERED     2

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1   52

  2  47.6673

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   51

  1   0.0751          3   0.0019          4   0.0080          5   4.7238          6   0.5660          7   0.1835          8  15.5752

U-STATISTIC            0.52174    DEGREES OF FREEDOM    1    1   52
APPROXIMATE F         47.66727    DEGREES OF FREEDOM    1   52.00

F MATRIX - DEGREES OF FREEDOM    1    52

            GROUP
              A
GROUP
     B    47.66727

*****************************************************************************************************

STEP NUMBER          2
VARIABLE ENTERED     8

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1   51

  2  43.6188          8  15.5752

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   50

  1   0.3563          3   0.0151          4   0.0227          5   0.1579          6   0.0491          7   0.0364

U-STATISTIC            0.39968    DEGREES OF FREEDOM    2    1   52
APPROXIMATE F         38.30165    DEGREES OF FREEDOM    2   51.00

F MATRIX - DEGREES OF FREEDOM    2    51

            GROUP
              A
GROUP
     B    38.30168

*****************************************************************************************************
```

```
STEP NUMBER        3
VARIABLE ENTERED   1

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1   50

  1   0.3563          2   3.8910          8  15.6382

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   49

  3   0.0173          4   0.0088          5   0.1701          6   0.2696          7   0.0211

U-STATISTIC            0.39685    DEGREES OF FREEDOM     3    1   52
APPROXIMATE F         25.33092    DEGREES OF FREEDOM     3   50.00

F MATRIX - DEGREES OF FREEDOM      3    50

             GROUP
                A
GROUP
    B     25.33057

*******************************************************************************************************

STEP NUMBER        4
VARIABLE ENTERED   6

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1   49

  1   0.5720          2   3.3151          6   0.2696          8  14.5170

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   48

  3   0.1368          4   0.5211          5   0.2741          7   0.0120

U-STATISTIC            0.39468    DEGREES OF FREEDOM     4    1   52
APPROXIMATE F         18.78806    DEGREES OF FREEDOM     4   49.00

F MATRIX - DEGREES OF FREEDOM      4    49

             GROUP
                A
GROUP
    B     18.78770

*******************************************************************************************************

STEP NUMBER        5
VARIABLE ENTERED   4

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1   48

  1   0.5081          2   3.2166          4   0.5211          6   0.7793          8  11.8491

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1   47

  3   0.1538          5   0.1563          7   0.0778

U-STATISTIC            0.39044    DEGREES OF FREEDOM     5    1   52
APPROXIMATE F         14.98779    DEGREES OF FREEDOM     5   48.00

F MATRIX - DEGREES OF FREEDOM      5    48

             GROUP
                A
GROUP
```

```
     B    14.98733

****************************************************************************************************

STEP NUMBER          6
VARIABLE ENTERED     5

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1    47

  1   0.5629          2   3.0270          4   0.3976          5   0.1563          6   0.7219          8   7.6949

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1    46

  3   0.1947          7   0.0804

U-STATISTIC           0.38914     DEGREES OF FREEDOM    6     1   52
APPROXIMATE F        12.29634     DEGREES OF FREEDOM    6    47.00

F MATRIX - DEGREES OF FREEDOM     6    47

            GROUP
               A
GROUP
     B    12.29597

****************************************************************************************************

STEP NUMBER          7
VARIABLE ENTERED     3

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1    46

  1   0.6726          2   2.7522          3   0.1947          4   0.3961          5   0.1972          6   0.8504          8   7.3142

VARIABLES NOT INCLUDED AND F TO ENTER - DEGREES OF FREEDOM     1    45

  7   0.8811

U-STATISTIC           0.38750     DEGREES OF FREEDOM    7     1   52
APPROXIMATE F        10.38695     DEGREES OF FREEDOM    7    46.00

F MATRIX - DEGREES OF FREEDOM     7    46

            GROUP
               A
GROUP
     B    10.38663

****************************************************************************************************

STEP NUMBER          8
VARIABLE ENTERED     7

VARIABLES INCLUDED AND F TO REMOVE - DEGREES OF FREEDOM     1    45

  1   0.7941          3   0.9949          5   0.3547          7   0.8811
  2   2.5477          4   0.8228          6   1.5602          8   5.4723

U-STATISTIC           0.38006     DEGREES OF FREEDOM    8     1   52
APPROXIMATE F         9.17524     DEGREES OF FREEDOM    8    45.00

F MATRIX - DEGREES OF FREEDOM     8    45

            GROUP
               A
GROUP
```

```
     B       9.17492

F LEVEL  INSUFFICIENT FOR FURTHER COMPUTATION

            FUNCTION
                  A                B
VARIABLE
  1         1.12544          1.45173
  2        -0.04698         -0.01417
  3         4.49524          7.25309
  4         1.67609          4.88811
  5         1.49667          2.12245
  6        -0.35767         -0.75064
  7        -0.21544         -0.52850
  8         0.84231          6.24854

CONSTANT
           -2.89865        -10.47528

      GROUP WITH             SQUARE OF DISTANCE FROM AND POSTERIOR
     LARGEST PROB.                     PROBABILITY FOR GROUP -

 GROUP                              A                  B
        A
  CASE
   1              A   11.285 0.796,    14.006 0.204,
   2              A    9.759 0.941,    15.281 0.059,
   3              A    3.079 0.994,    13.325 0.006,
   4              A    1.659 0.996,    12.518 0.004,
   5              A    2.545 0.999,    16.772 0.001,
   6              A   11.018 0.998,    23.790 0.002,
   7              A    2.408 0.997,    13.743 0.003,
   8              A    1.916 0.999,    15.114 0.001,
   9              A    1.310 0.992,    11.044 0.008,
  10              A    1.184 0.994,    11.513 0.006,
  11              A    3.956 0.976,    11.353 0.024,
  12              A    2.504 0.986,    11.079 0.014,
  13              B   34.635 0.034,    27.926 0.966,
  14              A    0.954 0.993,    10.953 0.007,
  15              A    2.438 0.970,     9.363 0.030,
  16              A    1.485 0.984,     9.676 0.016,
  17              A    1.679 0.995,    12.264 0.005,
  18              A   16.383 0.998,    28.747 0.002,
  19              A   40.648 0.998,    52.940 0.002,
  20              A    5.833 0.738,     7.904 0.262,
  21              A    5.720 0.913,    10.425 0.087,
  22              A    1.233 0.994,    11.367 0.006,
  23              A    3.266 0.994,    13.385 0.006,
  24              A    2.679 0.994,    12.859 0.006,
  25              A    1.730 0.967,     8.481 0.033,
  26              A    2.381 0.914,     7.098 0.086,
  27              A    0.796 0.998,    13.286 0.002,
  28              A    1.478 0.998,    13.469 0.002,
  29              A    0.938 0.998,    13.124 0.002,
  30              A    4.399 0.998,    16.711 0.002,
  31              A    2.350 0.995,    12.802 0.005,
  32              A    4.280 0.811,     7.195 0.189,
  33              A    1.554 0.995,    12.192 0.005,
  34              A    4.669 0.998,    16.889 0.002,
  35              A    1.718 0.998,    14.551 0.002,
  36              A    7.954 0.998,    20.150 0.002,
  37              A   29.507 0.848,    32.943 0.152,
  38              A    2.794 0.974,    10.024 0.026,
  39              A    3.957 0.996,    15.225 0.004,
  40              A    1.333 0.982,     9.291 0.018,
  41              A    2.019 0.995,    12.565 0.005,
```

```
42            A   6.302 0.997,  18.068 0.003,

GROUP                   A            B
    B
 CASE
  1           B  21.885 0.000,   3.846 1.000,
  2           B  19.465 0.003,   7.598 0.997,
  3           A  15.960 0.956,  22.099 0.044,
  4           B  42.375 0.000,  25.459 1.000,
  5           A   7.152 0.824,  10.243 0.176,
  6           B   7.903 0.172,   4.758 0.828,
  7           B  15.054 0.197,  12.240 0.803,
  8           A   3.695 0.925,   8.708 0.075,
  9           B  21.033 0.007,  11.050 0.993,
 10           B  45.081 0.000,  23.453 1.000,
 11           B  35.437 0.000,  11.099 1.000,
 12           B  40.275 0.001,  25.710 0.999,

         NUMBER OF CASES CLASSIFIED INTO GROUP -
               A       B
GROUP
       A      41       1
       B       3       9
```

```
SUMMARY TABLE

 STEP        VARIABLE          F VALUE TO          NUMBER OF         U-STATISTIC
NUMBER   ENTERED  REMOVED   ENTER OR REMOVE   VARIABLES INCLUDED

   1        2                  47.6673               1               0.5217
   2        8                  15.5752               2               0.3997
   3        1                   0.3563               3               0.3968
   4        6                   0.2696               4               0.3947
   5        4                   0.5211               5               0.3904
   6        5                   0.1563               6               0.3891
   7        3                   0.1947               7               0.3875
   8        7                   0.8811               8               0.3801

      EIGENVALUES

         1.63109     0.00001     0.00000     0.00000    -0.00000    -0.00000    -0.00000    -0.00000

      CUMULATIVE PROPORTICN CF TOTAL DISPERSION

         1.00000     1.00000     1.00000     1.00000     1.00000     1.00000     1.00000     1.00000

      CANONICAL CORRELATICNS

         0.78736     0.00291     0.00157     0.00092     0.00018     0.00064     0.00064     0.00104

COEFFICIENTS FOR CANONICAL VARIABLE -

ORIGINAL      1            2            3            4            5            6            7            8
VARIABLE
   1       0.10824     -0.56604      0.21136     -0.13818      0.11312     -0.02377      0.06072      0.07840
   2       0.01088      0.03338     -0.01126      0.00127      0.00261     -0.00014      0.00069     -0.00150
   3       0.91484     -1.40911     -2.36101     -3.20436     -0.86386      1.95822      0.54830     -0.66621
   4       1.06548     -2.16088     -5.02430      2.68509     -0.09756      0.14274      0.86542     -0.42151
   5       0.20758     -0.20230      0.34646     -0.42690      0.24151     -0.10850      0.50168     -1.64539
   6      -0.13036      0.26984      0.44760     -0.01320     -0.13428      0.04328     -0.01402      0.06364
   7      -0.10385      0.12899      0.31281      0.31666      0.28742      0.10729     -0.15775      0.06026
   8       1.79337      0.19745      1.01881      0.78773     -1.60443      0.30986     -3.04712      1.23312

GRCUP           CANONICAL VARIABLES EVALUATED AT GROUP MEANS
   1      -0.66990     -0.00000     -0.00000     -0.00000      0.00000      0.00000      0.00000     -0.00000
   2       2.34465     -0.00000      0.00000      0.00000     -0.00000      0.00000      0.00000     -0.00000

CHECK ON FINAL U-STATISTIC          0.38007
```

```
POINTS PLOTTED ON THE FOLLWING GRAPH

    X = FIRST CANONICAL VARIABLE
    Y = SECOND CANONICAL VARIABLE
    CASE NUMBER FOLLOWED BY * INDICATES THE POINT IS OFF THE GRAPH

GROUP       A  MEAN COORDINATES  -0.670  -0.000

CASE       X       Y    CASE       X       Y    CASE       X       Y    CASE       X       Y

   1   0.386  -1.716      11  -0.389  -1.545       2  -0.078  -1.002      12  -0.585  -1.058       3  -0.862   0.136      13   1.950   0.725
   4  -0.964  -0.513      14  -0.821  -0.201       5  -1.522   1.060      15  -0.311  -0.642       6  -1.281   0.620      16  -0.521  -0.036
   7  -1.043   0.133      17  -0.918   0.601       8  -1.352   0.974      18  -1.213   0.637       9  -0.777  -0.151      19  -1.201   0.178
  10  -0.876  -0.144      20   0.494  -1.108

  21   0.057  -1.852      31  -0.896   0.178      22  -0.843  -0.212      32   0.354  -1.089      23  -0.841  -0.177      33  -0.927  -0.521
  24  -0.851  -0.103      34  -1.189   1.446      25  -0.283   0.231      35  -1.291   1.029      26   0.055  -0.556      36  -1.185   0.972
  27  -1.234   0.422      37   0.268  -0.248      28  -1.152   0.147      38  -0.362   0.412      29  -1.184   0.072      39  -1.032   1.730
  30  -1.205  -0.261      40  -0.483   0.721

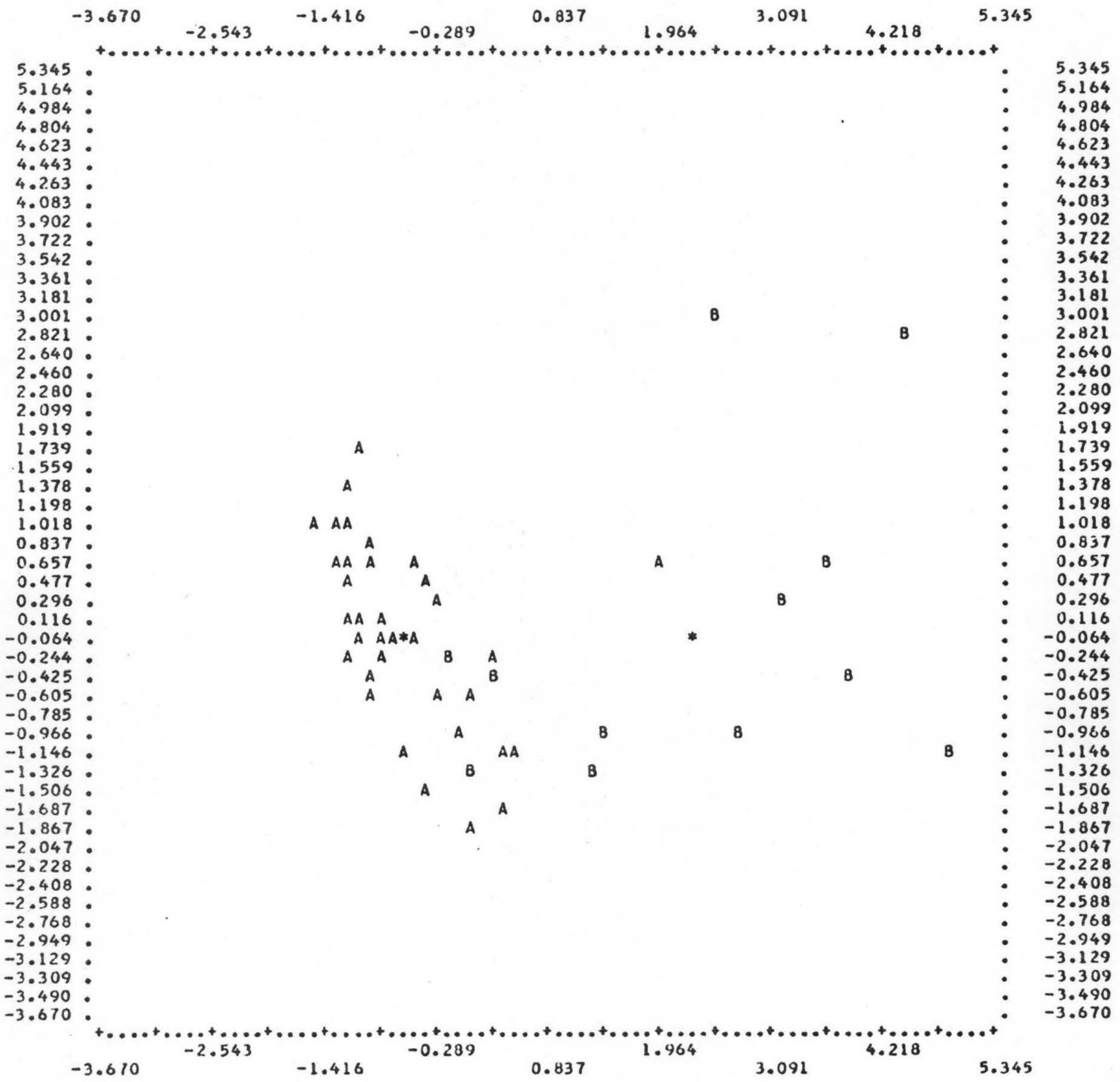
  41  -0.912   0.836
  42  -1.114  -0.124

GROUP       B  MEAN COORDINATES   2.345  -0.000

CASE       X       Y

   1   3.829  -0.435      11   4.874  -1.169
   2   2.806  -0.986      12   3.253   0.302
   3  -0.181  -0.166
   4   3.643   0.686
   5   0.325  -0.499
   6   1.359  -0.942
   7   1.304  -1.258
   8   0.006  -1.346
   9   2.493   2.919
  10   4.425   2.894

OVERLAP IS INDICATED BY $, GROUP MEANS BY *.
```



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FINISH CARD ENCOUNTERED, JOB TERMINATED
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