direction of motion may arise, and something given in pure intuition from which the concept of the velocity of motion may arise. But if these "somethings" are neither direction, nor velocity, nor some a priori geometry, what are they? That is the great puzzle of pure intuition.

Kant provides some examples of the kinds of things ~~that~~ to which he gives the name of "relative space." In one example, that of a ball rolling on a table inside the cabin of a ship, the cabin serves as a "space at rest" in which the motion of the ball is perceived. In a second example, he gives this cabin a window from which the riverbank can be observed. In this example, the ball is at rest relative to the riverbank (thus making "the outdoors" the "space at rest"), and the "space of the cabin" is in motion (in a direction opposite to that of the first example).

In this way, he points out, one is always at liberty to assign part of the motion to the "thing" and another part of the motion to the "space" in which the thing is found, at least insofar as rectilinear motion is concerned [:159-160]. This liberty, he warns us, is not present in the same respect when it comes to ~~rect~~ non-rectilinear motion [:160]. However, he promises to deal with this problem "in the sequel." In particular, he warns us that in Mechanics this "indifference" in assigning motion to the thing or its material space will ~~b~~ "not be any longer ~~be~~ so entirely indifferent" [:160].

The Composition of Motion

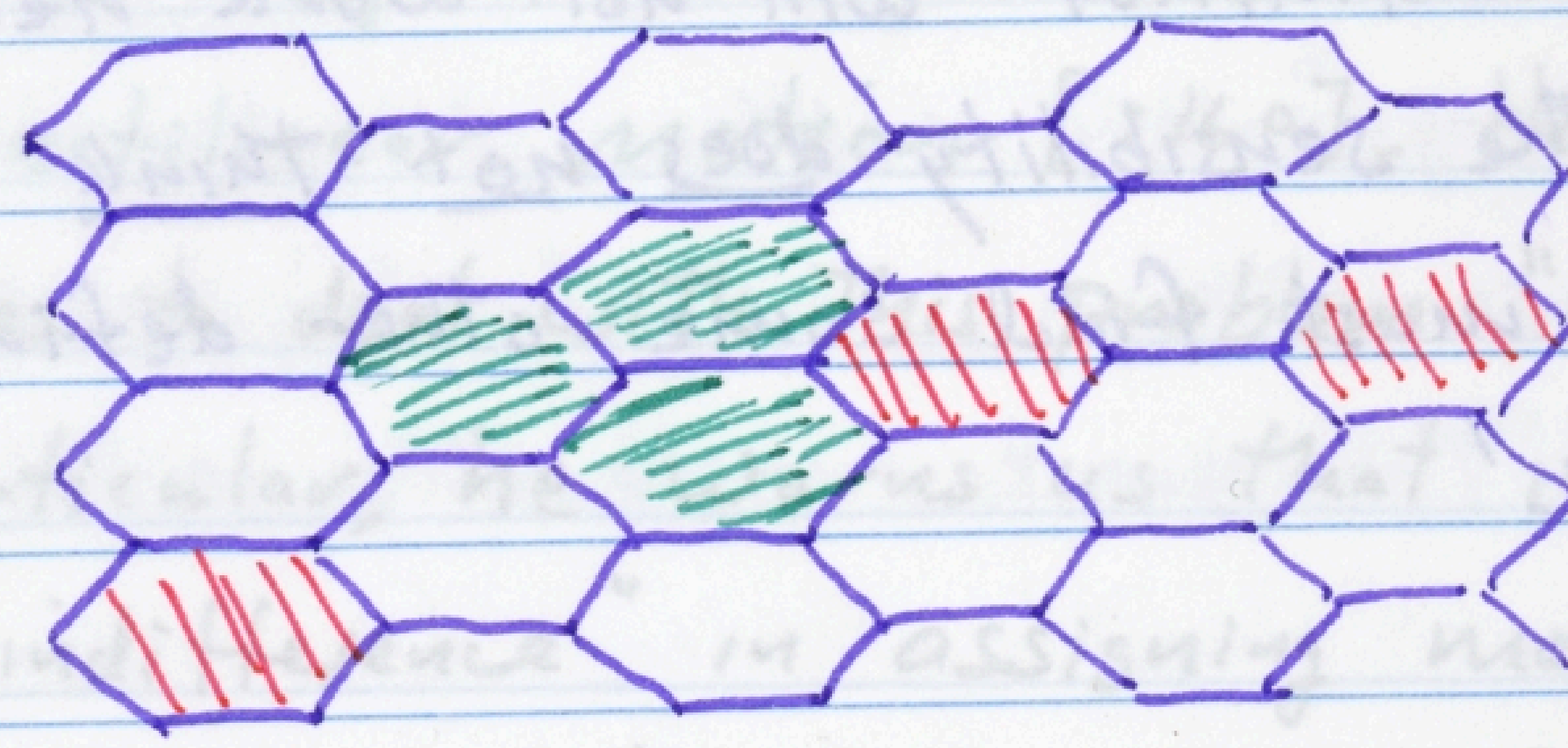
~~Phoronomy~~ Phoronomy is the metaphysics of motion (or, more accurately, the conception of motion) viewed as composition by aggragation ("quantity"). In this metaphysics, the concept of the extension of the thing moving may not be introduced because this would combine the concept of motion with the concept of the "body" in motion. Consequently, in phoronomy motion must be viewed as motion of a point [:160].

This immediately raises the question of: what, precisely, is a point? A "geometrical point" is a concept defined in terms of a limit to a process (e.g., "shrinking a ball down to nothing"), but such a "definition" will not work here because the sensibility does not think. Therefore, we must find some other definition of a "point."

The obvious starting point is to take Kant very seriously and propose that a "point" is matter cognized without any cognition of that matter as extended. The very notion of an extended body is the cognition of that body in terms of parts - each part occupying a different "place" and every place "in" that body being reachable from any other place by a "trajectory," or succession of "places," such that each place in this succession lies "in" the extended body.

Therefore: a material point is a representation of matter such that this representation is a singular cognition. [This is my deduction; Kant does not explicitly say this].

As an illustration of this concept, let us represent a particular manifold of sensations at a single moment of time by using a "grid" as follows:

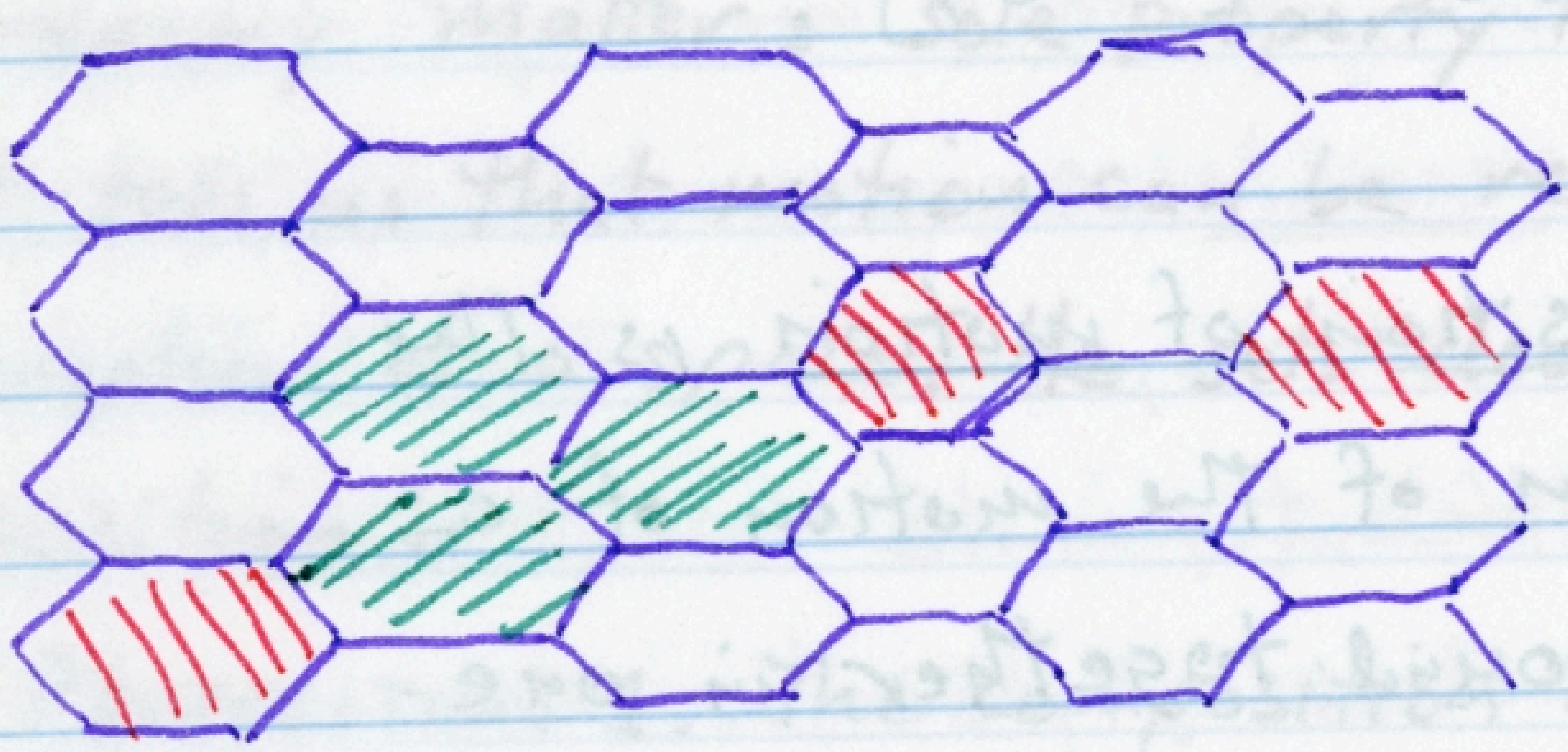


In this representation, let the green "cross-hatched" hexagons represent individual sense "elements" registering a common degree of sensation. In this "picture," if the three green hexagons are cognized as a singular structure (such that removal of any one of the 3 sense elements from the structure destroys the cognition of that structure), then these three sense elements define a single "point."

Note: There is one subtle but still fairly obvious question arising from this "picture" of a point. The three green hexagons are drawn adjacent to each other. Does this adjacency represent a second implied condition on a point? The answer to this question is not yet deduced;

for instance, consider the three red hexagons: these are not adjacent, but if they can be cognized as a singular cognition, then the union of these "red" hexagons must also be considered to define a "point."

Now, to this notion of how a point is to be represented, we add a second question. Suppose the previous manifold in sensation is immediately followed ~~by the~~ (in the "next moment of time") by the following manifold. Do the three green hexagons still represent the "same" point?



Again, we are not ready to deduce the answer to this question. If the answer is "yes," we can call the difference between these two manifolds a ~~[illegible]~~ "motion." If the answer is "no," then it is not "motion" in the perspective of ~~Phor~~ phoronomy.

Now let us return to Kant's doctrine. Since Quantity is composition as aggragation, the doctrine of phoronomy is the doctrine of the composition (as aggragate) of motions into <u>one</u> motion. This composition is an act of <u>construction</u> by Intuition.

"To construct the conception of a <u>composite</u> <u>motion</u> means to present <u>a</u> <u>priori</u> in intuition a motion so far as it arises from two or more given [motions] united in one movable" [:157].

"The <u>composition</u> <u>of</u> <u>motion</u> is the presentation of the motion of a point as bound together in one with two or more motions of the same" [:160].

The remainder of the Division of Phoronomy is basically a treatment of how this can be constructed. It is a somewhat difficult doctrine owing to the abstractness of "motion" as a "thing itself."

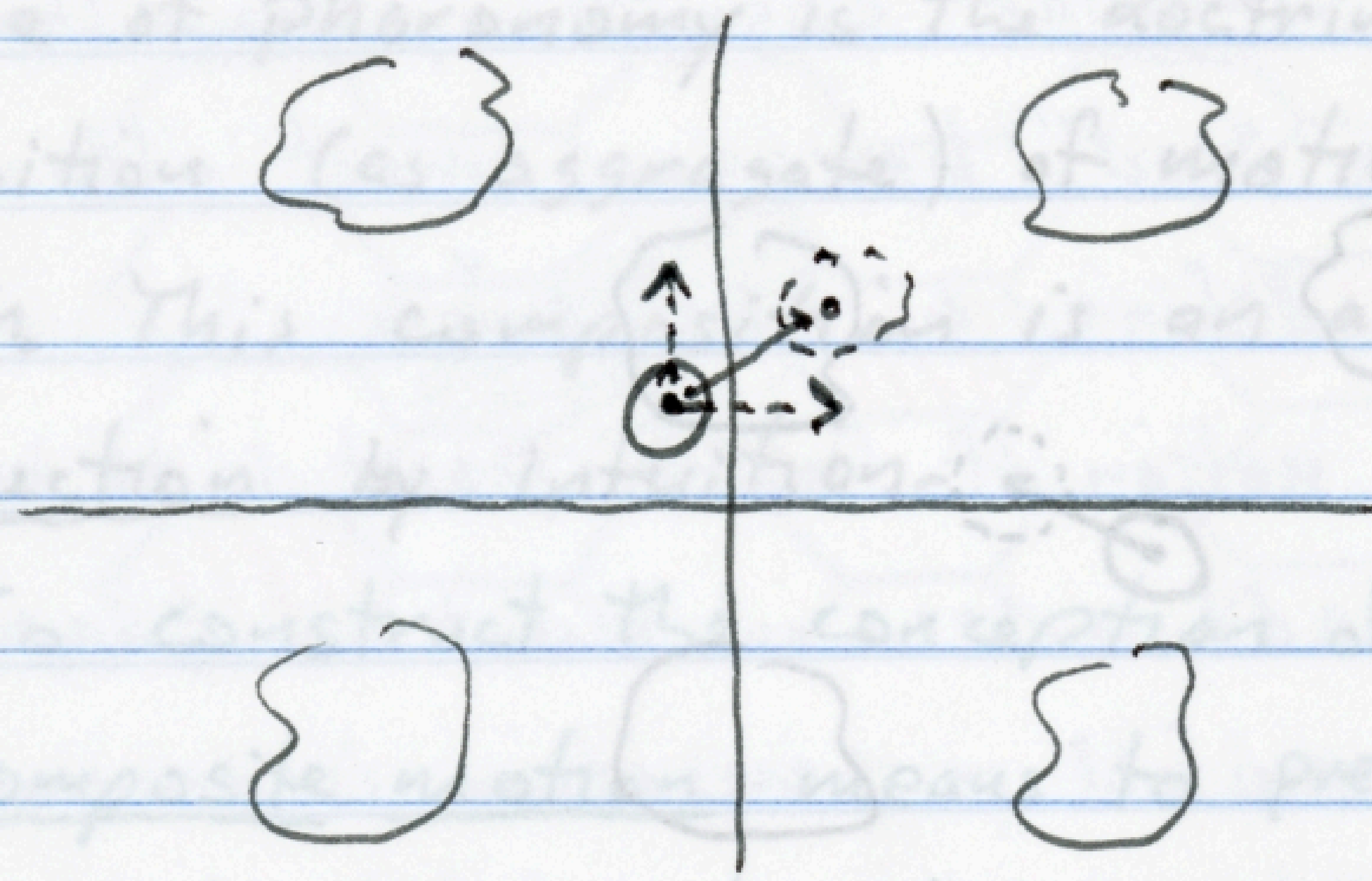
As an illustration, suppose we have the experience of seeing a thing moving relative to a background, e.g.

How can this single motion be regarded as the composition of <u>two</u> motions? Since in phoronomy matter's sole property is movability, Kant tells us that motion can be regarded only as description of a space. He sets down the following proposition:

"PROPOSITION 1. The composition of two motions in one and the same point can only be conceived by one of them being presented in absolute space, but, instead of the other, a motion of an equal velocity in the contrary direction of the relative space [being presented] as identical with it" [:161]

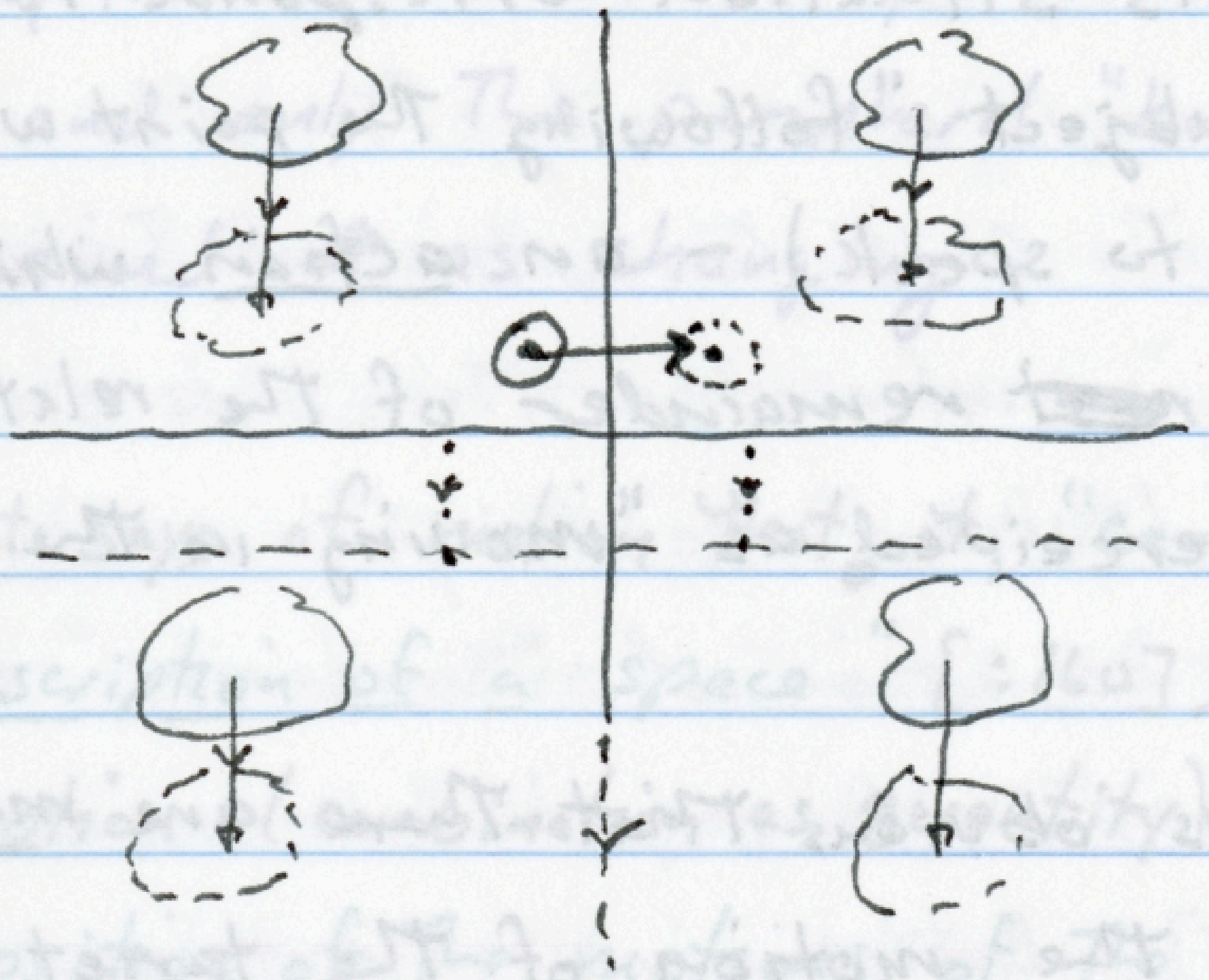
Now, what does this clumsy statement mean?

First, consider the usual treatment of this problem in vector mathematics.



We resolve the single motion into the vector sum of two constituent vectors. However, does such a representation have *real* meaning? No. We *never* have a *sensible* experience of this sort because these vectors, at each moment of *time*, describe two different points – neither of which is the *actual* moving point.

The situation is quite different, however, if the axes shown above are taken to represent the ~~m~~ "place" of the material space with respect to (subjective) absolute space. In this case, we can represent the single motion (in relative – material – space) as the composition of the motion of both the point *and* the relative space, e.g.



The motion of the original point is now "fixed" to that point at all moments of time, and the "second motion" accounting for the change in external relations in relative space is represented in the motion of the entire relative space with respect to (subjective) absolute space.

Kant presents us with three (somewhat difficult to follow) "demonstrations" of this proposition [:162–164]. What he is doing, in a sense, is fixing the real observations to the "rest frame" of the "observer" (i.e., the thinking Subject). It seems to me that, metaphysically, the "absolute space" (which is motionless) is – in a real sense – the "rest frame" of the noumenal I of the Subject.

Expressed in terms of the sensorimotor homologues, this situation corresponds to the observing Subject "following the point with his eyes" (so to speak) - an action which results in the ~~rest~~ remainder of the relative space being perceived as "moving in the visual field."

Further, it is obvious that there are many ways in which the motion of the target point could be "resolved" in this "picture." For example, the Subject could follow the target point perfectly (with its eyes), in which case the target point is "absolutely at rest" and the entire phenomenon is perceived as motion of the relative space itself.

Comment: The "real significance" of a perception of this sort is inseparably bound to what could be called the Subject's "focus of attention." The target point "moves" if the subject chooses to think it moves. If the Subject's "attention" is so "riveted" to the target point (e.g., keeps it centered in the fovea), The empirical <u>intuition</u> is one in which the perception of the target point changes <u>not</u> <u>at</u> <u>all</u>, and only the peripheral "background" is perceived as changing.

Kant says of motion that it, "can only be viewed <u>as</u> <u>description</u> <u>of</u> <u>a</u> space" [:160]. The <u>composition</u> of motion (combination as quantity) is, "the composition of the motions of the same point according to its direction and velocity, i.e., the presentation of a single motion as one that comprises within it two or perhaps several motions in one, at the same time, in the same point... but not in so far as they produce the latter as causes produce effects." [:162] In other words, we do not say "the" motion of a point is "caused" or "produced" by the motions ~~of~~ of which "the" motion is <u>said</u> to be composed.

Kant gives a mathematical definition of velocity (i.e., $C = S/T$) [:155] but it is clear to me that this "definition" is given for convenience and to clarify that Kant is talking about <u>spatial</u> velocity — and then only for "rectilinear" motion. He does not actually make any numerical use of this equation; insofar as how to interpret the concept of velocity, Piaget's observations in The Child's Conception of Time are probably more informative than this classical expression of (average) velocity.

However, the cognition of "similarity" and "equality" are available in intuition as congruity [:165]. But congruity of two motions combined in a third can never be presented in material (relative) space. Consequently, Kant says, geometrical construction of motions can never be possible except through congruences and, therefore, is not possible in a relative (material) space unless moving causes are also introduced. [:165].

Kant makes the following interesting point about composition in quantity. He points out that while it is true that a "larger" space can be viewed as composed of an aggragate of smaller spaces, the same is not true of "larger" velocities. It is "not necessarily implied... that

two equal velocities may be combined in the same way as two equal spaces, for it is not in itself obvious that a given velocity consists of smaller [velocities]; and in the same way that a rapidity consists of slownesses" [:165].
He continues, "For the parts of the velocity are not outside one another... and if the former are to be considered as quantity, the conception of their quantity, as it is intensive, must be constructed in a different manner to that of the extensive quantity of space." [:165]

It is well to remember here that the foundations of a natural science must be capable of being represented mathematically, which for Kant means "capable of representation in sensible intuition." And, as phoronomy abstracts from matter everything except its movability, the composition of motion can take place only at a "point" and, therefore, phoronomy can deal only with "rectilinear" motion. For curvilinear motion, we must add something else — "moving causes" — and pass on to another division.