Purpose: This experiment will experimentally determine the scattering parameters of the Hybrid or "Magic" Tee Directional Coupler under the condition that ports 3 and 4 of the coupler are matched loaded.

Theoretical Discussion:

When ports 3 & 4 of the coupler are matched (forcing $S_{33} = S_{44} = 0$), the scattering matrix for the hybrid Tee may be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \text{[EQ. 1]}$$

The unitary property for a lossless structure requires

$$[S^*]_t [S] = [I] \quad \text{[EQ. 2]}$$

Now, $[S^*]_t$ may be expressed as

$$[S^*]_t = \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} \quad \text{[EQ. 3]}$$

Plugging EQ. 1 & EQ. 3 into EQ. 2 yields the following system of equations:

$$S_{11}^* S_{11} + S_{12}^* S_{12} + S_{13}^* S_{13} + S_{14}^* S_{14} = 1 \quad \text{[EQ. 2.1]}$$
$$S_{11}^* S_{12} + S_{12}^* S_{22} + S_{13}^* S_{13} - S_{14}^* S_{14} = 0 \quad \text{[EQ. 2.2]}$$
$$S_{11}^* S_{13} + S_{12}^* S_{13} = 0 \quad \text{[EQ. 2.3]}$$
$$S_{11}^* S_{14} - S_{12}^* S_{14} = 0 \quad \text{[EQ. 2.4]}$$

$$S_{12}^* S_{11} + S_{22}^* S_{12} + S_{13}^* S_{13} - S_{14}^* S_{14} = 0 \quad \text{[EQ. 2.5]}$$
$$S_{12}^* S_{12} + S_{22}^* S_{22} + S_{13}^* S_{13} + S_{14}^* S_{14} = 1 \quad \text{[EQ. 2.6]}$$

EE 554 — [S] PARAMETERS OF HYBRID TEE (CONT) — WELLS — MAR. 18, 1975

$$S_{12}^* S_{13} + S_{22}^* S_{13} = 0 \quad \text{[EQ. 2.7]}$$
$$S_{12}^* S_{14} - S_{22}^* S_{14} = 0 \quad \text{[EQ. 2.8]}$$

$$S_{13}^* S_{11} + S_{13}^* S_{12} = 0 \quad \text{[EQ. 2.9]}$$
$$S_{13}^* S_{12} + S_{13}^* S_{22} = 0 \quad \text{[EQ. 2.10]}$$
$$2 |S_{13}^*|^2 = 1 \quad \text{[EQ. 2.11]}$$
$$S_{13}^* S_{14} - S_{13}^* S_{14} = 0 \quad \text{[EQ. 2.12]}$$

$$S_{14}^* S_{11} - S_{14}^* S_{12} = 0 \quad \text{[EQ. ~~2.13~~ 2.13]}$$
$$S_{14}^* S_{12} - S_{14}^* S_{22} = 0 \quad \text{[EQ. 2.14]}$$
$$S_{14}^* S_{13} - S_{14}^* S_{13} = 0 \quad \text{[EQ. 2.15]}$$
$$2 |S_{14}^*|^2 = 1 \quad \text{[EQ. 2.16]}$$

Equations 2.11 and 2.16 imply

$$|S_{13}^*| = \pm\sqrt{1/2} = |S_{13}|$$
$$|S_{14}^*| = \pm\sqrt{1/2} = |S_{14}|$$

Equations 2.4 and 2.8 require

$$S_{11}^* = S_{12}^* = S_{22}^* \Rightarrow |S_{11}|^2 = |S_{12}|^2 = |S_{22}|^2$$

From equation 2.3 we know $S_{11}^* = -S_{12}^*$
However, $S_{11}^* = S_{12}^* \Rightarrow S_{12}^* = S_{12}^* = S_{22}^* = 0$
so

$$S_{11} = S_{12} = S_{22} = 0$$

Therefore, the scattering matrix may be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sqrt{1/2} & \sqrt{1/2} \\ 0 & 0 & \sqrt{1/2} & -\sqrt{1/2} \\ \sqrt{1/2} & \sqrt{1/2} & 0 & 0 \\ \sqrt{1/2} & -\sqrt{1/2} & 0 & 0 \end{bmatrix} \quad \text{[EQ. 4]}$$

under the conditions that ports 3 & 4 are matched terminated and the structure is lossless.

Equation 4 tells us several things:

1) Ports 1 & 2 are isolated from each other
2) Ports 3 & 4 are isolated from each other
3) ~~Energy~~ Ports 1 & 2 are matched
4) Energy incident upon either port 1 or port 2 will divide

EE554 [S] PARAMETERS OF HYBRID TEE (CONT) WELLS MARCH 18, 1975

equally between ports 3 & 4.

5) Energy incident upon either port 3 or port 4 will divide equally between ports 1 & 2.

These properties are symbolized in figure 1.

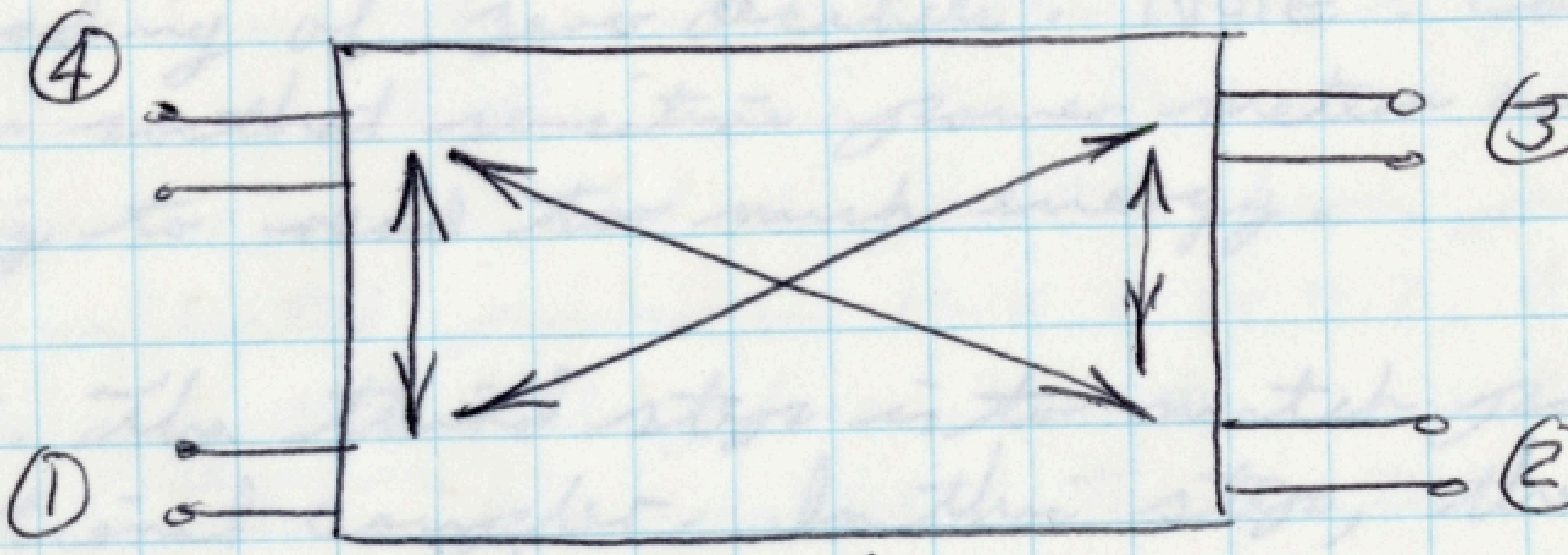

FIG. 1

PROCEDURE: In the following experiment, a hybrid tee directional coupler will have ports 3 & 4 matched by means of waveguide tuners. Energy will be incident upon one port while the other ports are terminated with either matched loads or else terminated by a power meter. The source of the incident wave is shown schematically in figure 2.

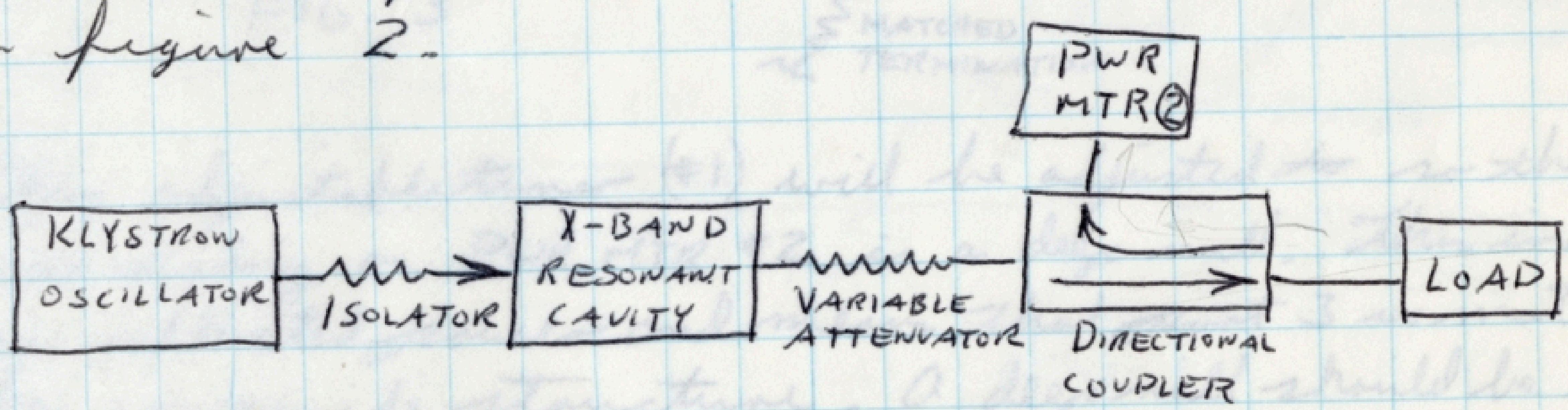

FIG. 2

STEP 1: The first step in the procedure will be to measure the ~~Klystron~~ Klystron frequency. To do this, POWER METER #1 will be placed as the load in figure 2. ~~Since, at~~ The resonant cavity will be adjusted so as to pass a minimum of energy to PWR MTR #1. (When the resonant cavity is adjusted such that its resonant frequency is the same as the Klystron frequency, the cavity will subtract a maximum amount of energy from the incident wave). After reading the Klystron frequency from the scale on the resonant cavity, the cavity will be readjusted so as to pass a larger amount of energy. This amount is not critical at this time.

EE354 [S] PARAMETERS OF HYBRID TEE (CONT) Wells MARCH 18, 1975

STEP 2. The second step is to zero the power meters. A short circuit will become the load in figure 2. With a short circuit load, all incident power will be reflected and read on PWR MTR #2 on the directional coupler. Both power meters will be set so that full incident power give a reading of zero decibels. NOTE: Caution should be taken so that sensitive power meters are not damaged by trying to read too much energy.

STEP 3. The third step is to match port 3 to the directional coupler. In this step, the load in figure 2 is shown in figure 3

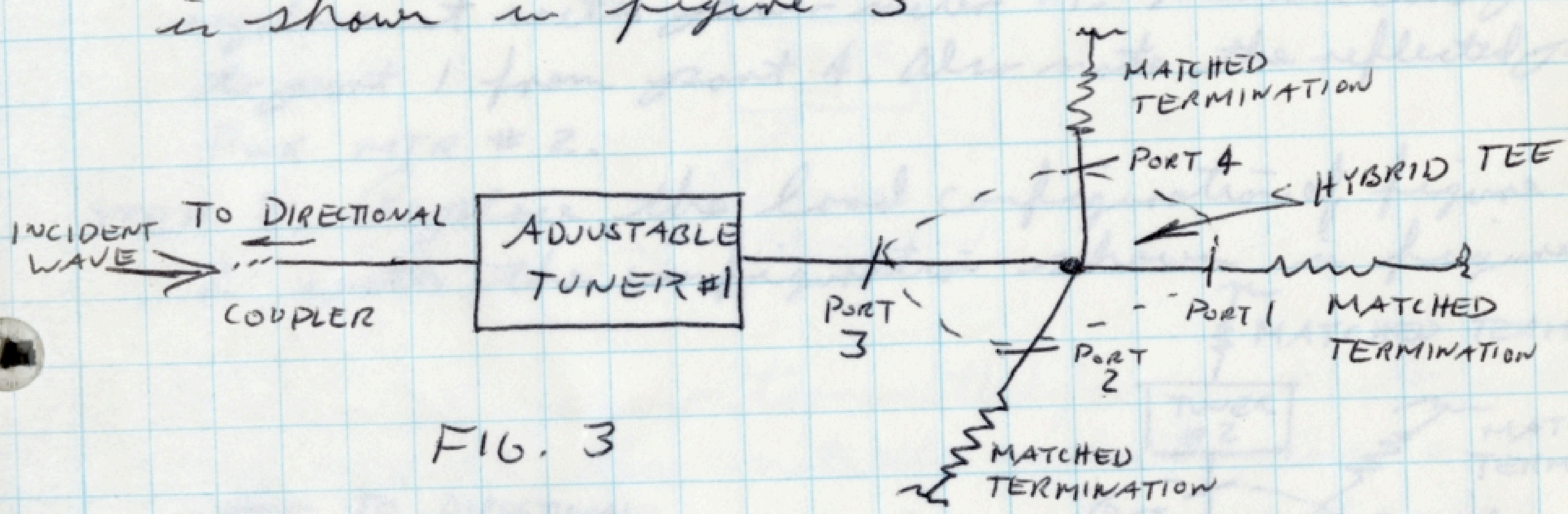

FIG. 3

The adjustable tuner (#1) will be adjusted to so that the power reading on PWR MTR #2 is a deep null. This indicates zero reflected power and means that port 3 is matched to the waveguide structure. A deep null should be in the area of 55 to 60 dB down.

STEP 4. This step repeats step 3 except the load in figure 2 is as shown in figure 4.

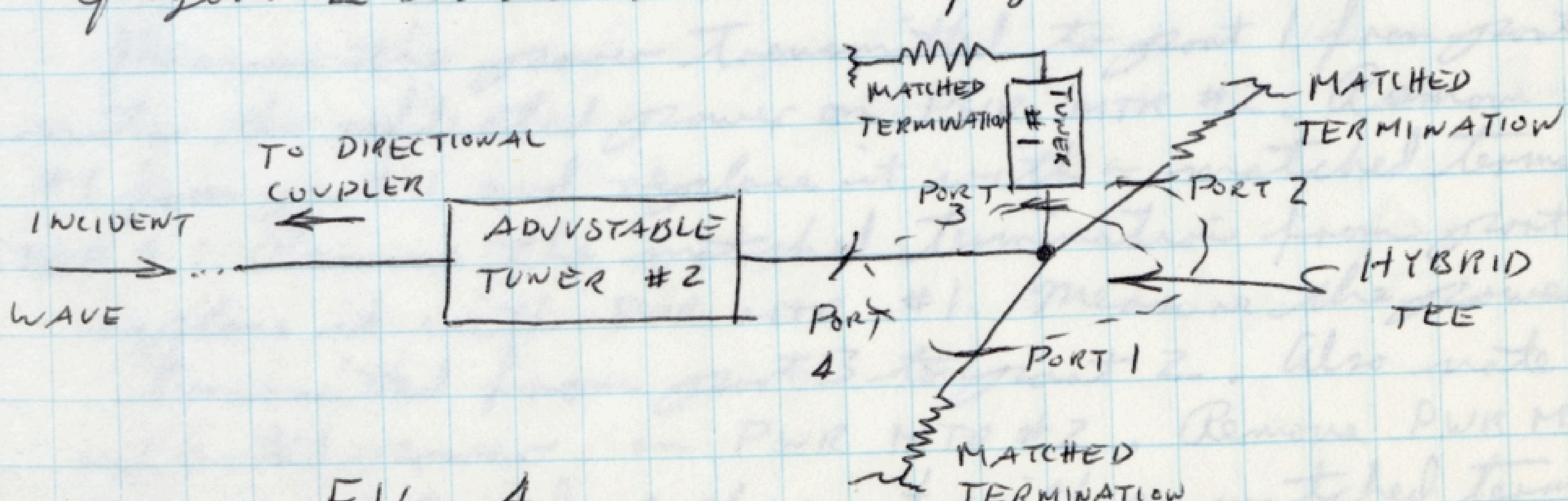

FIG. 4

NOTE: DO NOT READJUST EITHER TUNER ONCE THEY HAVE BEEN ADJUSTED TO MATCH THE PORT THEY FEED

EE 554 [S] PARAMETERS OF HYBRID TEE (CONT) Wells

MAR. 18, 1975

~~#~~

STEP 5. USING ~~THE~~ AS THE LOAD the ~~circuit~~ configuration of figure 4, replace the matched termination on port 3 and measure the transmitted power using PWR MTR #1. Also, note the reflected power on PWR MTR #2.

STEP 6. Replace the matched termination back on port 3 and then remove the matched termination from port 2. In its place put ~~the indicator~~ PWR MTR #1 and measure the power transmitted to port 2 from port 4. Also, note the reflected power on PWR MTR #2. Remove PWR MTR #1 from port 2 and replace it with a matched termination.

STEP 7. ~~Replace~~ Remove the matched termination from port 1 and replace it with power meter #1. Measure the power transmitted to port 1 from port 4. Also note the reflected power on PWR MTR #2.

STEP 8. Replace the load configuration of figure 4 in figure 2 with the configuration shown in figure 5 below.

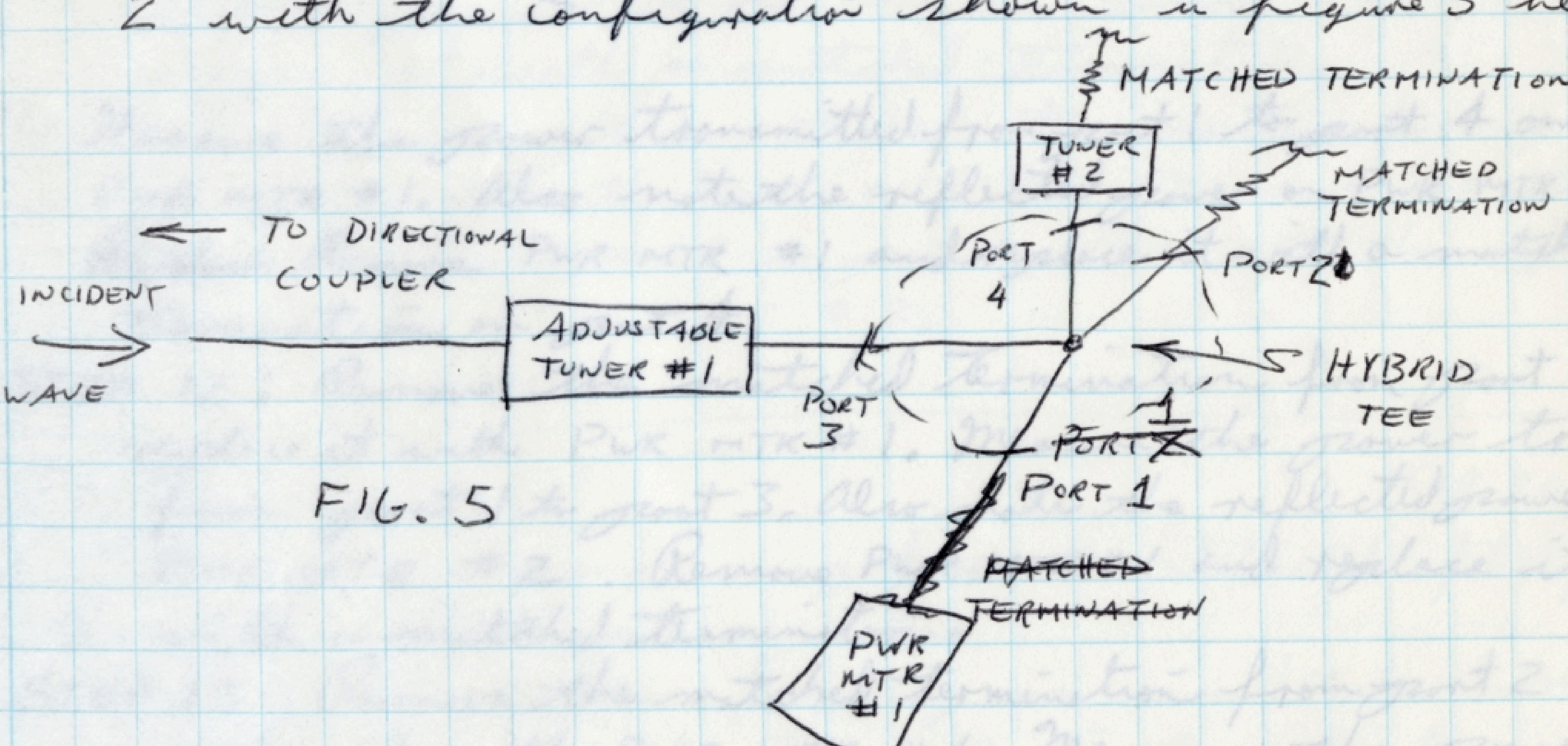

FIG. 5

Measure the power transmitted to port 1 from port 3. Also note the reflected power on PWR MTR #2. Remove PWR MTR #1 from port 1 and replace it with a matched termination.

STEP 9: Remove the matched termination from port 2 and replace it with PWR MTR #1. Measure the power transmitted from port 3 to port 2. Also note the reflected power on PWR MTR #2, Remove PWR MTR #1 from port 2 and replace it with a matched termination.

STEP 10: Remove the matched termination from port 4 and replace it with PWR MTR #1. Measure the power transmitted

EE 554 [S] PARAMETERS OF HYBRID TEE (CONT) Wells MAR. 18, 1975

from port 3 to port 4. Also note the reflected power on PWR MTR #2.

STEP 11: In figure 2, replace the load configuration of figure 5 with the configuration of figure 6.

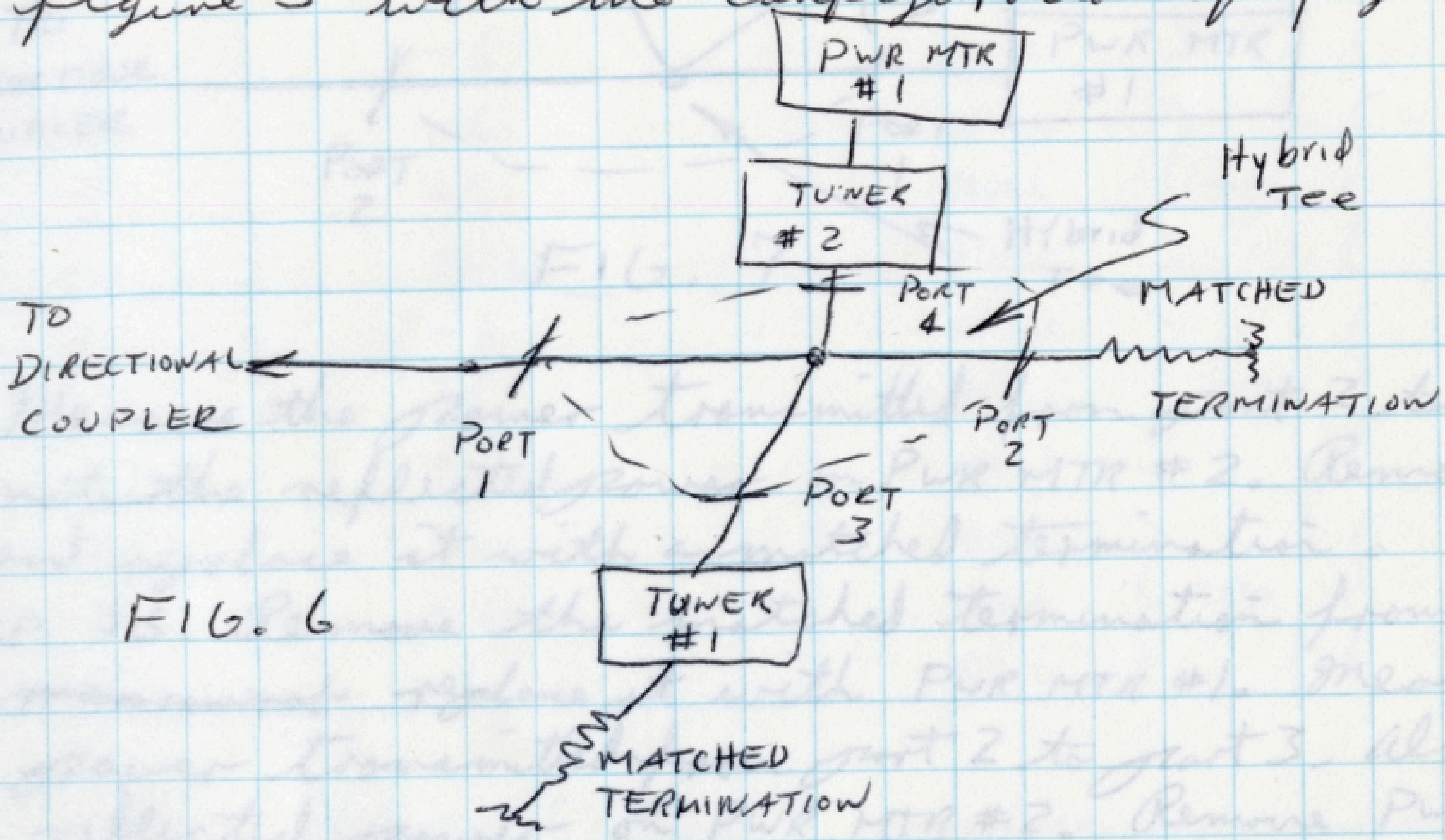

FIG. 6

Measure the power transmitted from port 1 to port 4 on PWR MTR #1. Also note the reflected power on PWR MTR #2. ~~Replace~~ Remove PWR MTR #1 and replace it with a matched termination on port 4.

STEP 12: Remove the matched termination from port 3 and replace it with PWR MTR #1. Measure the power transmitted from port 1 to port 3. Also note the reflected power on PWR MTR #2. Remove PWR MTR #1 and replace it with a matched termination.

STEP 13: Remove the matched termination from port 2 and replace it with PWR MTR #1. Measure the power transmitted ~~p~~ from port 1 to port 2. Also note the reflected power on ~~port~~ PWR MTR #2. Remove PWR MTR #1.

STEP 14: In figure 2, replace the load configuration of figure 6 with the load configuration shown in figure 7 on page 7.

EE 554 [S] PARAMETERS OF HYBRID TEE (CONT) Wells

MAR. 18, 1975

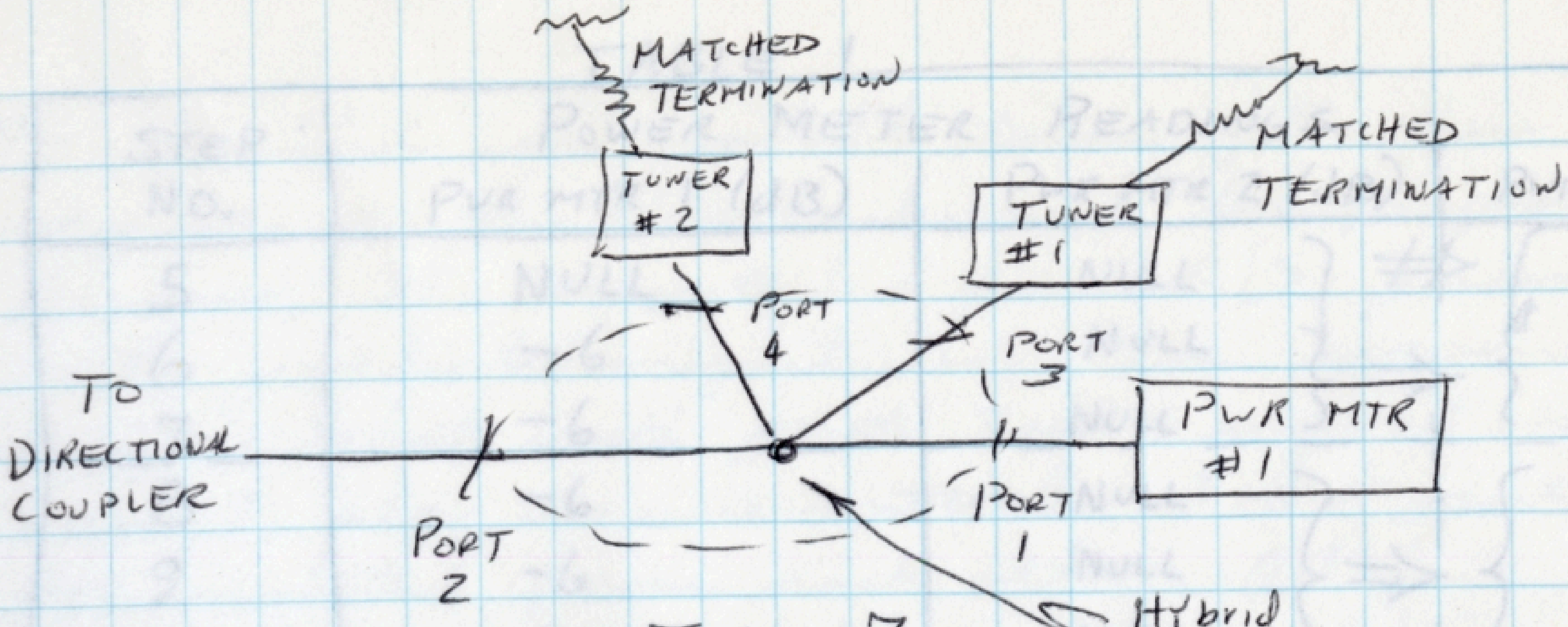

FIG. 7

Measure the power transmitted from port 2 to port 1. Also note the reflected power on PWR MTR #2. Remove PWR MTR #1 and replace it with a matched termination.

STEP 15. Remove the matched termination from port 3 and ~~measured~~ replace it with PWR MTR #1. Measure the power transmitted from port 2 to port 3. Also note the reflected power on PWR MTR #2. Remove PWR MTR #1 and replace it with a matched termination.

STEP 16. Remove the matched termination from port 4 and replace it with PWR MTR #1. Measure the power transmitted from port 2 to port 4. Also note the reflected power on PWR MTR #2.

END OF PROCEDURE.

PERFORMANCE: MAR. 20, 1975

The following is a summary of results obtained after following the above outlined procedures.

STEP. 1. The Klystron frequency was measured to be 9.84 GHz

STEP 2 was carried out. 0 dB ~~indicates~~ indicates full incident power.

STEP 3 was carried out. The dB reading for a null is noise sensitive and fluctuates in the vicinity from −50 dB to −60 dB. A reading in this area will be recorded and designated by use of the term "NULL".

STEP 4 was carried out.

Steps 5 through 16 were carried out and the measurements are summarized in table 1.

EE 554 [S] PARAMETERS OF HYBRID TEE (CONT) Wells MAR. 20, 1975

TABLE 1

STEP NO.	POWER METER READINGS: PWR MTR 1 (dB)	POWER METER READINGS: PWR MTR 2 (dB)	$P_{READ}/P_{INCIDENT}$: PWR MTR 1	$P_{READ}/P_{INCIDENT}$: PWR MTR 2
5	NULL	NULL	0	0
6	−6	NULL	1/2	0
7	−6	NULL	1/2	0
8	−6	NULL	1/2	0
9	−6	NULL	1/2	0
10	NULL	NULL	0	0
11	−6	NULL	1/2	0
12	−6	NULL	1/2	0
13	NULL	NULL	0	0
14	NULL	NULL	0	0
15	−6	NULL	1/2	0
16	−6	NULL	1/2	0

DISCUSSION: Steps 5 thru 7 comprise a measurement of power distributions from port 4 to the other 3 ports. The fraction of incident power delivered to the indicating meters are indicated in the $P_{READ}/P_{INCIDENT}$ columns. The readings of steps 5-7 indicate that port 4 is matched. Also, since $|S_{ij}|^2 = P_{ij}$ the measurements show

$$|S_{44}| = 0 \;,\; |S_{43}| = 0 \;,\; |S_{42}| = |S_{41}| = \sqrt{\tfrac{1}{2}}$$

Steps 8-10 are measurements of power distribution from port 3 to the other ports. Results of these measurements indicate that

$$|S_{33}| = 0, \; |S_{34}| = 0, \; |S_{31}| = |S_{32}| = \sqrt{1/2}$$

Steps 11-13 are measurements of power distribution from port 1 to the other ports. The results indicate

$$|S_{11}| = 0, \; |S_{12}| = 0, \; |S_{13}| = |S_{14}| = \sqrt{1/2}$$

Steps 14-16 are measurements of power distribution from port 2 to the other ports. Results of these measurements indicate

$$|S_{22}| = 0, \; |S_{21}| = 0, \; |S_{23}| = |S_{24}| = \sqrt{1/2}$$

EE554 [S] PARAMETERS OF HYBRID TEE (CONT) Wells MAR, 20, 1975

No information was available to me after these readings in regards to the sign values of the [S] parameters. However, the magnitudes for each [S] parameter agrees with predicted magnitudes. The experiment also verifies that with ports 3 and 4 matched that ports 1 and 2 also become matched and that ports 1 and 2 are isolated from each other. All five theoretical predictions on pages two and three of this notebook are experimentally verified.